2nd Tutorial:

Periodic Systems

Hands-on workshop 2013

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Outline of this tutorial

1. Periodic systems with DFT
   - The crystal structure
   - Sampling of the Brillouin zone
   - Band structure and density of states
   - Minimizing energy with respect to unit cell parameters
2. Systems of interest
   - Bulk Silicon
   - Silicon surface
   - Mn-doped GaAs
3. Overview

Periodic systems with DFT

- The crystal structure
- Sampling of the Brillouin zone
- Band structure and density of states
- Find minimal lattice constant

Periodic systems with DFT

- The crystal structure
- Sampling of the Brillouin zone
- Band structure and density of states
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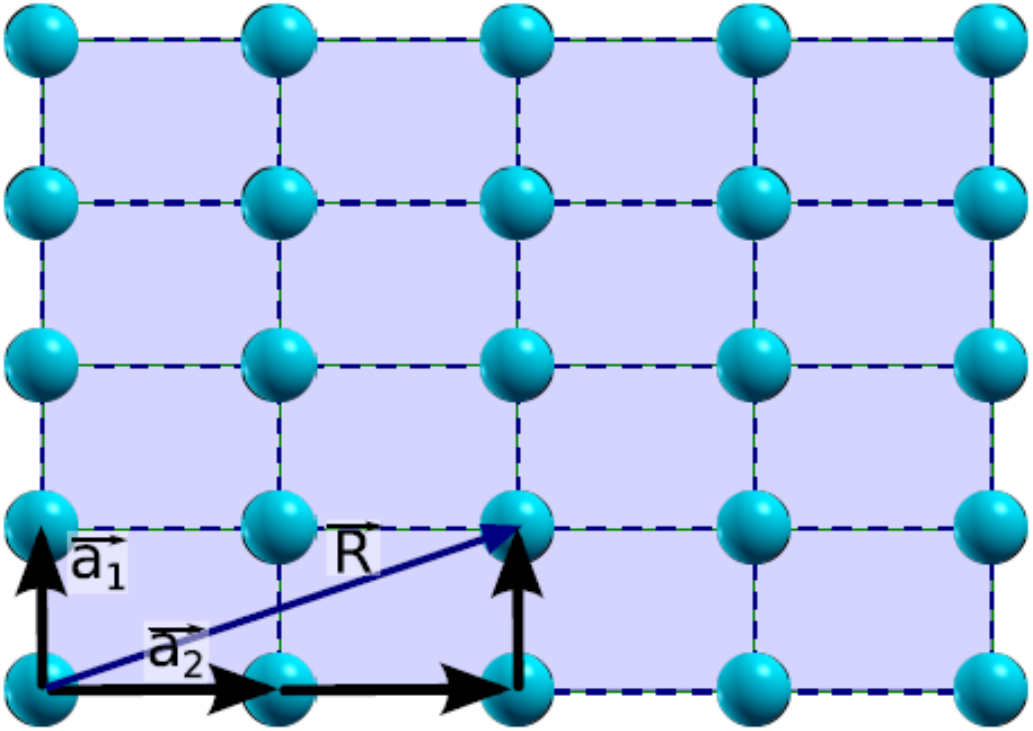
A 2-dimensional example



$$V(\vec{r} + \vec{R}) = V(\vec{r})$$

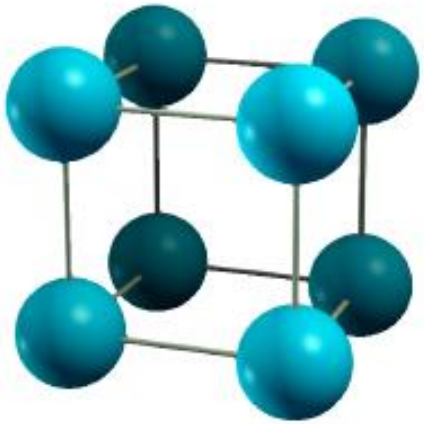
$$\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2$$

with

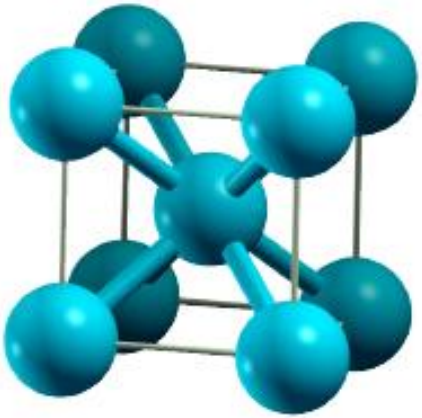
$\vec{a}_{1,2}$: primitive vectors

$n_{1,2}$: integers

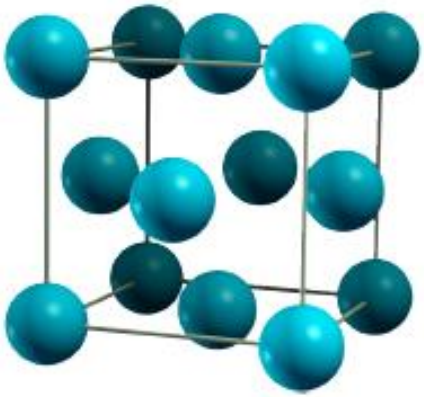
The crystal lattice: First examples



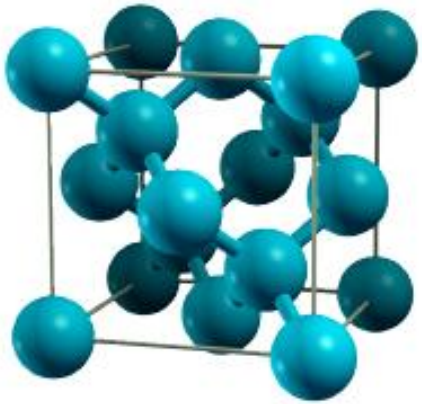
simple cubic



BCC

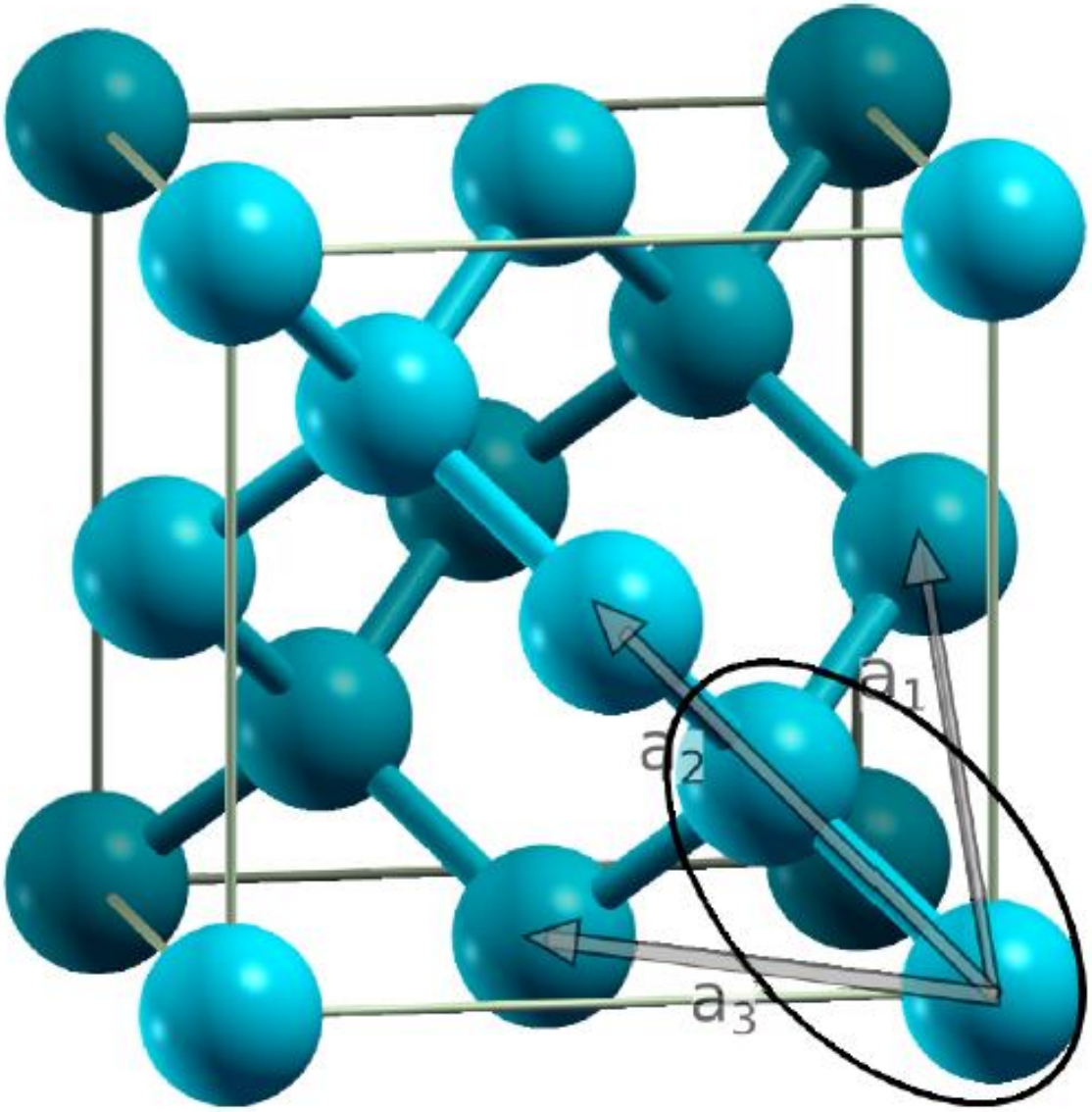


FCC



diamond

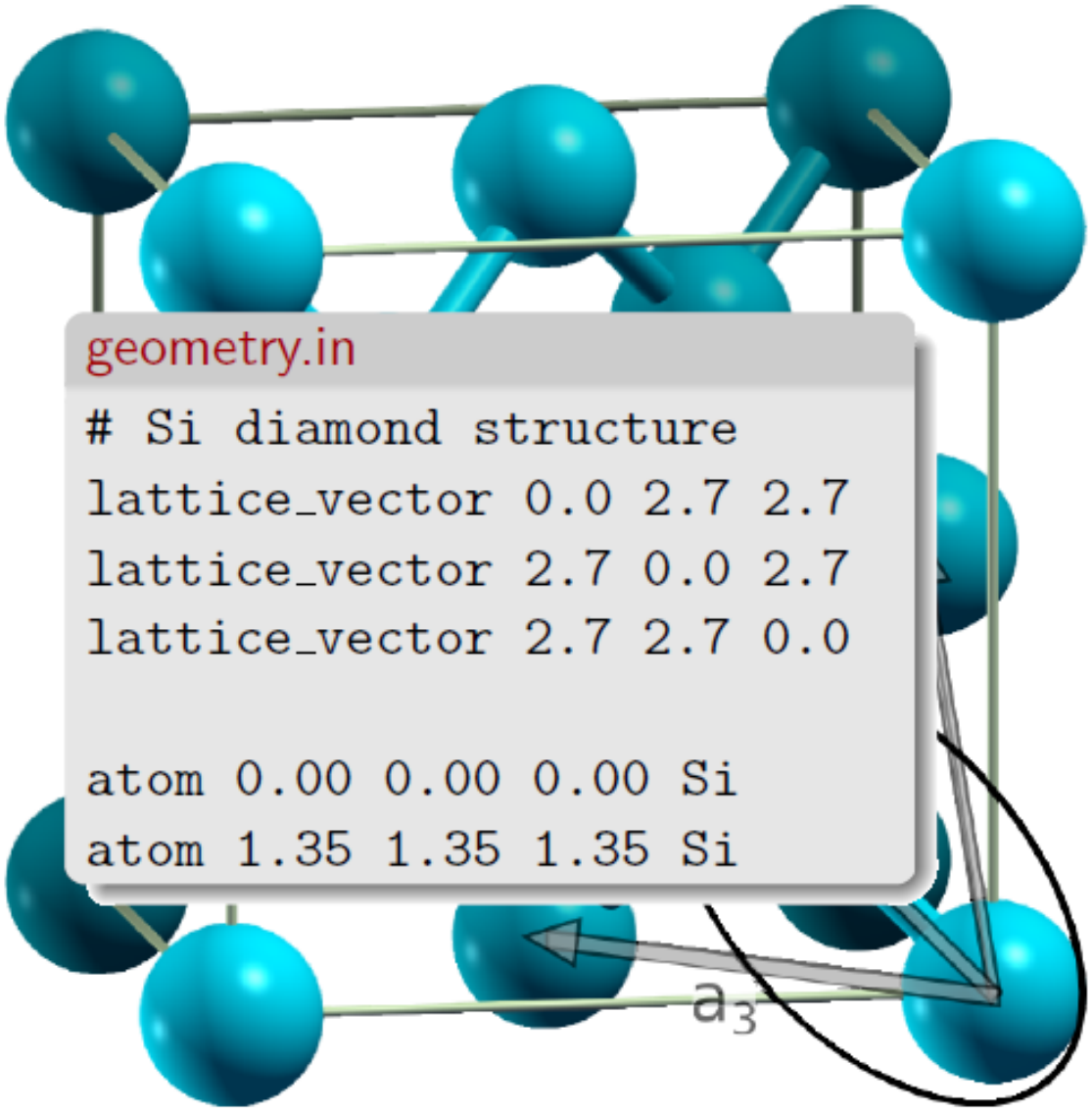
How to specify the crystal structure for a calculation?



The geometry.in file

- specify the primitive unit vectors
- give coordinates of the atoms in the basis

How to specify the crystal structure for a calculation?



geometry.in

```
# Si diamond structure
lattice_vector 0.0 2.7 2.7
lattice_vector 2.7 0.0 2.7
lattice_vector 2.7 2.7 0.0

atom 0.00 0.00 0.00 Si
atom 1.35 1.35 1.35 Si
```

The geometry.in file

- specify the primitive unit vectors
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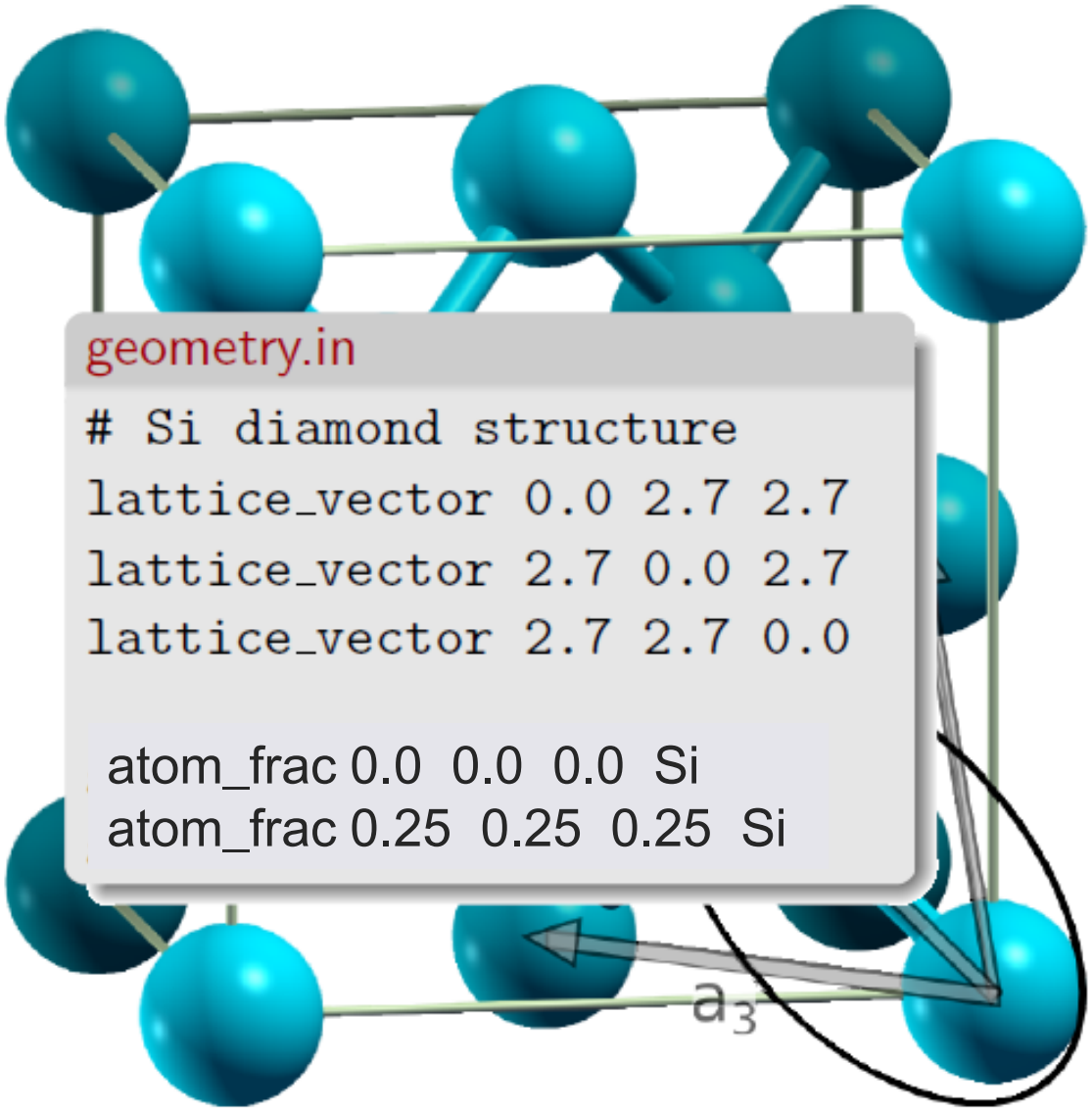
How to specify the crystal structure for a calculation?



geometry.in

```
# Si diamond structure
lattice_vector 0.0 2.7 2.7
lattice_vector 2.7 0.0 2.7
lattice_vector 2.7 2.7 0.0

atom_frac 0.0  0.0  0.0  Si
atom_frac 0.25  0.25  0.25  Si
```

The geometry.in file

- specify the primitive unit vectors
- give coordinates of the atoms in the basis

Bloch's Theorem

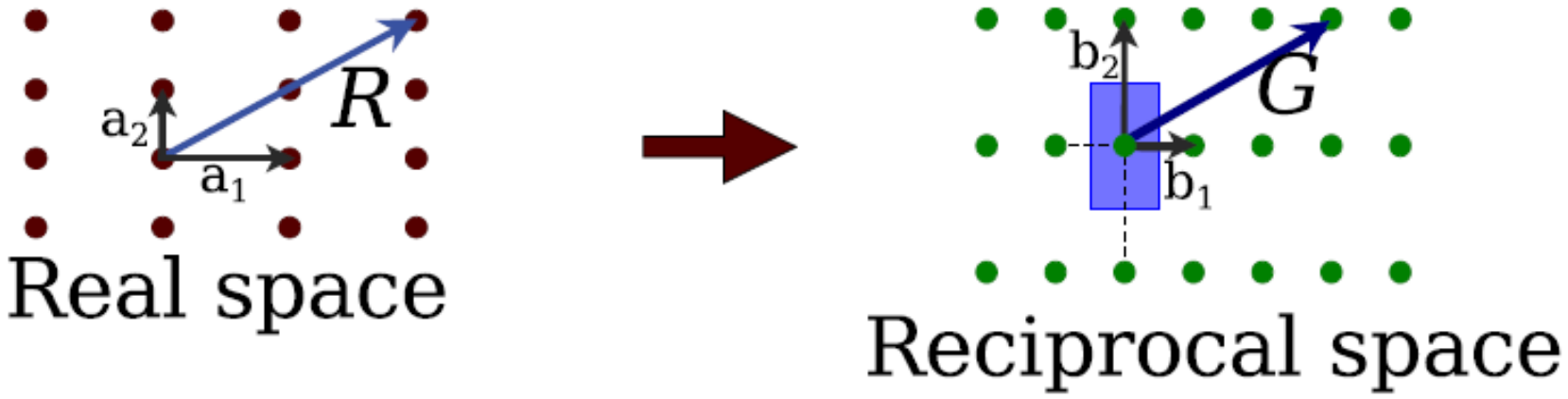
- Bloch's theorem $\Rightarrow$ give conserved quantum number k for the single particle state $\psi_{n,\vec{k}}(\vec{r}) = u_{n,\vec{k}}(\vec{r})\, e^{i\vec{k}\vec{r}}$
- Kohn-Sham orbital $\psi_{n,k}(\vec{r})$ depends on its quantum number n and on the point k in the first Brillouin zone (1BZ)
- The quantum number n is discrete, but k is continous.

The electronic density $\rho(\vec{r})$

$$\rho(\vec{r}) = \frac{1}{V_{BZ}} \sum_{n=1}^{N_{el}} \int_{Brillouin\ zone} |\psi_{n,k}(\vec{r})|^2 \, d^3k$$

The Brillouin zone

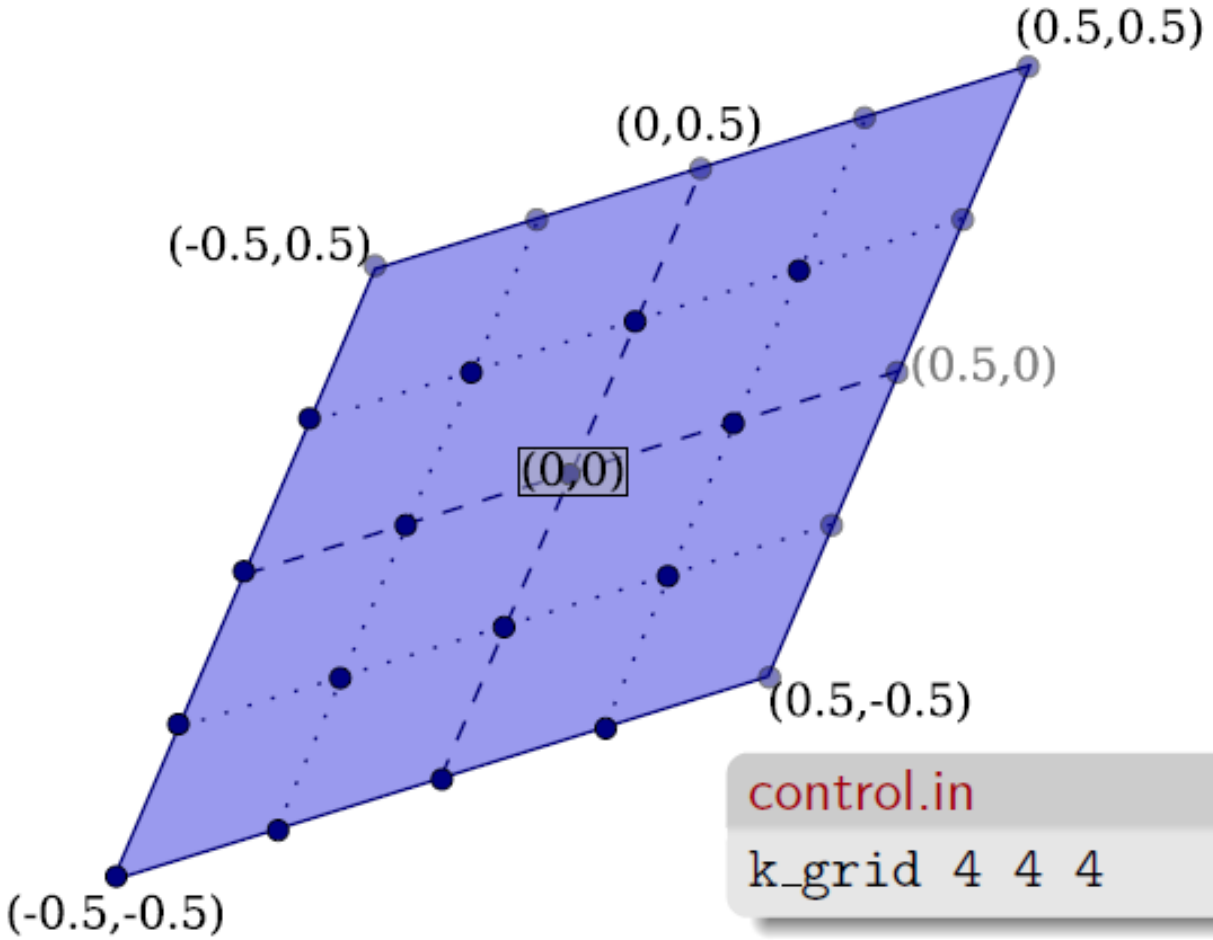
$$b_l = 2\pi \frac{a_m \times a_n}{\Omega}$$



- In practice, calculations are performed on a grid of points in the 1BZ

$$\psi_{n,\vec{k}+\vec{G}}(\vec{r}) = \psi_{n',\vec{k}}$$

The grid in the Brillouin zone



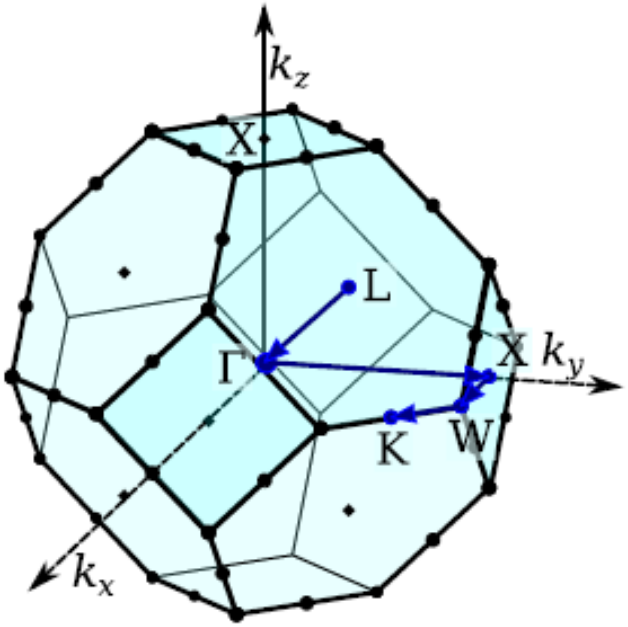
control.in

```
k_grid 4 4 4
```

Bandstructure: example silicon

Kohn-Sham equation

$$\hat{h}_k \psi_{n,k}(\vec{r}) = \epsilon_{n,k} \psi_{n,k}(\vec{r})$$



- find SCF solution
- choose a path in the Brillouin Zone
  typical: along high symmetry lines
- plot the Kohn-Sham eigenvalues ($\epsilon(k)$)

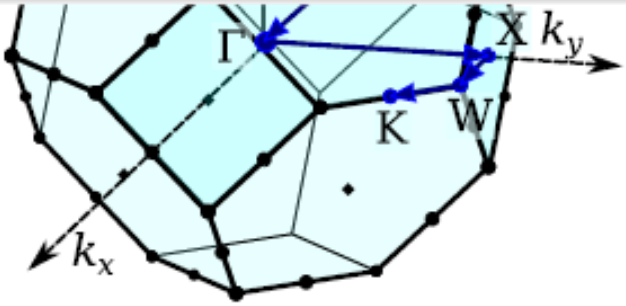
Bandstructure: example silicon

Kohn-Sham equation

$$\hat{h}_k \psi_{n,k}(\vec{r}) = \epsilon_{n,k} \psi_{n,k}(\vec{r})$$

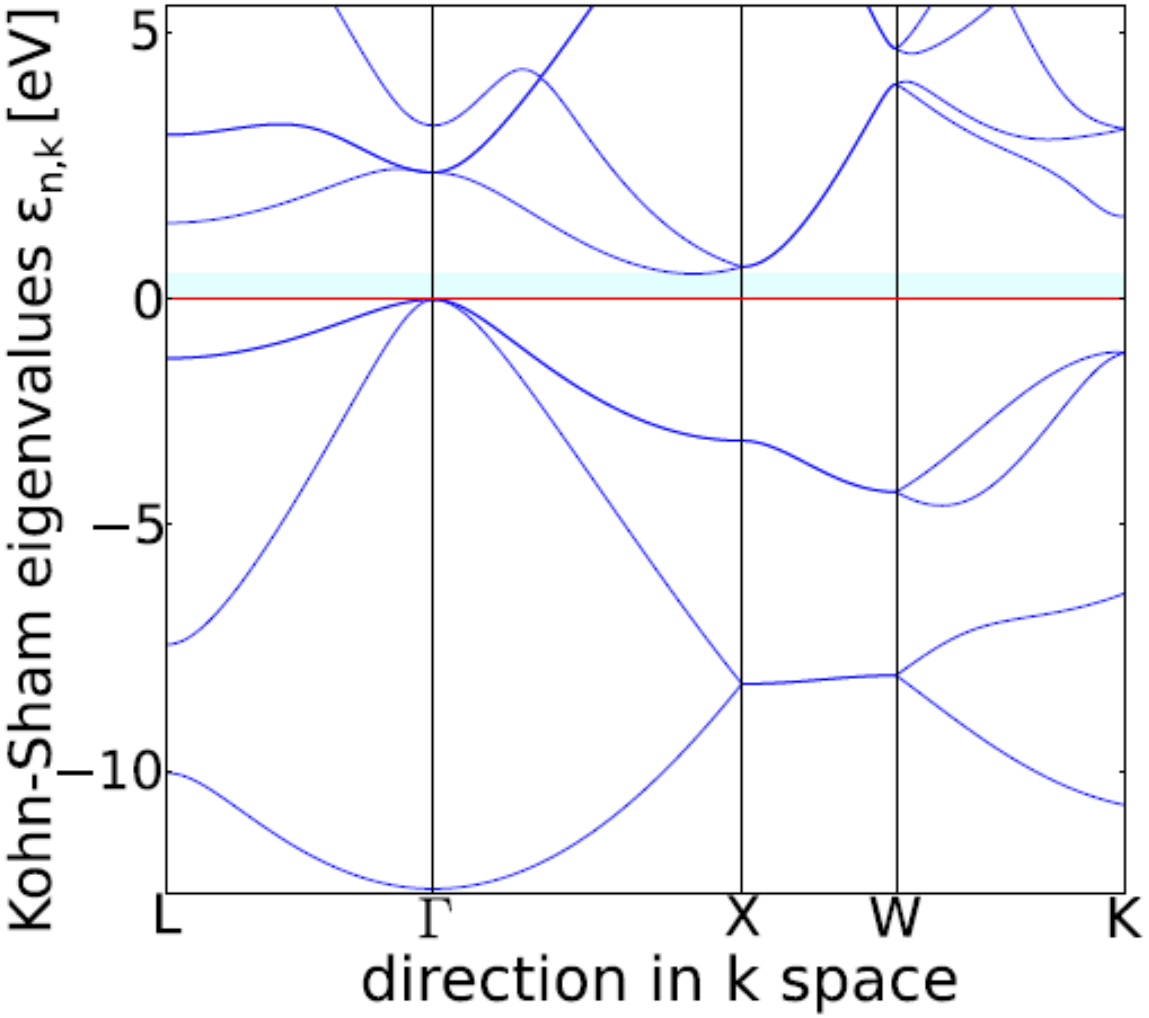
control.in

```
output band  0.5 0.5 0.5  0.0 0.0 0.0  50  L Gamma
output band  0.0 0.0 0.0  0.0 0.5 0.5  50  Gamma X
...
```



- typical: along high symmetry lines
- plot the Kohn-Sham eigenvalues ($\epsilon(k)$)

The electronic Band structure of Silicon



- semiconductor
- indirect band gap

The density of states (DOS)

- number of states n within a given energy interval $[\epsilon - \Delta\epsilon, \epsilon + \Delta\epsilon]$

$$n = \int_{\epsilon-\Delta\epsilon}^{\epsilon+\Delta\epsilon} g(\epsilon)\ d\epsilon$$

  $g(\epsilon)$ is the density of states
- $g(\epsilon)$ in a free atom or molecule is

$$g(\epsilon) = \sum_i \delta(\epsilon_i - \epsilon)$$

- in a periodic system the number of states per energy is averaged over k

$$g(\epsilon) = \frac{1}{V_{BZ}} \sum_i \int_{BZ} d^3k\ \delta(\epsilon_{i,k} - \epsilon)$$

Density of states: Broadening and k-points

Density of states (DOS)

$$g(\epsilon) = \frac{1}{V_{BZ}} \sum_i \int_{BZ} d^3k \; \delta(\epsilon_{i,k} - \epsilon)$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \frac{1}{N_k} \sum_i \sum_k \exp\left[-\frac{1}{2}\left(\frac{\epsilon - \epsilon_{k,i}}{\sigma}\right)^2\right]$$

where σ is the Gaussian broadening

The density of states (DOS)

- What are the units?

The density of states (DOS)

- What are the units?

number of states

The density of states (DOS)

- What are the units?

number of states per energy unit

The density of states (DOS)

- What are the units?

number of states per energy unit per volume unit

The density of states (DOS)

- What are the units?

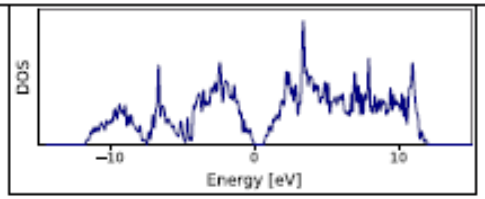
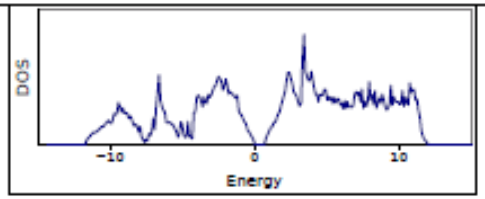
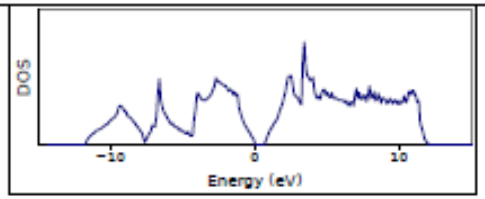
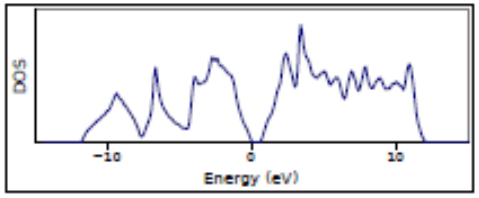
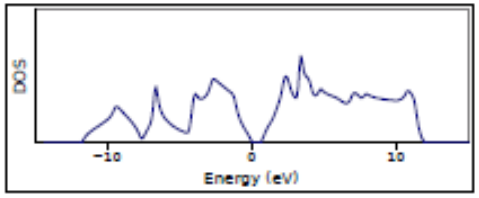
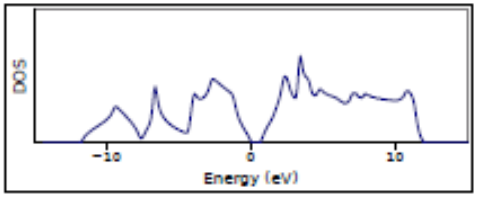
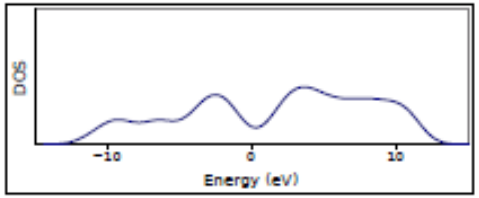
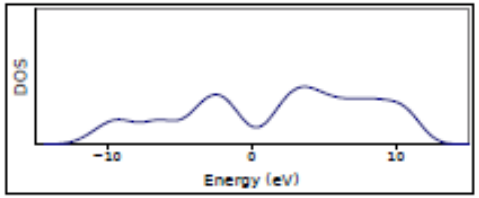
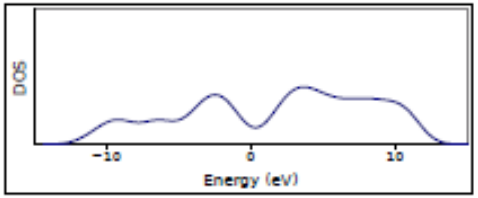
In FHI-aims and many other codes:

number of states per energy unit per *unit cell*

Density of states: Broadening and k-points

$$g(\epsilon) = \frac{1}{\sqrt{2\pi}\sigma}\frac{1}{N_k}\sum_i\sum_k \exp\left[-\frac{1}{2}\left(\frac{\epsilon-\epsilon_{k,i}}{\sigma}\right)^2\right]$$

where σ is the Gaussian broadening

σ/grid	20x20x20	40x40x40	60x60x60
0.01			
0.1			
1.0			

Minimizing energy with respect to unit cell parameters

Minimizing energy with respect to unit cell parameters

Strain tensor (3x3, symmetric):

$$\vec{R}'_\alpha = (\mathbf{1}+\boldsymbol{\varepsilon})\vec{R}_\alpha, \quad \vec{a}'_i = (\mathbf{1}+\boldsymbol{\varepsilon})\vec{a}_i$$

$\vec{R}_\alpha$: atomic positions

$\vec{a}_i$: lattice vectors

Direct minimization of the energy:

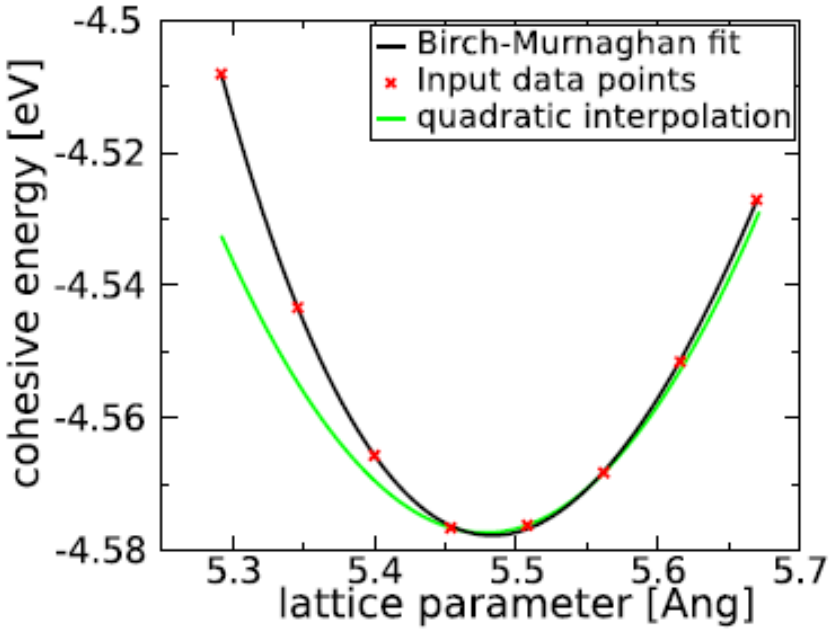
stress tensor

$$\sigma_{ij} = \frac{1}{V}\frac{\partial E(\vec{R}'_\alpha, \vec{a}'_i)}{\partial \varepsilon_{ij}} = 0$$

In FHI-aims, analytic stress tensor is implemented for LDA, GGA, and hybrid functionals

Interpolation of single-point calculations

Birch-Murnaghan Fit versus quadratic interpolation



V : Volume

E_0 : Equilibrium energy

V_0 : optimum Volume

B_0 : Bulk modulus

B_0' : derivative of B_0 (w.r.t. pressure)

Birch-Murnaghan equation of state

$$E(V) = E_0 + \frac{B_0 V}{B_0'}\left(\frac{(V_0/V)^{B_0'}}{B_0' - 1} + 1\right) - \frac{B_0 V_0}{B_0' - 1}$$

Bulk Silicon

Motivation

VOLUME 45, NUMBER 12 PHYSICAL REVIEW LETTERS 22 SEPTEMBER 1980

Microscopic Theory of the Phase Transformation and Lattice Dynamics of Si

M. T. Yin and Marvin L. Cohen
Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 14 July 1980)

An *ab initio* calculation for the solid-solid phase transformation, static structural properties, and the lattice dynamics of Si is presented. A density-functional pseudopotential scheme is used with the atomic number as the only input. The detailed properties of the diamond to β-tin transition are accurately reproduced. The phonon frequencies and mode-Grüneisen parameters at Γ and X, along with the lattice constant, bulk modulus, and cohesive energy, are calculated and found to be in excellent agreement with experiment.

PACS numbers: 63.20.Dj, 61.50.Lt

Motivation

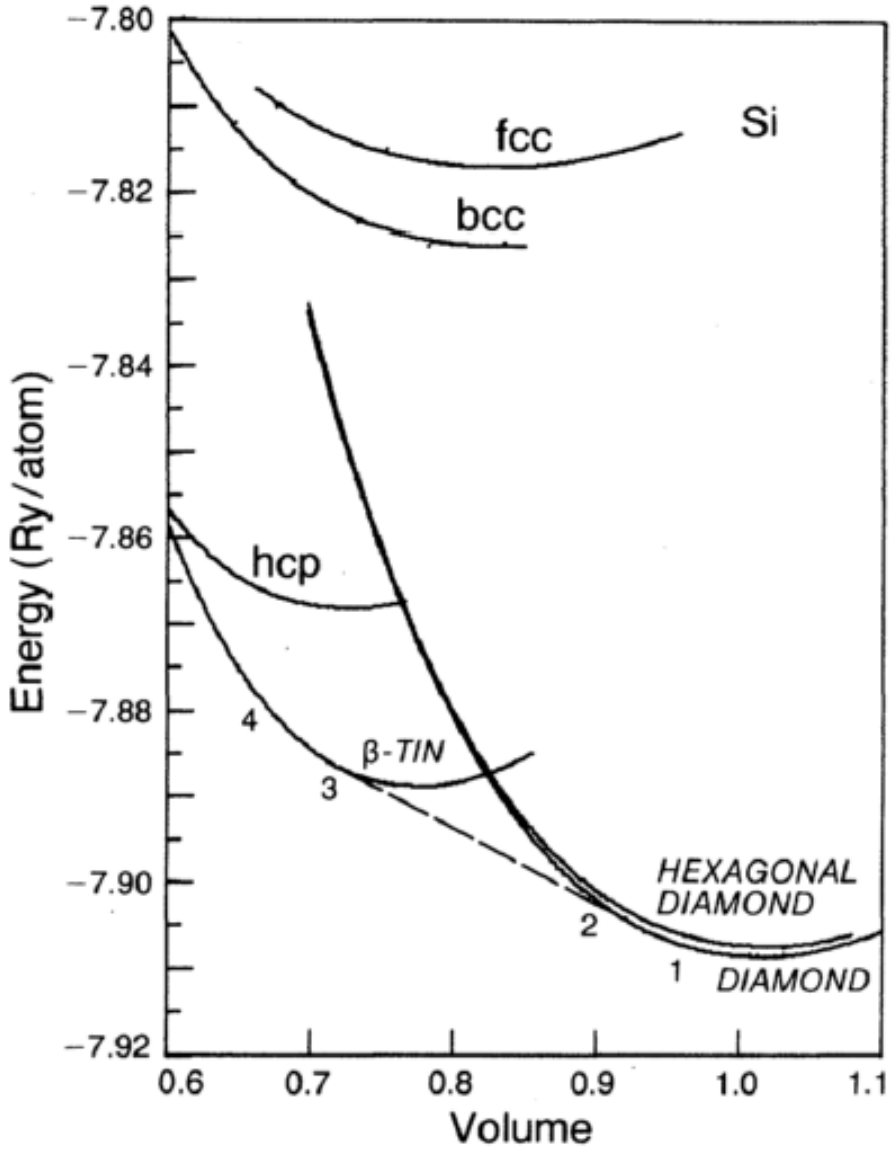
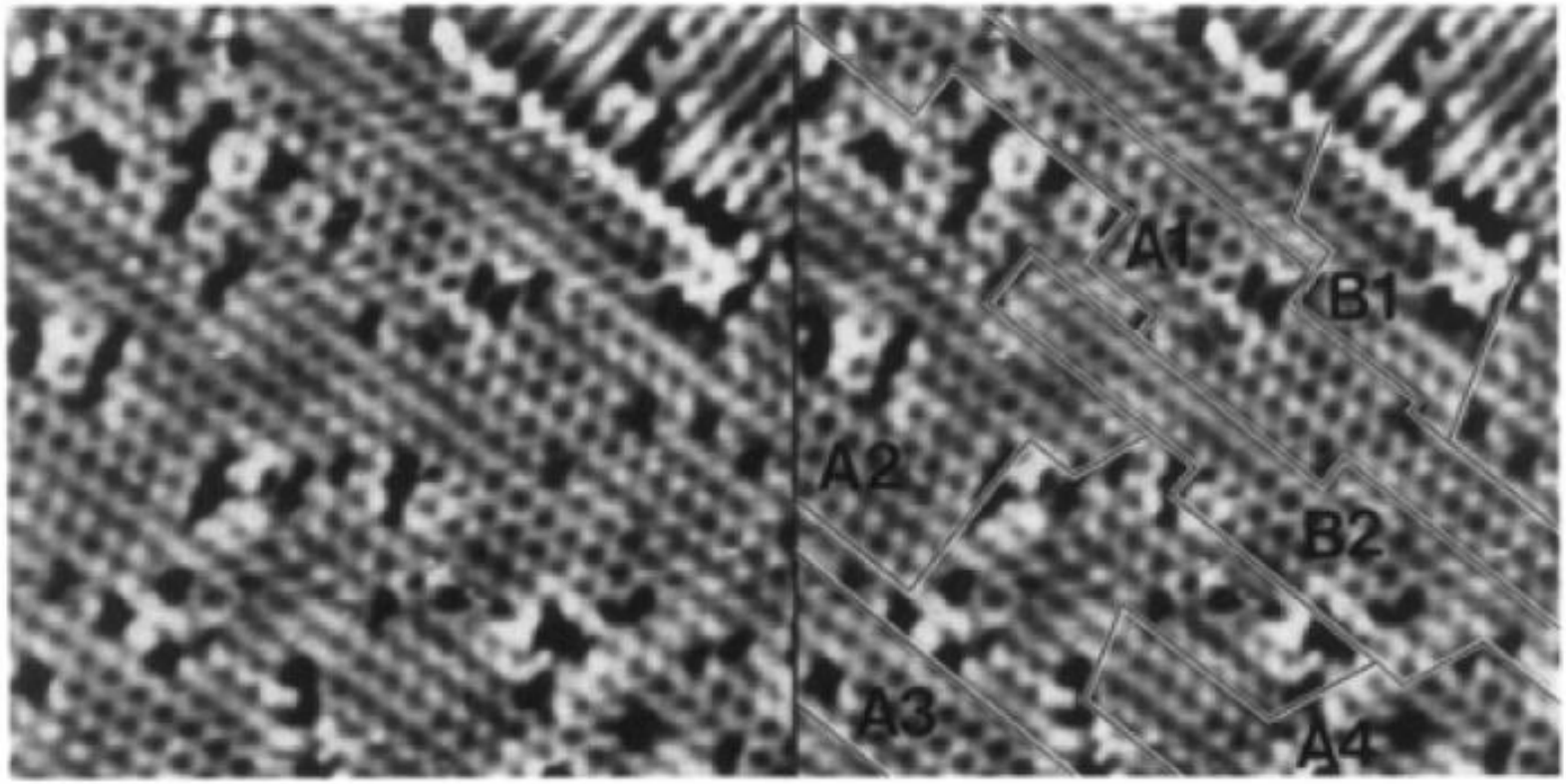


FIG. 1. The diamond, hexagonal diamond, and β-tin, hcp, bcc, and fcc structural energies (in units of Ry/atom) as a function of the atomic volume [normalized to the measured free volume (Ref. 16)] for Si. The dashed line is the common tangent of the energy curves for the diamond and the β-tin structures.

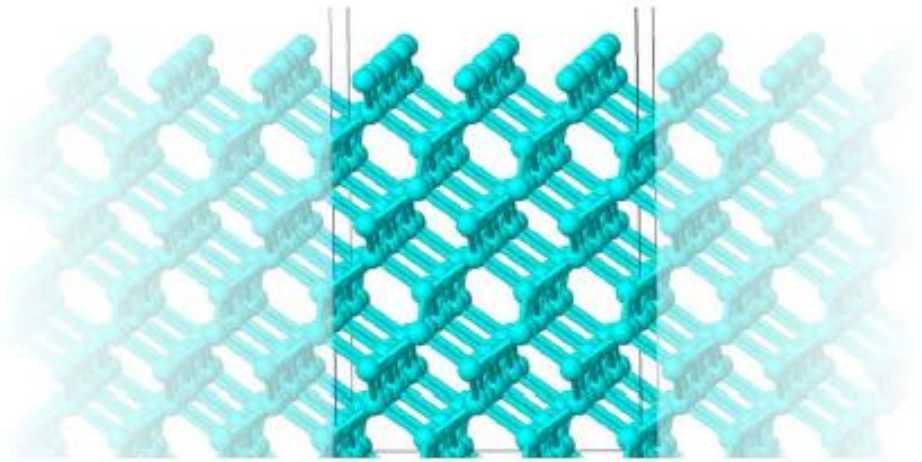
Silicon surface

Motivation



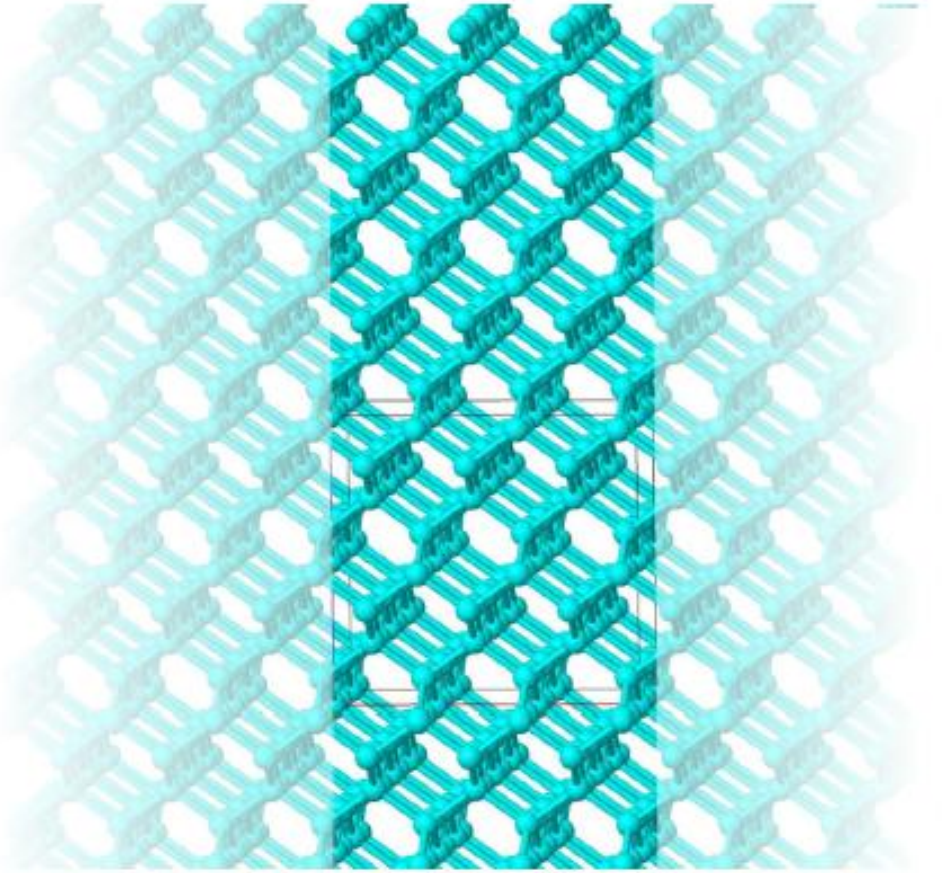
R.A. Wolkow, PRL 68,2636 (1992)

Supercell approach



- Surface: periodic in two directions
- Start from bulk geometry
- insert vacuum
  here: increase lattice vector in z-direction
- saturate the dangling bonds on the bottom layer with hydrogen
- choose vacuum large enough (no interaction between slabs)

Supercell approach



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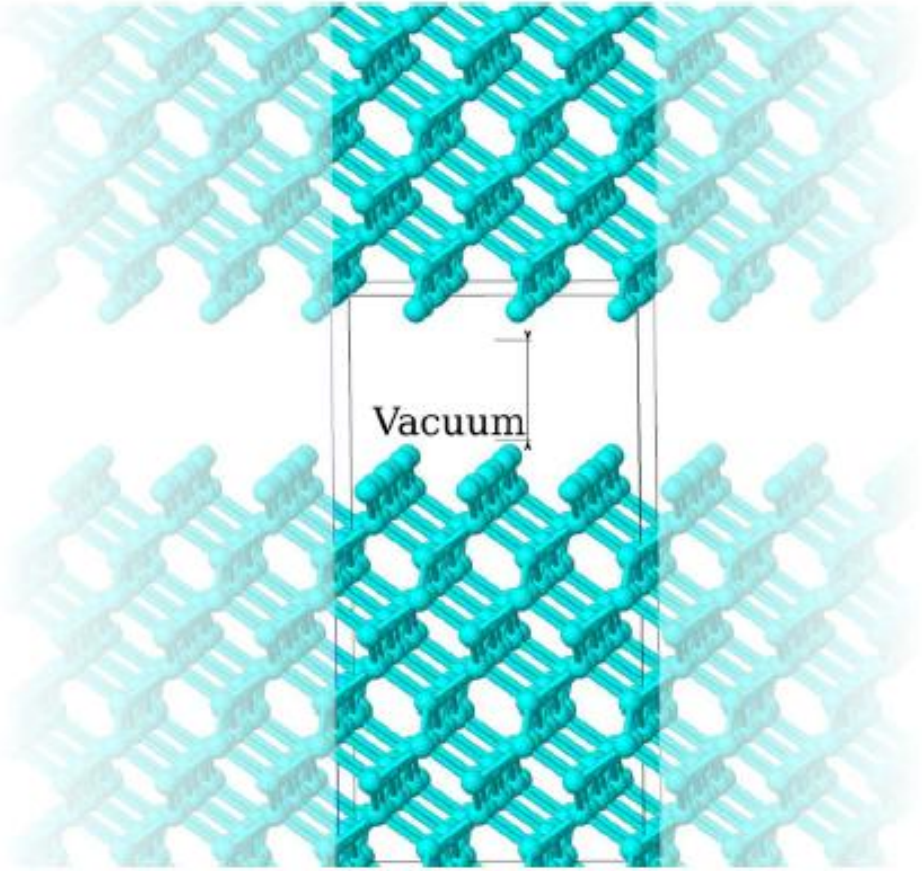

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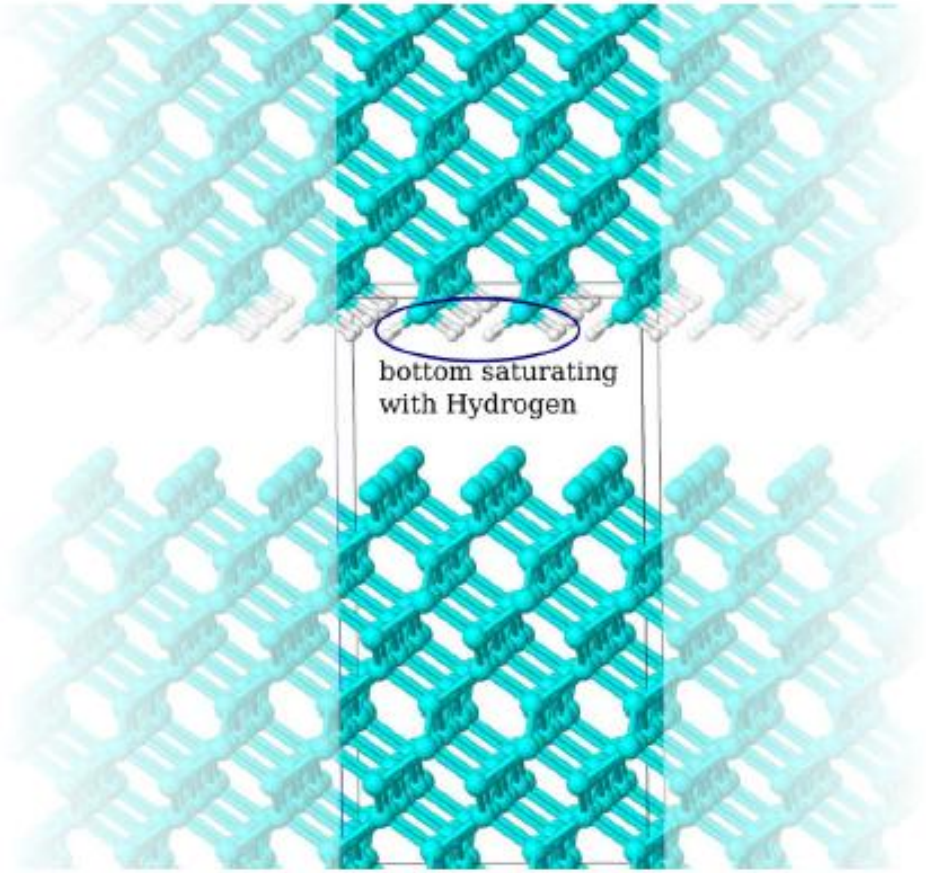

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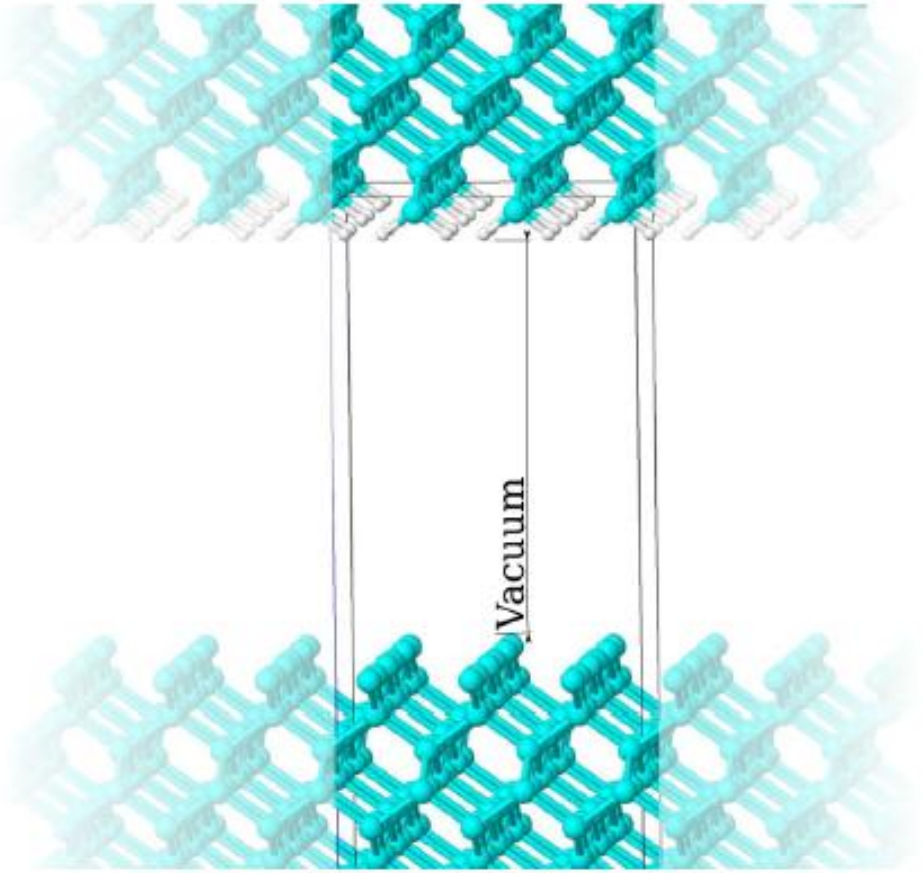

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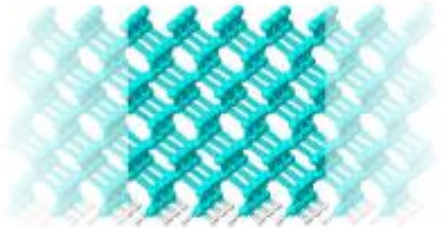

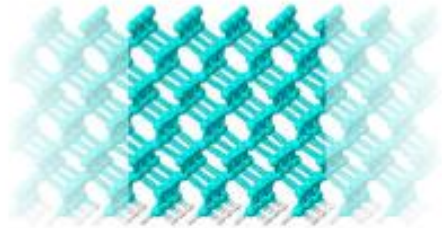

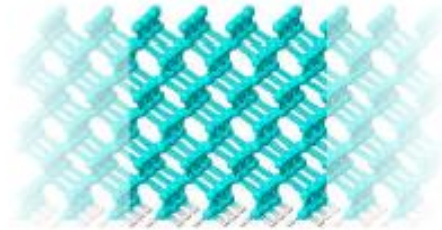

- Surface: periodic in two directions
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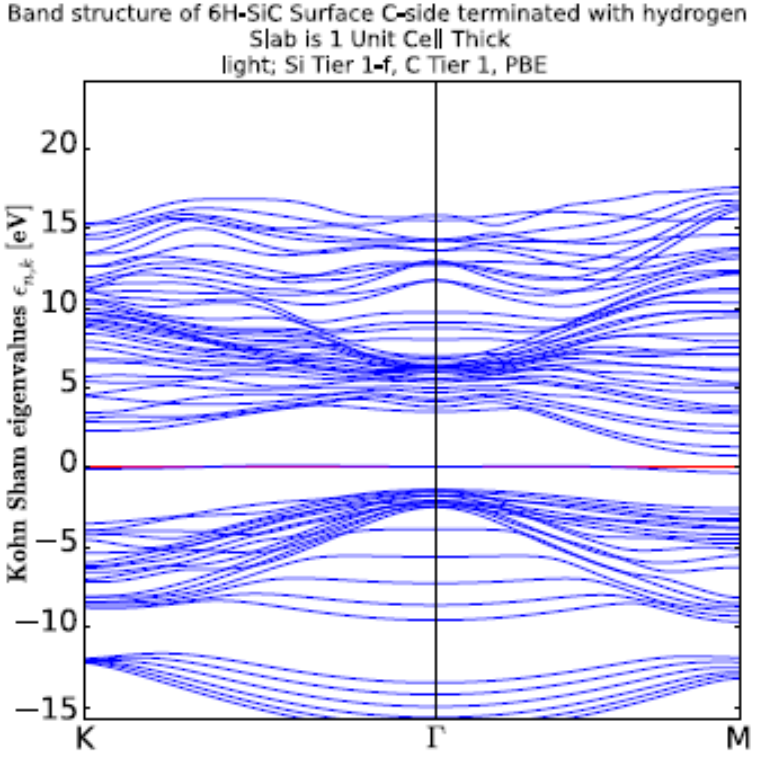
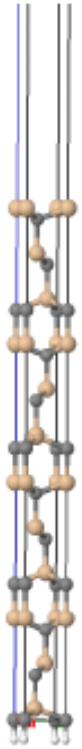
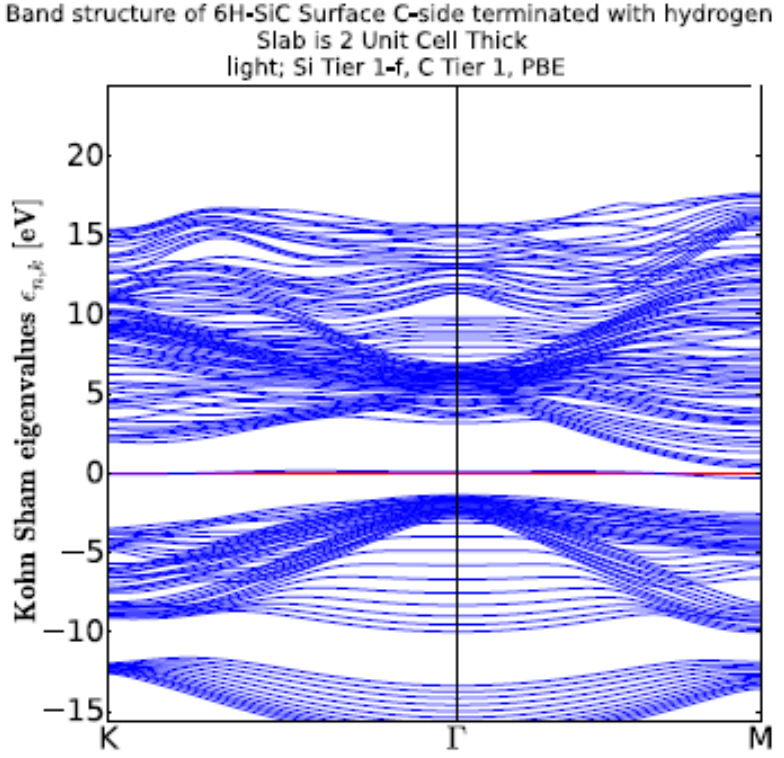
The projected band structure

An example: Hexagonal silicon carbide



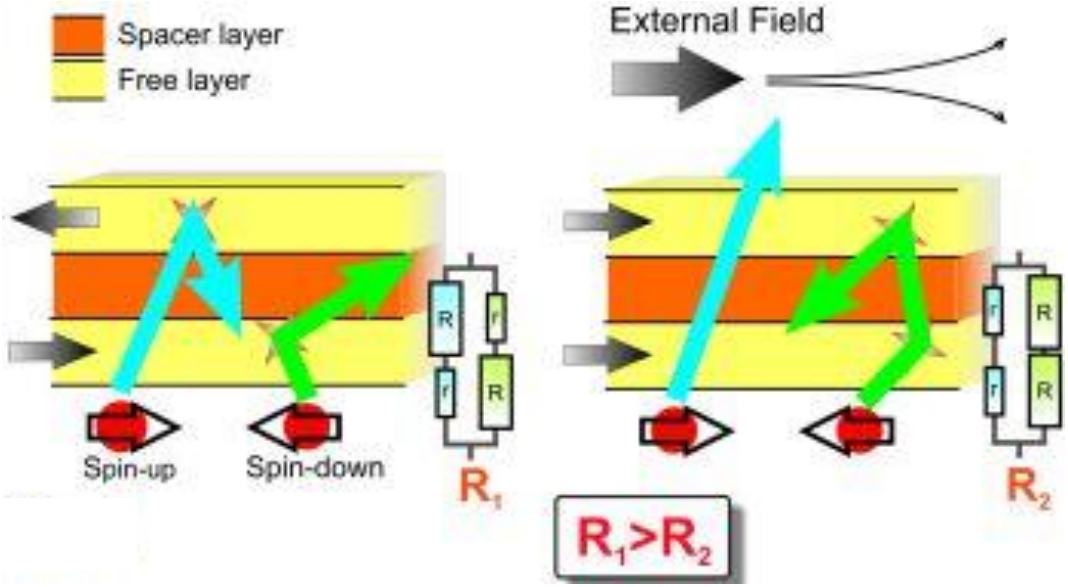


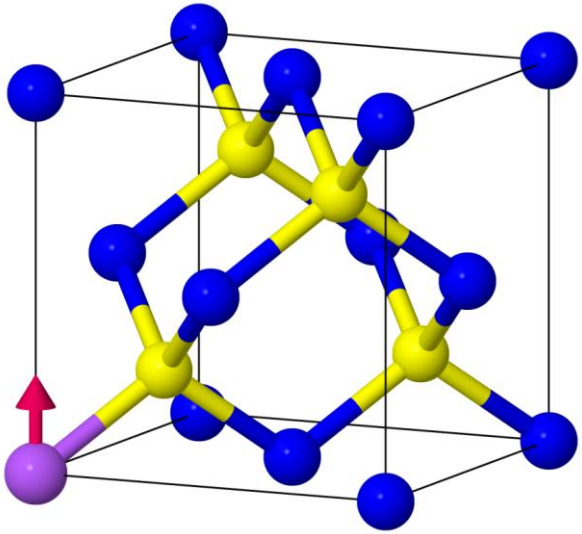


Mn-doped GaAs

Manipulating currents with spin and *vice versa*

Giant magneto-resistance (GMR):





- Non-volatile random-access memory (low power; GaMnN)
- Ferromagnetic-paramagnetic transition induced by electric field
- Turning ferromagnetism on-off optically

Overview

1. (Problems I to V) introduces basic bulk properties and convergence tests
   - Problem I: Generation and visualization of bulk structures
   - Problem II: Energy convergence tests
   - Problem III: Phase stability and cohesive properties
   - Problem IV: Unit cell relaxation
   - Problem V: Electronic structure and density of states
2. (Problems VI to VII) discusses surface calculations
   - Problem VI: Electronic structure of crystal surfaces
   - Problem VII: Relaxing surface structures
3. (Problems VIII to IX) covers magnetism and collinear spin calculations on Mn-doped GaAs
   - Problem VIII: Magnetic Ga_3MnAs_4
   - Problem IX: Ferromagnetic and antiferromagnetic $Ga_{0.75}Mn_{0.25}As$

Before we start:

```
change directory:
cd /afs/ictp/public/shared/smr2475
./setup-config.sh
logout
login again
```