

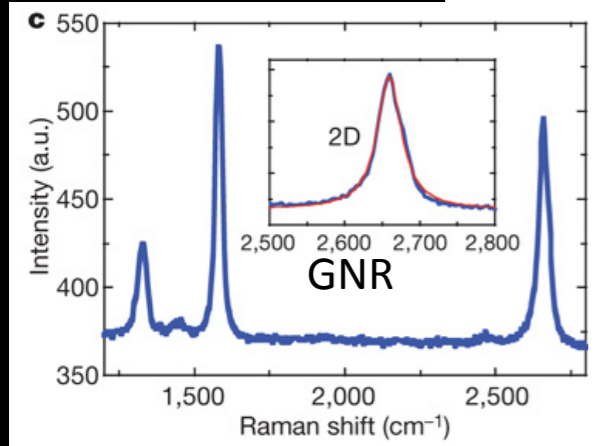
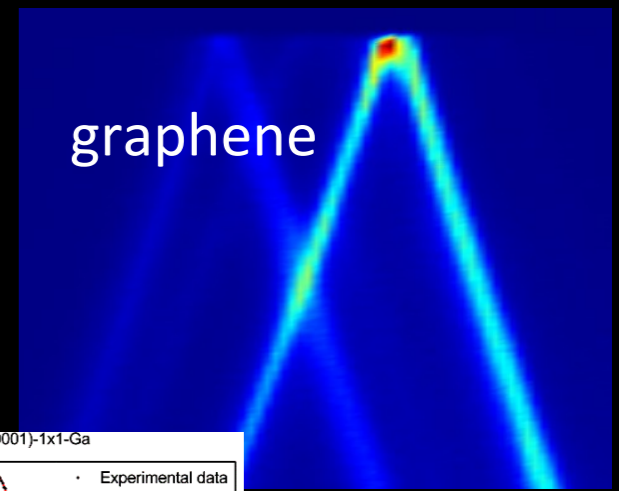
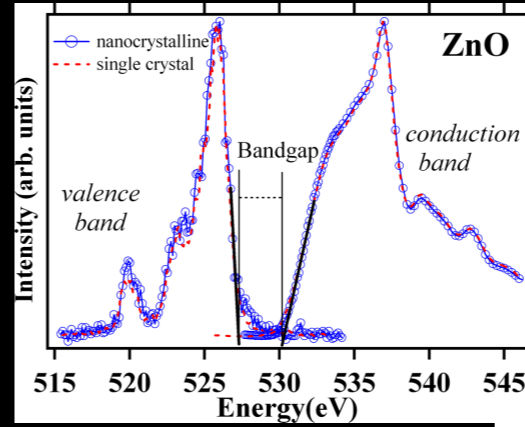
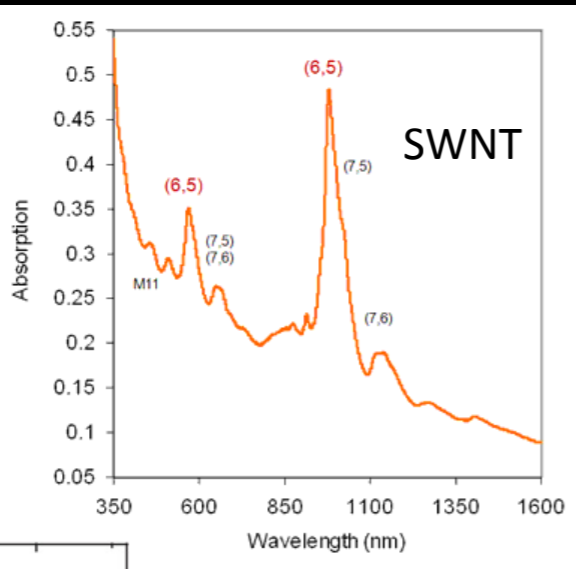
# simulating molecular excitations using quantum mechanics and digital computers

Stefano Baroni

SISSA - Scuola Internazionale Superiore di Studi Avanzati

lecture given at the *Workshop on Density Functional Theory and Beyond: Computational Materials Science for Real Materials*,  
August 6-15, 2013, the *Abdus Salam International Centre for Theoretical Physics*, Trieste

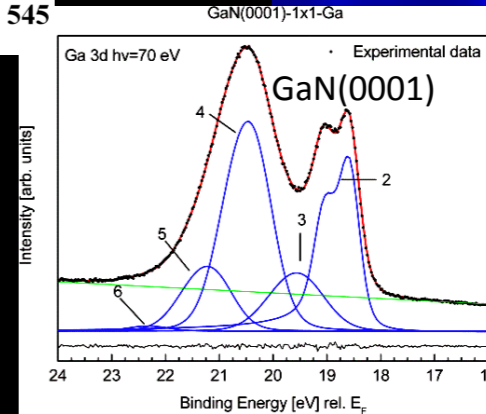
absorption  
lattice and  
molecular  
vibrations



Raman

XAS/XES

electronic  
transitions

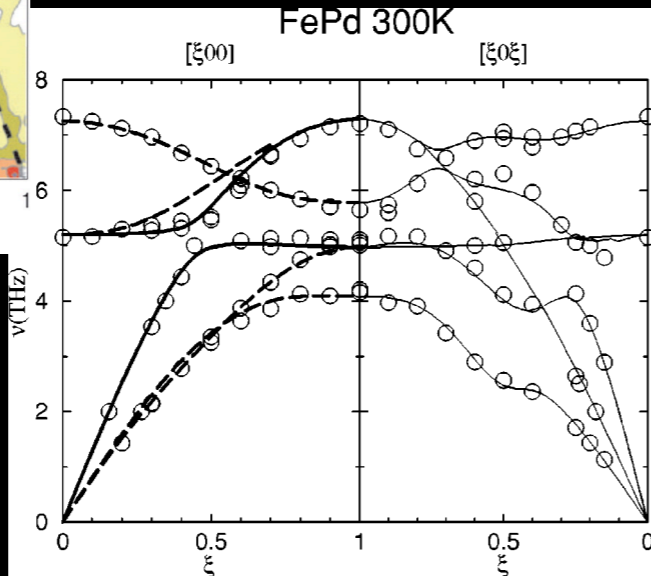
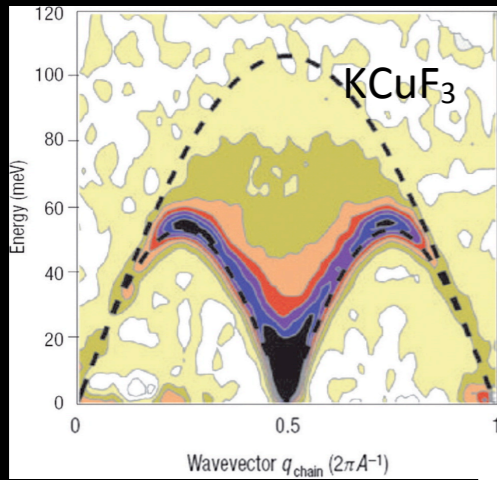


XPS

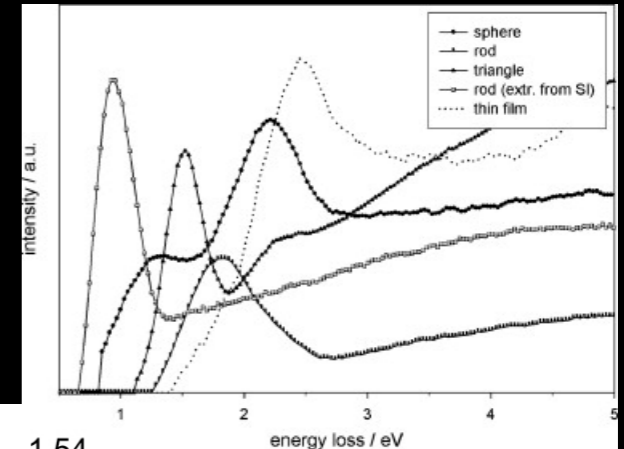
ARPES

# spectroscopies

spin fluctuations

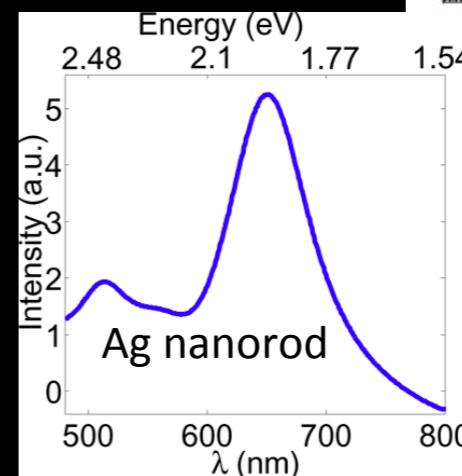


eels



inelastic  
neutron  
scattering

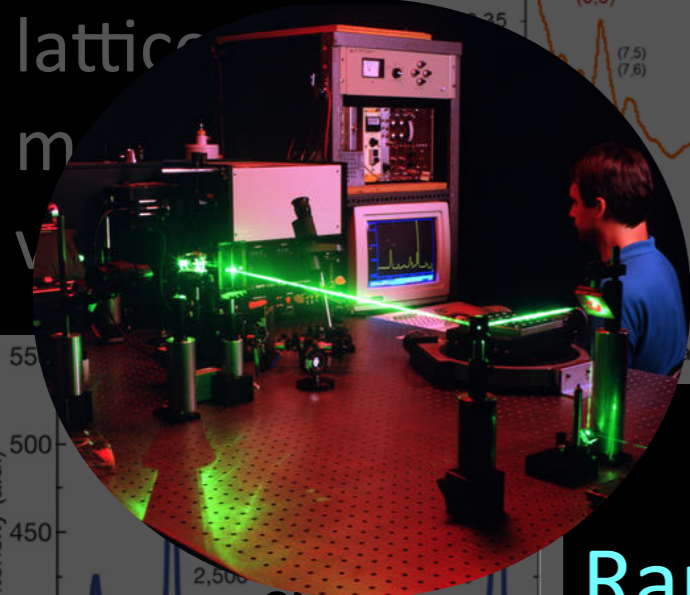
lattice vibrations



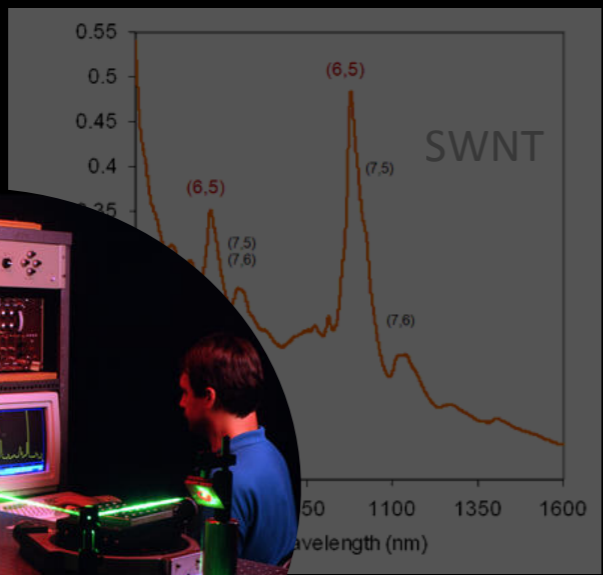
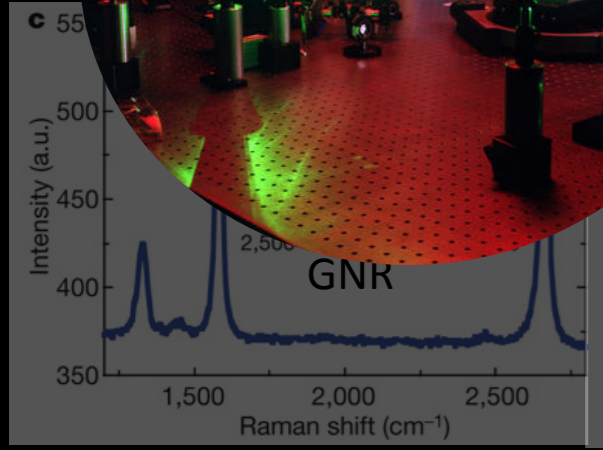
charge fluctuations



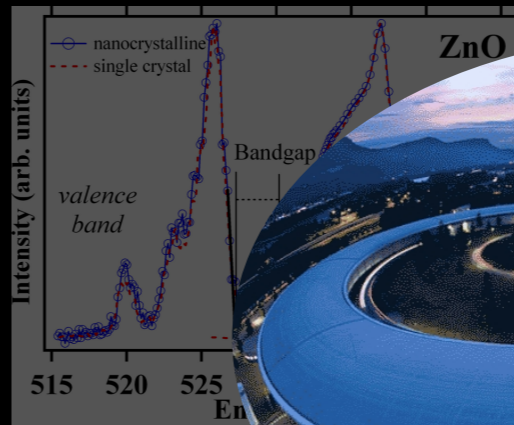
absorption



Raman



XAS/XES

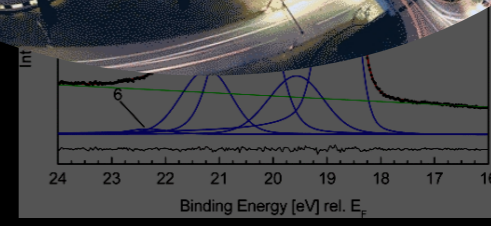


electronic transitions

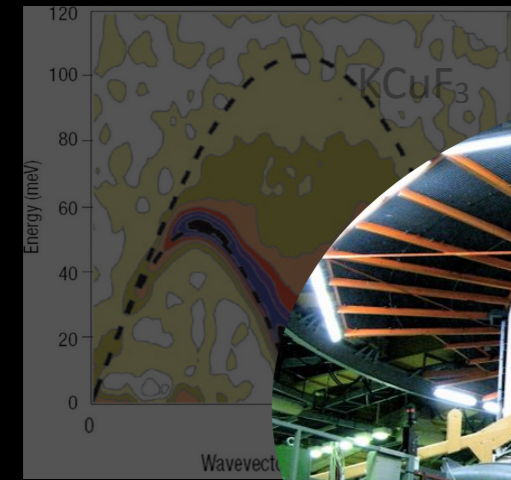


ARPES

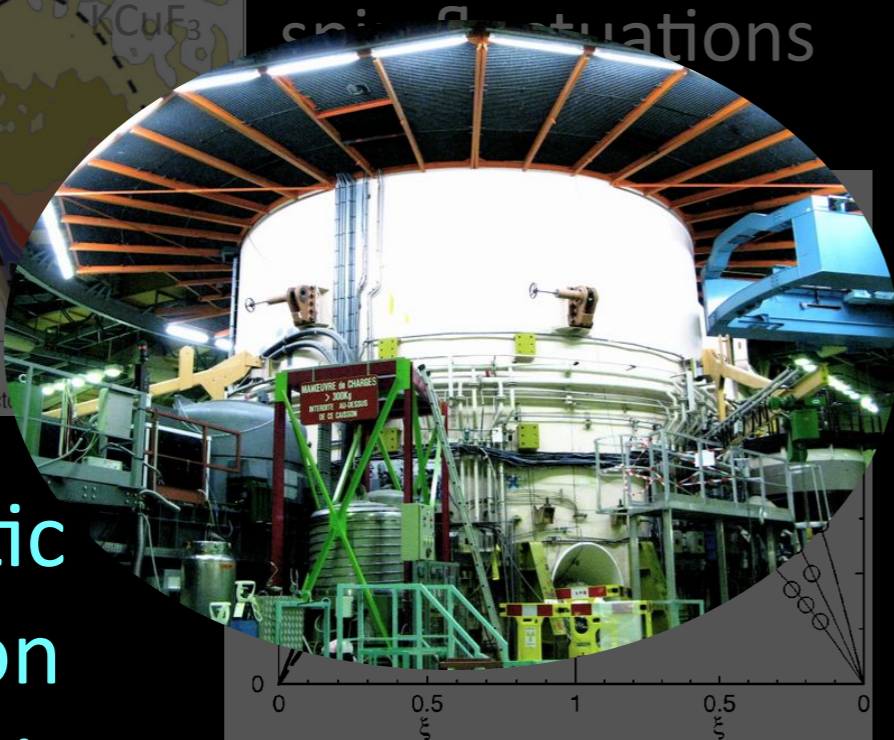
XPS



# spectroscopies

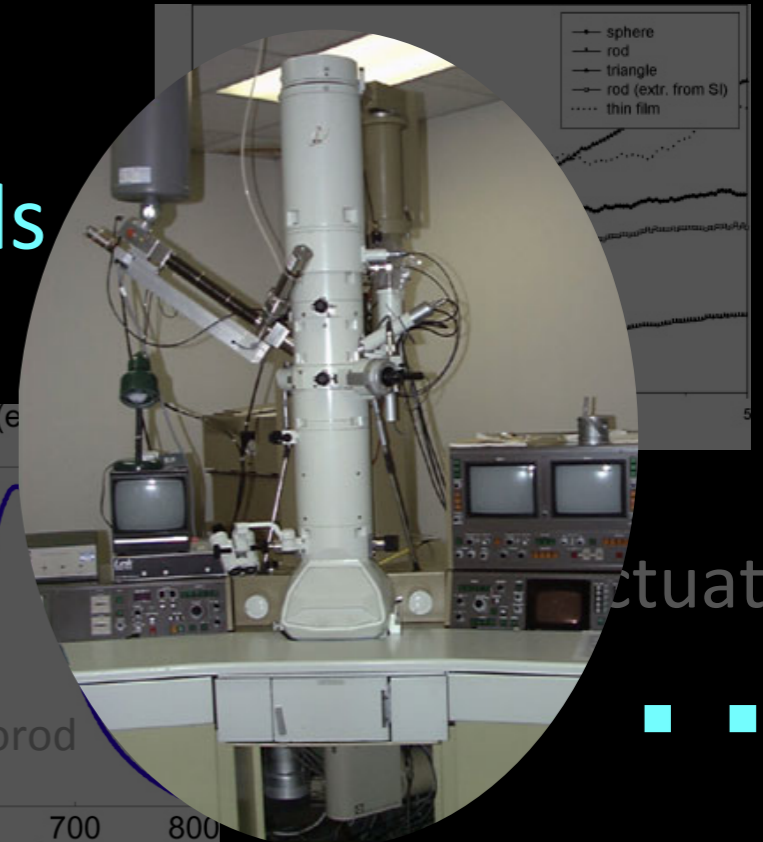
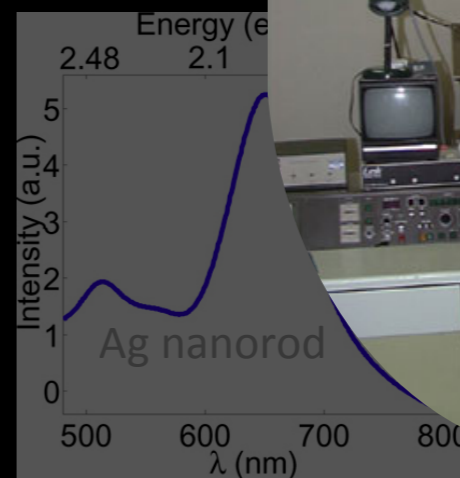


inelastic neutron scattering



lattice vibrations

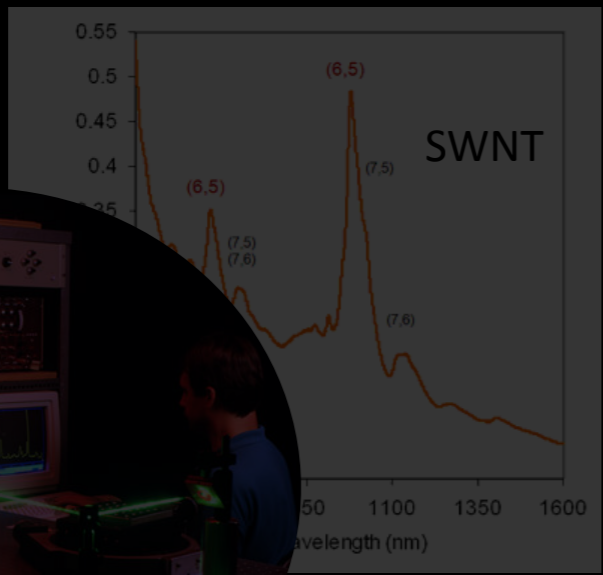
eels



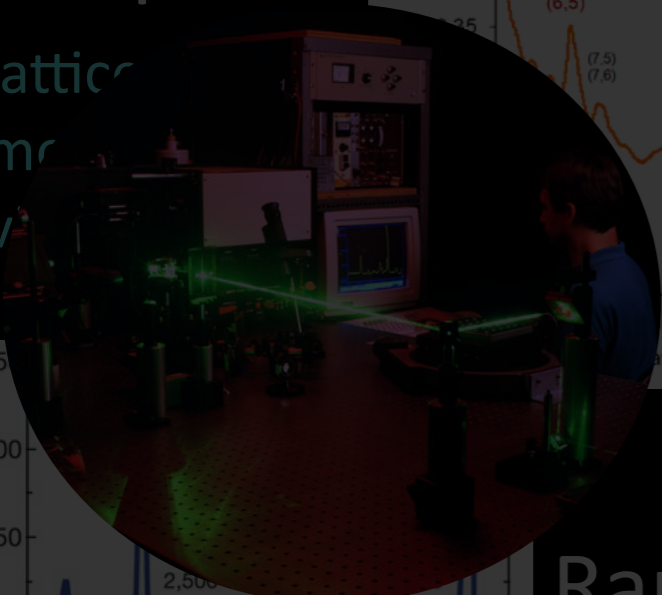
microscopy



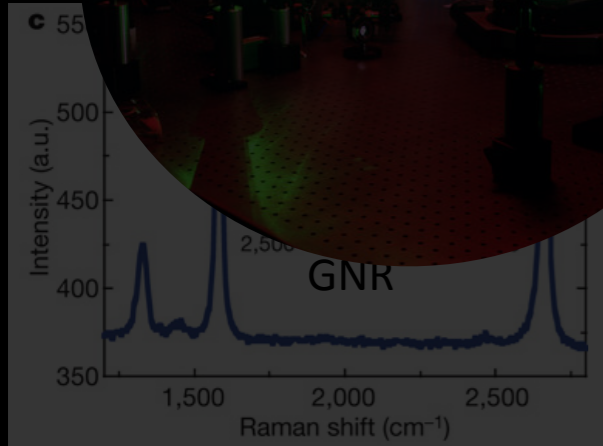
absorption



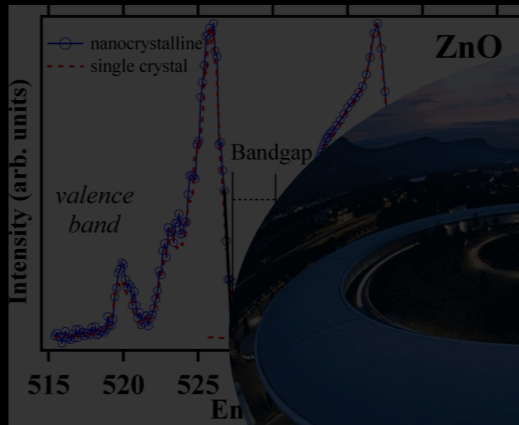
lattice  
mo  
v



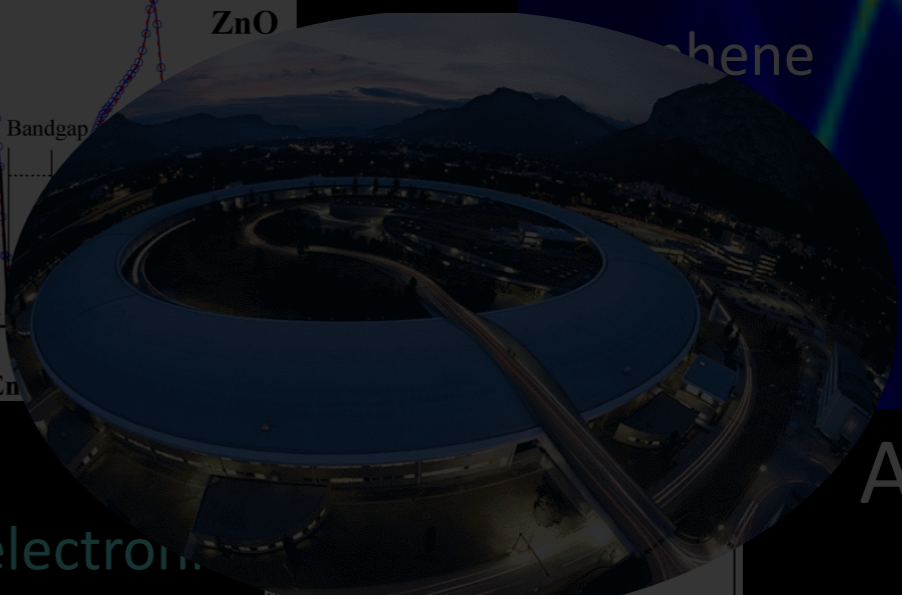
Raman



XAS/XES

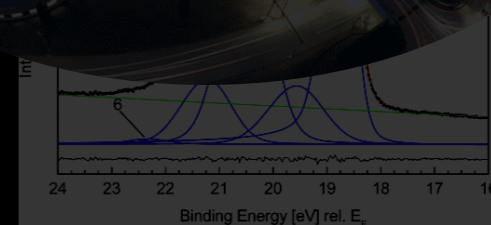


electron  
transitions

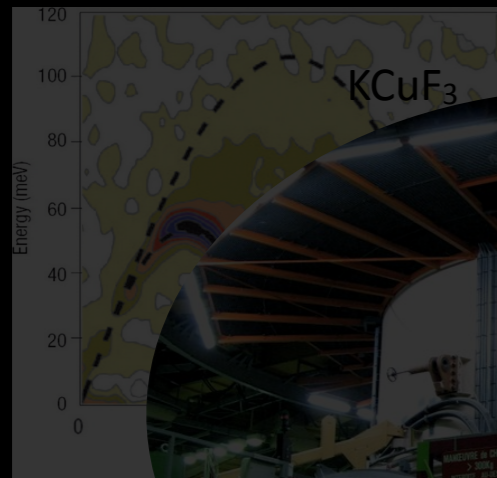


ARPES

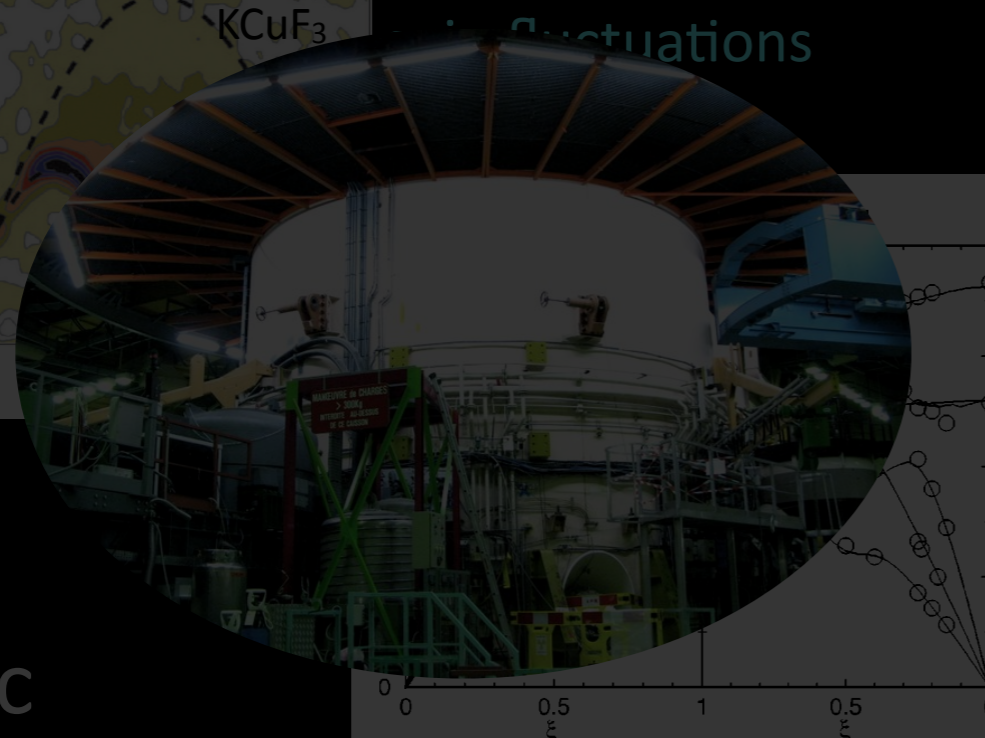
XPS



# spectroscopies



excitations

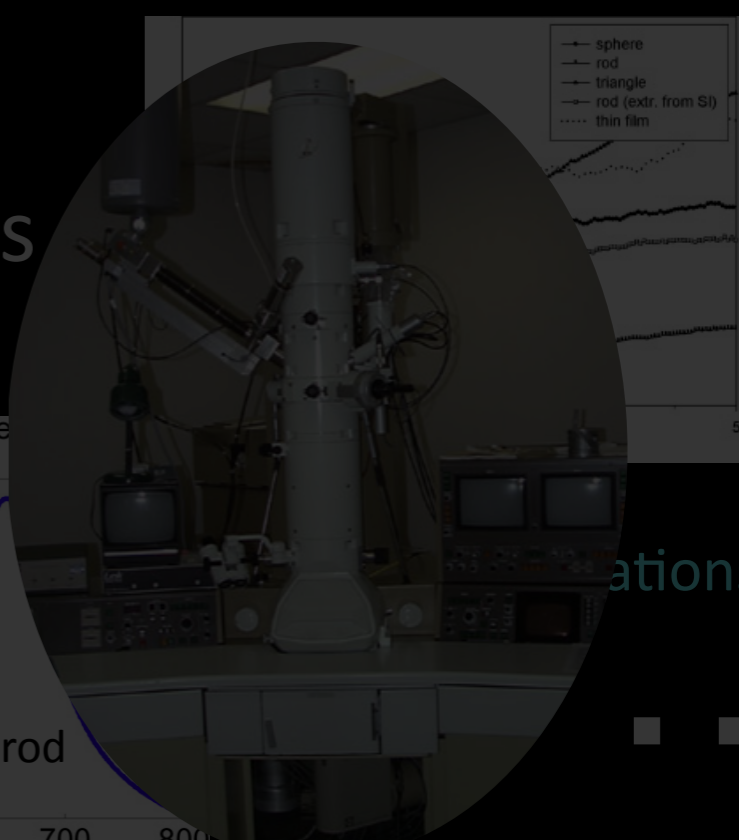
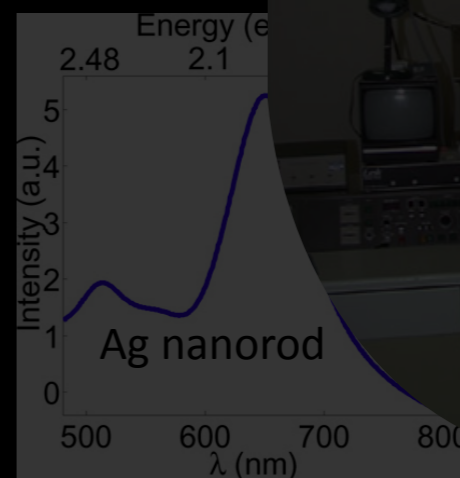


inelastic

neutron scattering

lattice vibrations

eels



excitations



# spectroscopies

probe



system

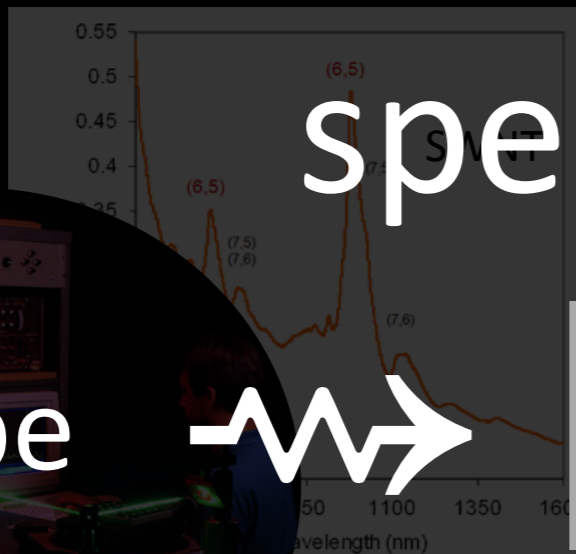
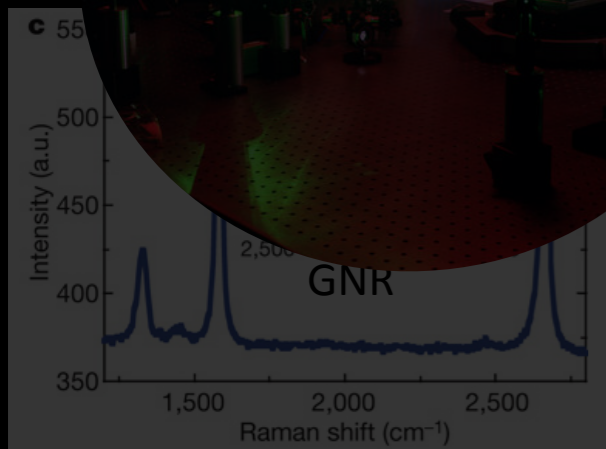


response

$$R(\omega) = \chi_{RP}(\omega)P(\omega)$$

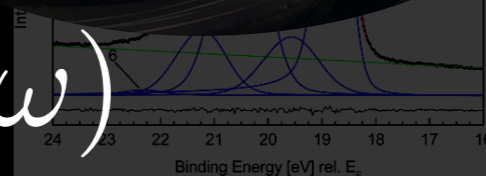
absorption

lattice  
m  
v



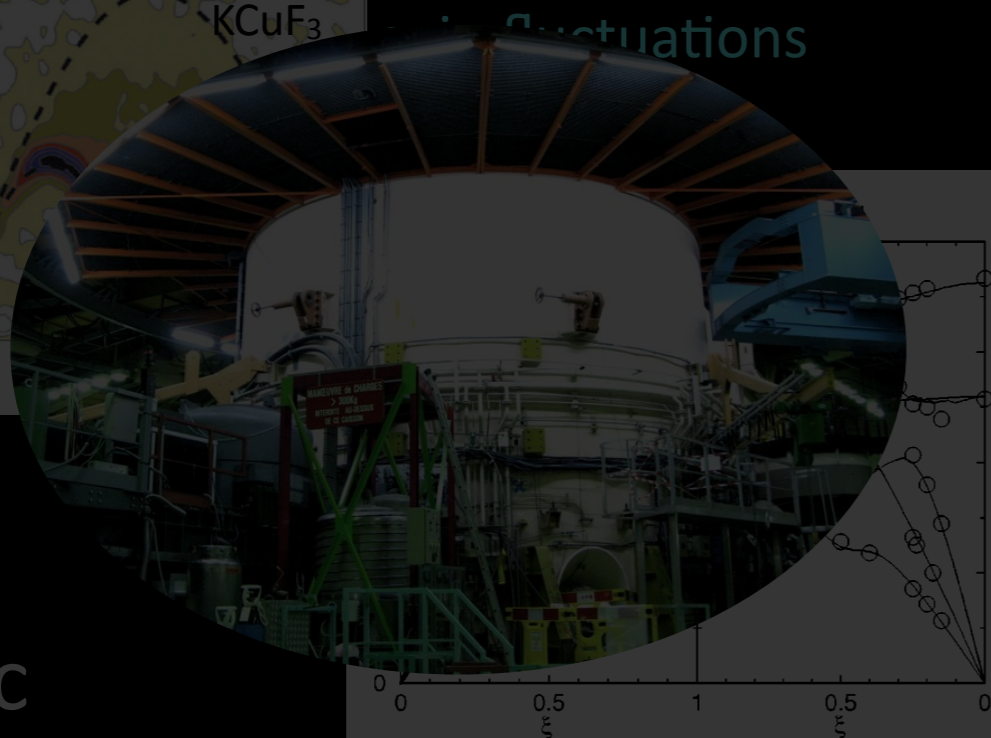
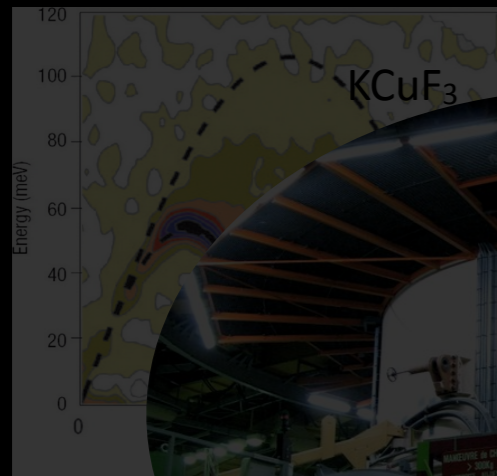
XAS/XLS

electron  
transitions



XPS

ARPES

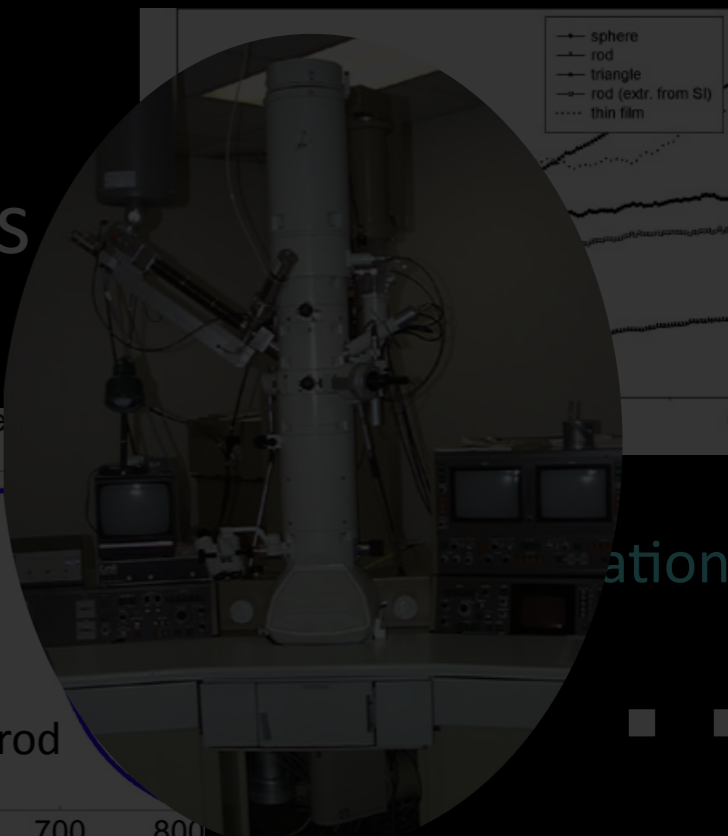
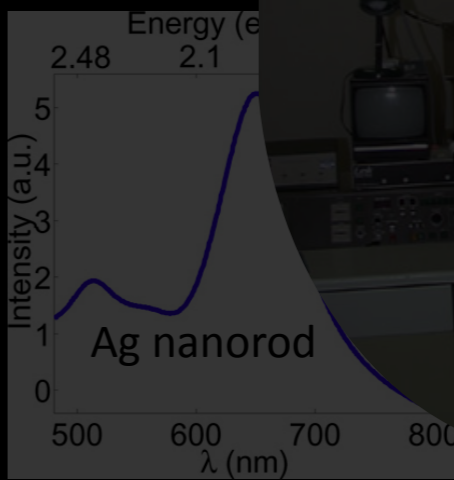


inelastic

neutron scattering

lattice vibrations

eels



ations



# spectroscopies

probe



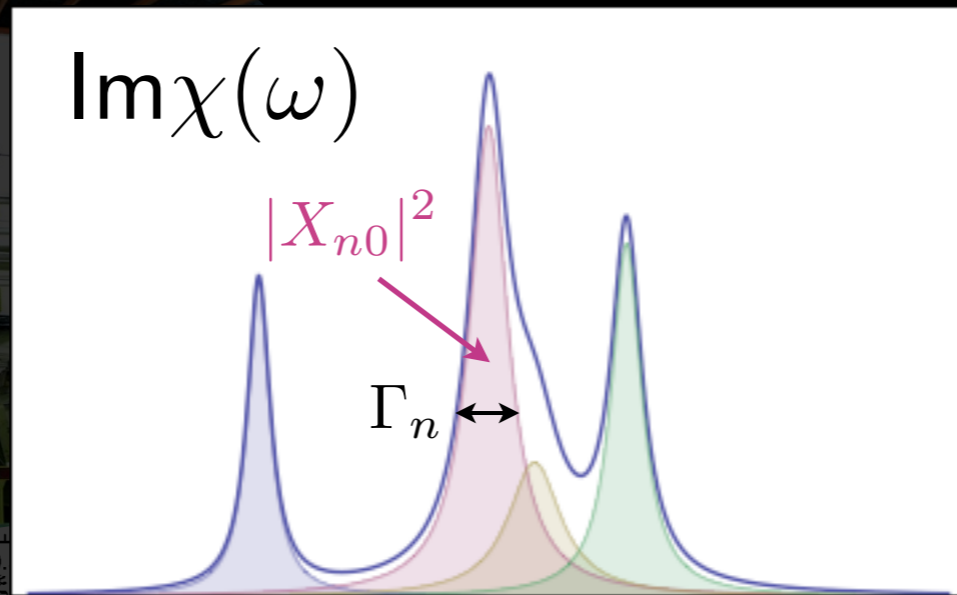
system



response

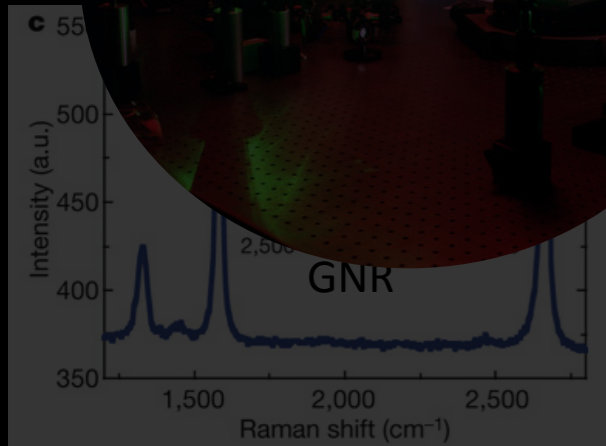
$$R(\omega) = \chi_{RP}(\omega)P(\omega)$$

$$\chi_{RP}(\omega) = \sum_{n \neq 0} \left[ \frac{R_{0n}P_{n0}}{\omega - E_{n0} + i\delta} - \frac{P_{0n}R_{n0}}{\omega + E_{n0} + i\delta} \right]$$



absorption

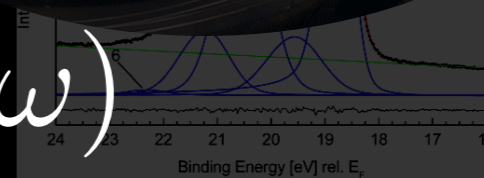
lattice  
m  
v



Raman

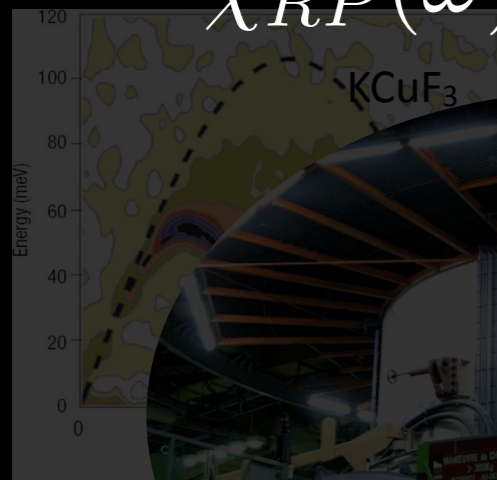
XAS/XLS

electron  
transitions



XPS

ARPES



inelastic

neutron scattering

lattice vibrations

↑ E<sub>n0</sub>

rod

ations



# probing the flickers of matter

| flicker             | probe                                |
|---------------------|--------------------------------------|
| molecular vibration | IR, Raman, INS,<br>HREELS, ...       |
| spin fluctuation    | INS, EPR, MCD, ...                   |
| charge fluctuation  | optical and PE<br>spectra, EELS, ... |
| ...                 | ...                                  |

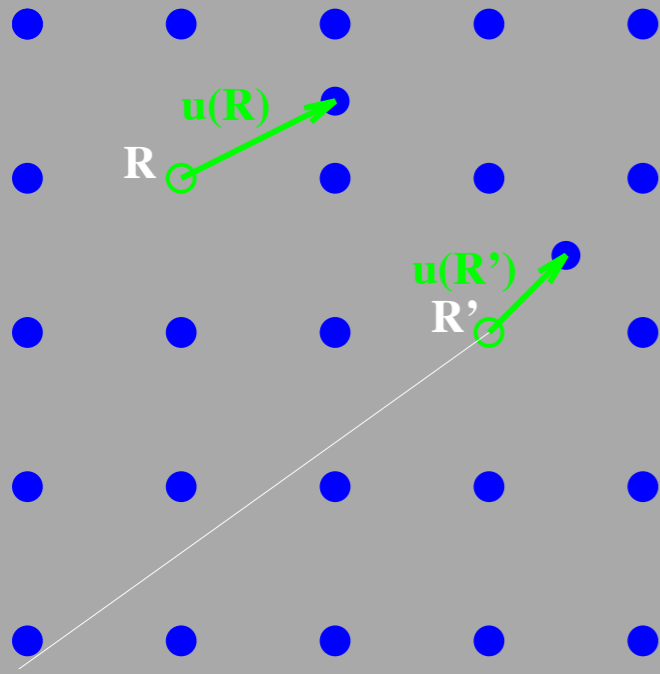
# probing the flickers of matter

| flicker             | probe                                | theory           |
|---------------------|--------------------------------------|------------------|
| molecular vibration | IR, Raman, INS,<br>HREELS, ...       | DFT              |
| spin fluctuation    | INS, EPR, MCD, ...                   | constrained DFT  |
| charge fluctuation  | optical and PE<br>spectra, EELS, ... | TDDFT, MBPT, ... |
| ...                 | ...                                  | ...              |



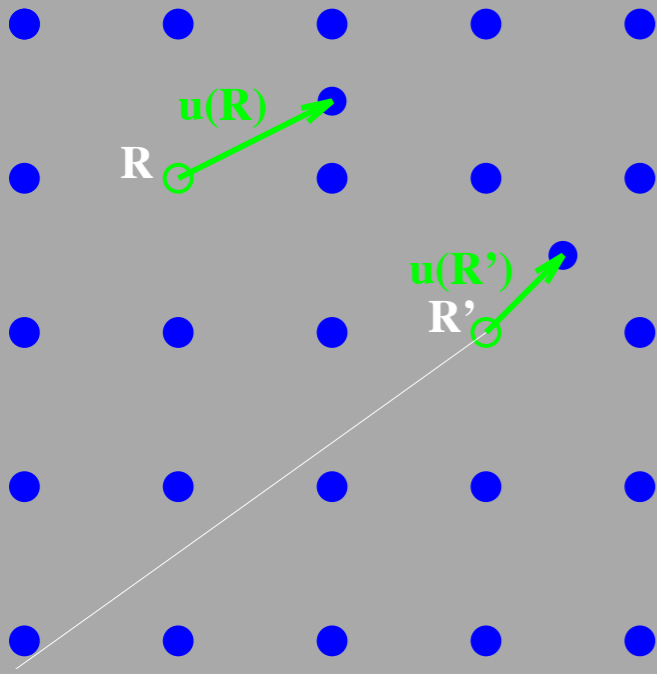
simulating atomic vibrations ...

# lattice dynamics



$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r} - \mathbf{R})}{\partial \mathbf{R}} + \dots$$

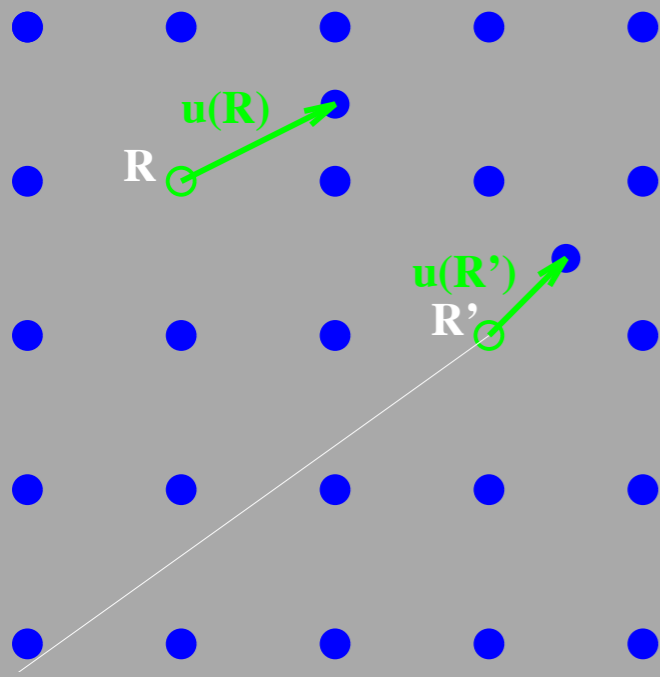
# lattice dynamics



$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r} - \mathbf{R})}{\partial \mathbf{R}} + \dots$$

$$E = E_0 + \frac{1}{2} \sum_{\mathbf{R}, \mathbf{R}'} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} \cdot \mathbf{u}(\mathbf{R}') + \dots$$

# lattice dynamics

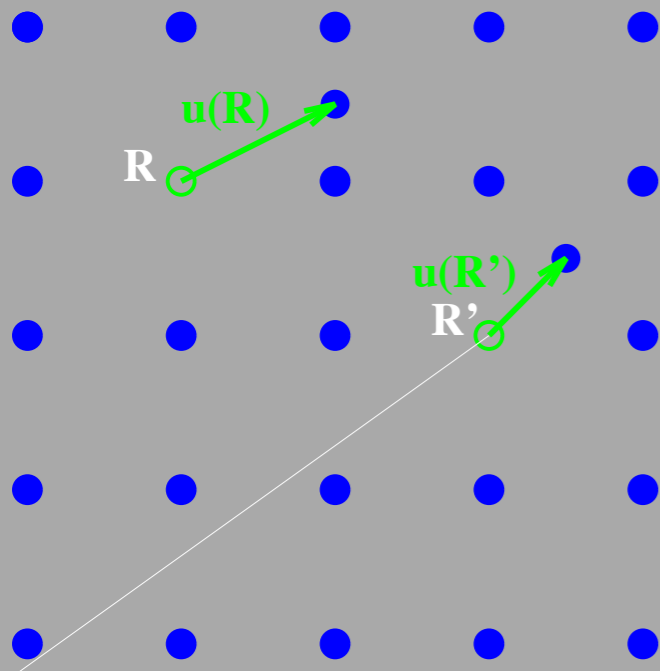


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$$\frac{\partial F(\mathbf{R})}{\partial \mathbf{u}(\mathbf{R}' )}$$

$$E = E_0 + \frac{1}{2} \sum_{\mathbf{R}, \mathbf{R}'} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} \cdot \mathbf{u}(\mathbf{R}') + \dots$$

# lattice dynamics



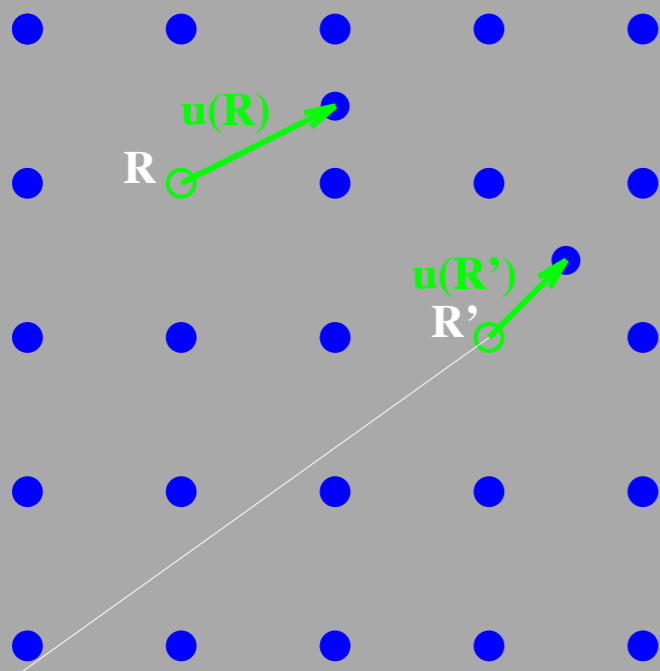
$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r} - \mathbf{R})}{\partial \mathbf{R}} + \dots$$

$$\frac{\partial F(\mathbf{R})}{\partial \mathbf{u}(\mathbf{R}' )}$$

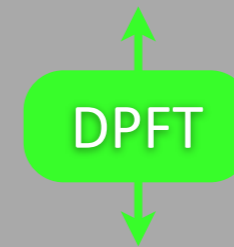
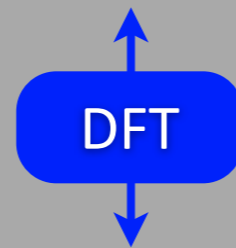
$$E = E_0 + \frac{1}{2} \sum_{\mathbf{R}, \mathbf{R}'} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} \cdot \mathbf{u}(\mathbf{R}') + \dots$$

$$\det \left[ \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} - \omega^2 M(\mathbf{R}) \delta_{\mathbf{R}, \mathbf{R}'} \right] = 0$$

# lattice dynamics



$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_{\mathbf{R}} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial v(\mathbf{r} - \mathbf{R})}{\partial \mathbf{R}} + \dots$$



$$\frac{\partial F(\mathbf{R})}{\partial \mathbf{u}(\mathbf{R}' )}$$

$$E = E_0 + \frac{1}{2} \sum_{\mathbf{R}, \mathbf{R}'} \mathbf{u}(\mathbf{R}) \cdot \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} \cdot \mathbf{u}(\mathbf{R}') + \dots$$

$$\det \left[ \frac{\partial^2 E}{\partial \mathbf{u}(\mathbf{R}) \partial \mathbf{u}(\mathbf{R}')} - \omega^2 M(\mathbf{R}) \delta_{\mathbf{R}, \mathbf{R}'} \right] = 0$$

# density-functional perturbation theory

$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_i u_i V_i'(\mathbf{r})$$

# density-functional perturbation theory

$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_i u_i V_i'(\mathbf{r})$$

$$\frac{\partial E(\mathbf{u})}{\partial u_i} = \int n_{\mathbf{u}}(\mathbf{r}) V_i'(\mathbf{r}) d\mathbf{r}$$

Hellmann-  
Feynman



# density-functional perturbation theory

$$V(\mathbf{r}) = V_0(\mathbf{r}) + \sum_i u_i V_i'(\mathbf{r})$$

$$\frac{\partial E(\mathbf{u})}{\partial u_i} = \int n_{\mathbf{u}}(\mathbf{r}) V_i'(\mathbf{r}) d\mathbf{r}$$

Hellmann-  
Feynman

$$\frac{\partial^2 E(\mathbf{u})}{\partial u_i \partial u_j} = \int \frac{\partial n_{\mathbf{u}}(\mathbf{r})}{\partial u_j} V_i'(\mathbf{r}) d\mathbf{r}$$

DFPT

# calculating the response

$$n(\mathbf{r}) = \sum_v |\phi_v(\mathbf{r})|^2$$

$$n'(\mathbf{r}) = 2\text{Re} \sum_v \phi_v^{\circ*}(\mathbf{r}) \phi'_v(\mathbf{r})$$

# calculating the response

$$n(\mathbf{r}) = \sum_v |\phi_v(\mathbf{r})|^2$$

$$\begin{aligned} n'(\mathbf{r}) &= 2\text{Re} \sum_v \phi_v^{\circ*}(\mathbf{r}) \phi'_v(\mathbf{r}) \\ &= 2\text{Re} \sum_{cv} \rho'_{vc} \phi_v^{\circ*}(\mathbf{r}) \phi_c^{\circ}(\mathbf{r}) \end{aligned}$$

$$\phi'_v = \sum_c \phi_c^{\circ} \frac{\langle \phi_c^{\circ} | V' | \phi_v^{\circ} \rangle}{\epsilon_v^{\circ} - \epsilon_c^{\circ}}$$

# calculating the response

$$n(\mathbf{r}) = \sum_v |\phi_v(\mathbf{r})|^2$$

$$\begin{aligned} n'(\mathbf{r}) &= 2\text{Re} \sum_v \phi_v^{\circ*}(\mathbf{r}) \phi'_v(\mathbf{r}) \\ &= 2\text{Re} \sum_{cv} \rho'_{vc} \phi_v^{\circ*}(\mathbf{r}) \phi_c^{\circ}(\mathbf{r}) \end{aligned}$$

$$\phi'_v = \sum_c \phi_c^{\circ} \frac{\langle \phi_c^{\circ} | V' | \phi_v^{\circ} \rangle}{\epsilon_v^{\circ} - \epsilon_c^{\circ}}$$

$$(H^{\circ} - \epsilon_v^{\circ}) \phi'_v = -P_c V' \phi_v^{\circ}$$

calculating the response

$$n'(\mathbf{r}) = 2\text{Re} \sum_v \phi_v^{\circ*}(\mathbf{r}) \phi'_v(\mathbf{r})$$

$$(H^{\circ} - \epsilon_v^{\circ}) \phi'_v = -P_c V' \phi_v^{\circ}$$

# DFPT: the equations

DFT

$$V_0(\mathbf{r}) \Leftrightarrow n(\mathbf{r})$$

$$V_{SCF}(\mathbf{r}) = V_0(\mathbf{r}) + \int \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' + \mu_{xc}(\mathbf{r})$$

$$n(\mathbf{r}) = \sum_{\epsilon_v < E_F} |\phi_v(\mathbf{r})|^2$$

$$(-\Delta + V_{SCF}(\mathbf{r}))\phi_v(\mathbf{r}) = \epsilon_v \phi_v(\mathbf{r})$$

# DFPT: the equations

DFT

$$V_0(\mathbf{r}) \rightleftharpoons n(\mathbf{r})$$

$$V_{SCF}(\mathbf{r}) = V_0(\mathbf{r}) + \int \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' + \mu_{xc}(\mathbf{r})$$

$$n(\mathbf{r}) = \sum_{\epsilon_v < E_F} |\phi_v(\mathbf{r})|^2$$

$$(-\Delta + V_{SCF}(\mathbf{r}))\phi_v(\mathbf{r}) = \epsilon_v \phi_v(\mathbf{r})$$

DFPT

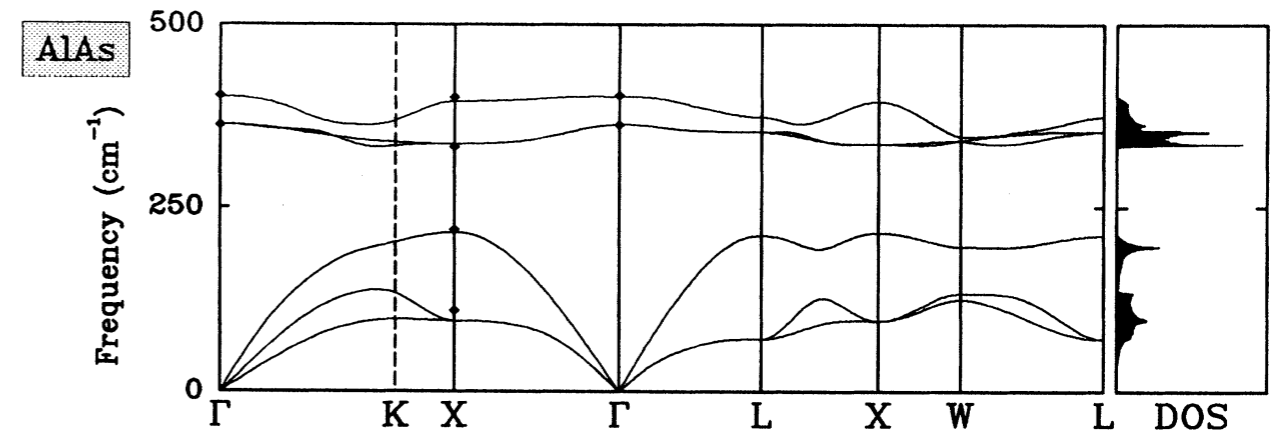
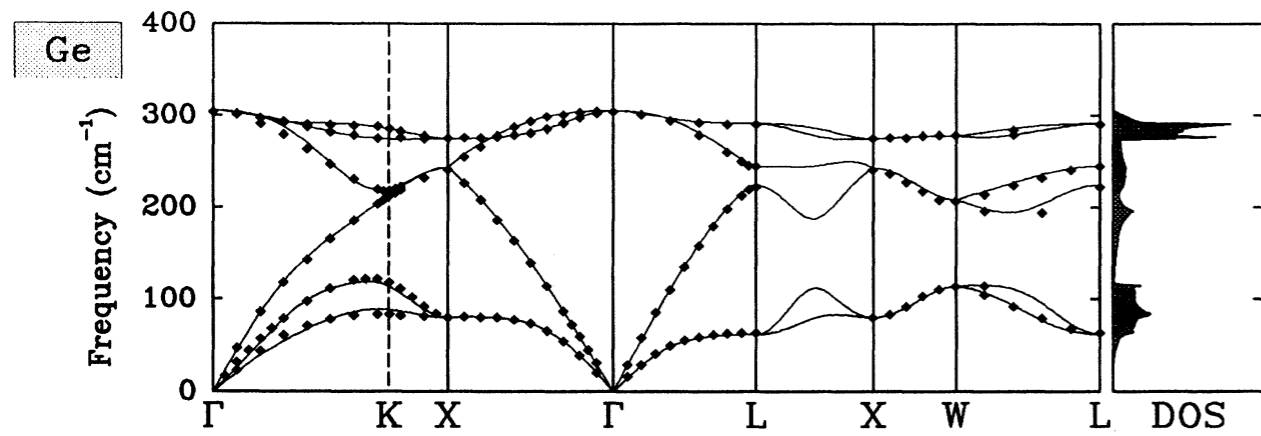
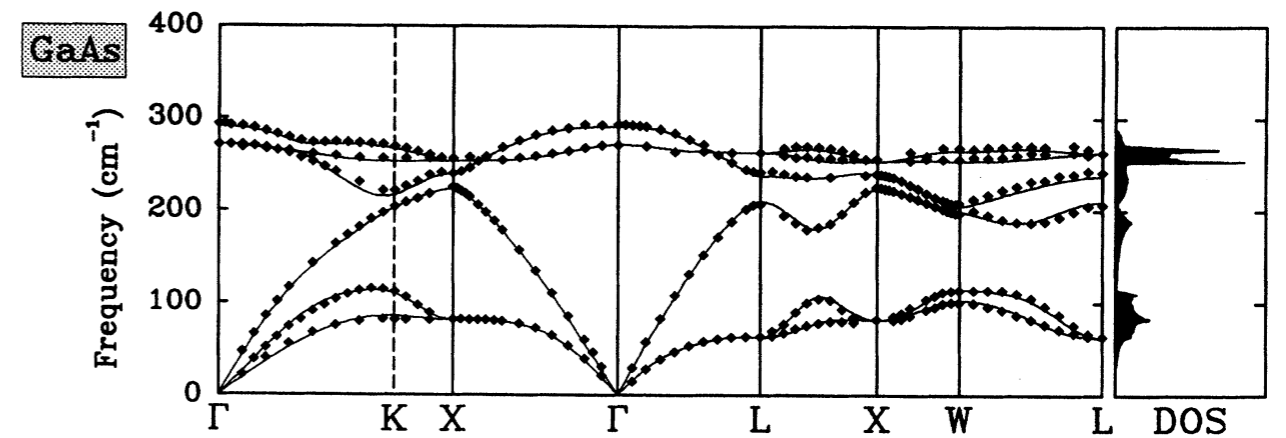
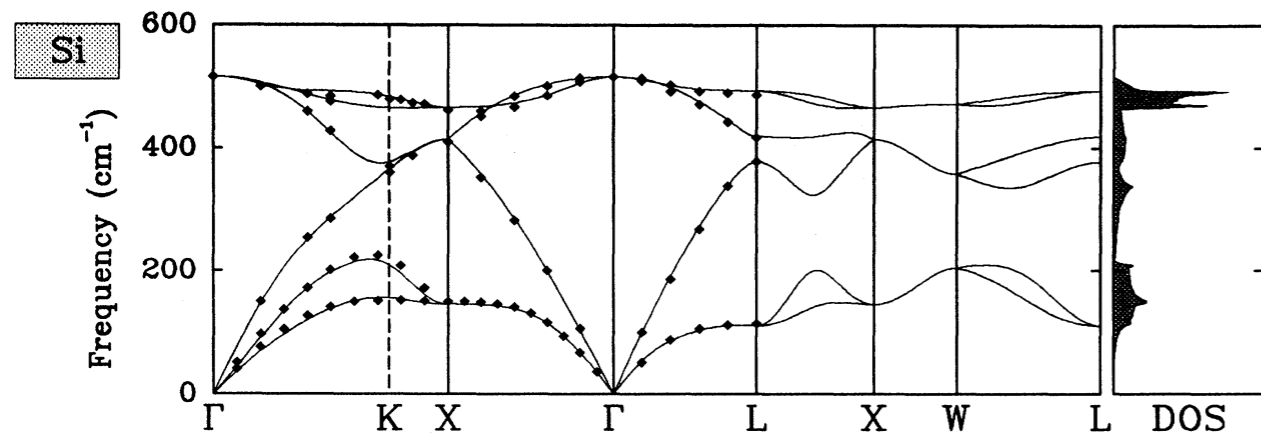
$$V'(\mathbf{r}) \rightleftharpoons n'(\mathbf{r})$$

$$V'_{SCF}(\mathbf{r}) = V'(\mathbf{r}) + \int \frac{n'(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' + \mu'_{xc}(\mathbf{r})$$

$$n'(\mathbf{r}) = 2 \operatorname{Re} \sum_{\epsilon_v < E_F} \phi_v^*(\mathbf{r}) \phi'_v(\mathbf{r})$$

$$(-\Delta + V_{SCF}(\mathbf{r}) - \epsilon_v)\phi'_v(\mathbf{r}) = P_c V'_{SCF}(\mathbf{r})\phi_v(\mathbf{r})$$

# phonons from DFPT



P. Giannozzi, S. de Gironcoli, P. Pavone, and SB, Phys. Rev. B **43**, 7231 (1991)



# applications done so far

- Dielectric properties
- Piezoelectric properties
- Elastic properties
- Phonon in crystals and alloys
- Phonon at surfaces, interfaces, superlattices, and nano-structures
- Raman and infrared activities
- Anharmonic couplings and vibrational line widths
- Mode softening and structural transitions
- Electron-phonon interaction and superconductivity
- Thermal expansion
- Isotopic effects on structural and dynamical properties
- Thermo-elasticity and other thermal properties of minerals
- ...

SB, A. Dal Corso, S. de Gironcoli, and P. Giannozzi, *Phonons and related crystal properties from density-functional perturbation theory*, *Rev. Mod. Phys.* **73**, 515 (2001)

# a sampler of recent applications

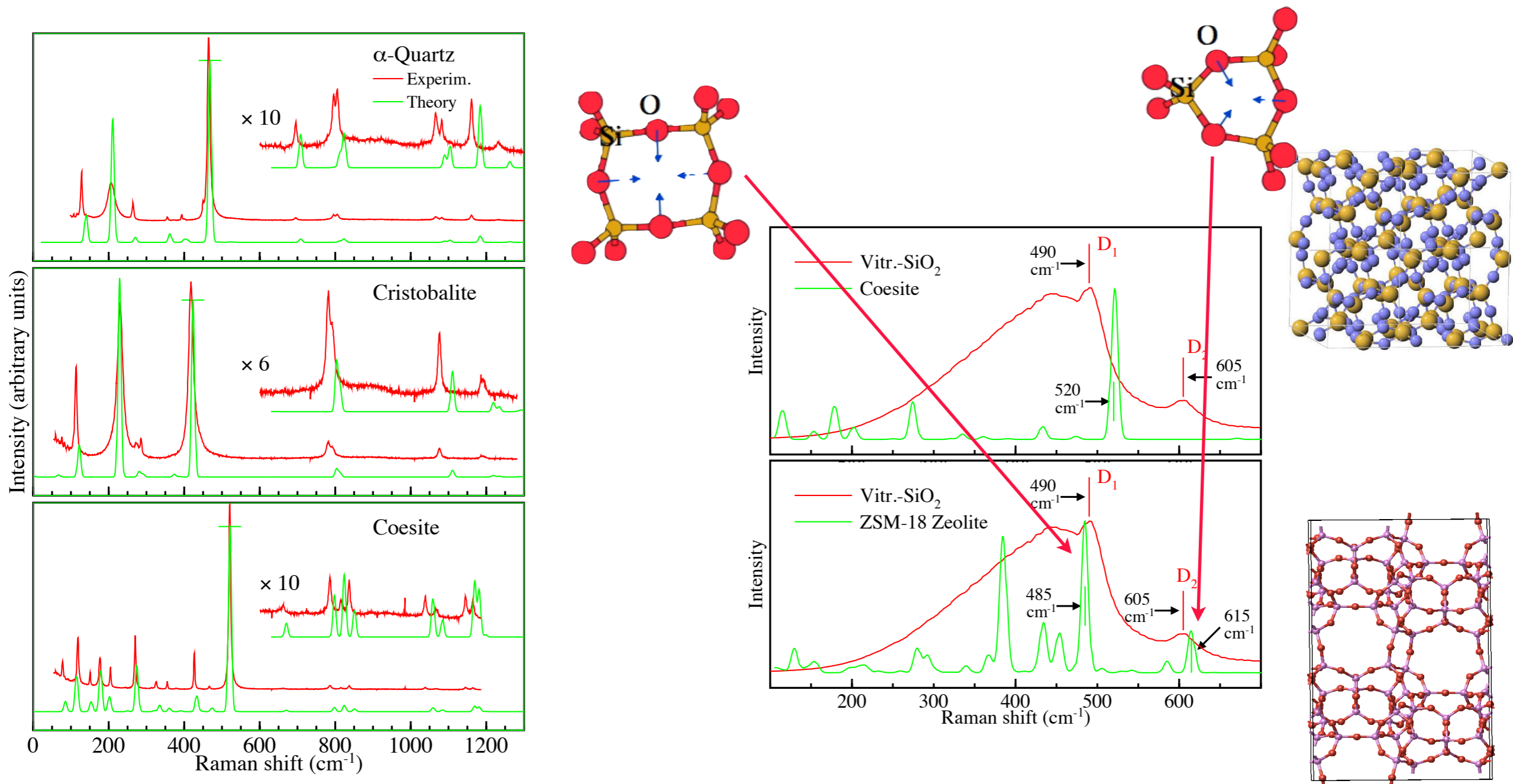
VOLUME 90, NUMBER 3

PHYSICAL REVIEW LETTERS

week ending  
24 JANUARY 2003

## First-Principles Calculation of Vibrational Raman Spectra in Large Systems: Signature of Small Rings in Crystalline $\text{SiO}_2$

Michele Lazzeri and Francesco Mauri



# a sampler of recent applications

J|A|C|S

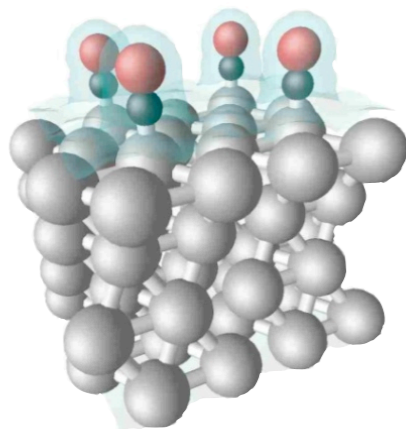
A R T I C L E S

Published on Web 08/17/2007

## Vibrational Recognition of Adsorption Sites for CO on Platinum and Platinum–Ruthenium Surfaces

Ismaila Dabo,<sup>\*,†</sup> Andrzej Wieckowski,<sup>‡</sup> and Nicola Marzari<sup>†</sup>

11046 J. AM. CHEM. SOC. ■ VOL. 129, NO. 36, 2007

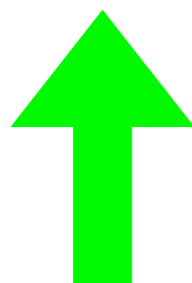


atop (CO@Pt<sub>1</sub>)

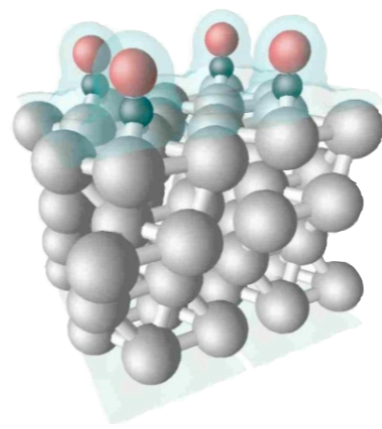
$E_{\text{DFT}} = +0.10 \text{ eV}$

$\nu_{\text{DFT}} = 2050 \text{ cm}^{-1}$

$\nu_{\text{exp}} = 2070 \text{ cm}^{-1}$



expt

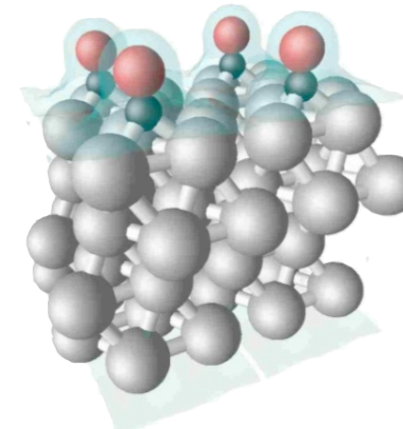


bridge (CO@Pt<sub>2</sub>)

$E_{\text{DFT}} = +0.03 \text{ eV}$

$\nu_{\text{DFT}} = 1845 \text{ cm}^{-1}$

$\nu_{\text{exp}} = 1830 \text{ cm}^{-1}$

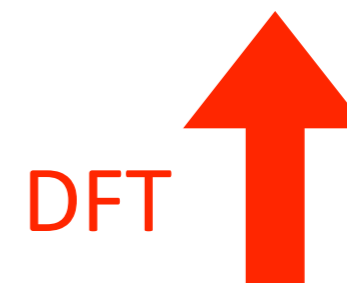


fcc (CO@Pt<sub>3</sub>)

$E_{\text{DFT}} = 0 \text{ eV}$

$\nu_{\text{DFT}} = 1743 \text{ cm}^{-1}$

$\nu_{\text{exp}} = 1780 \text{ cm}^{-1}$



DFT

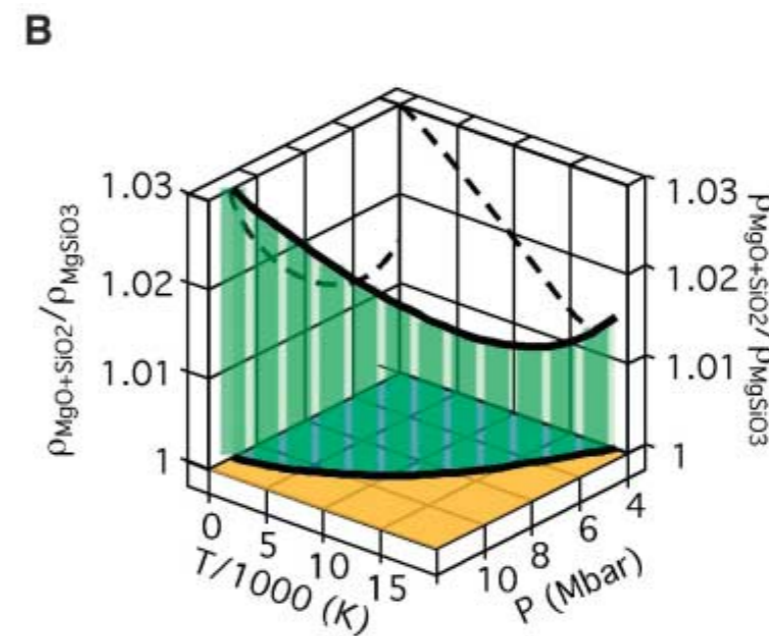
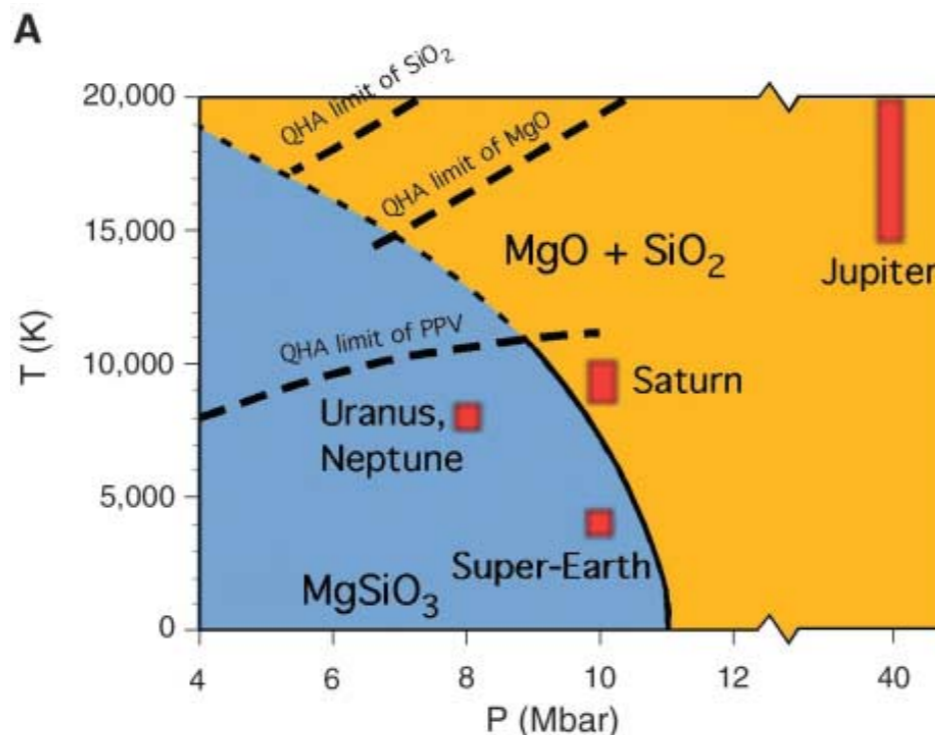
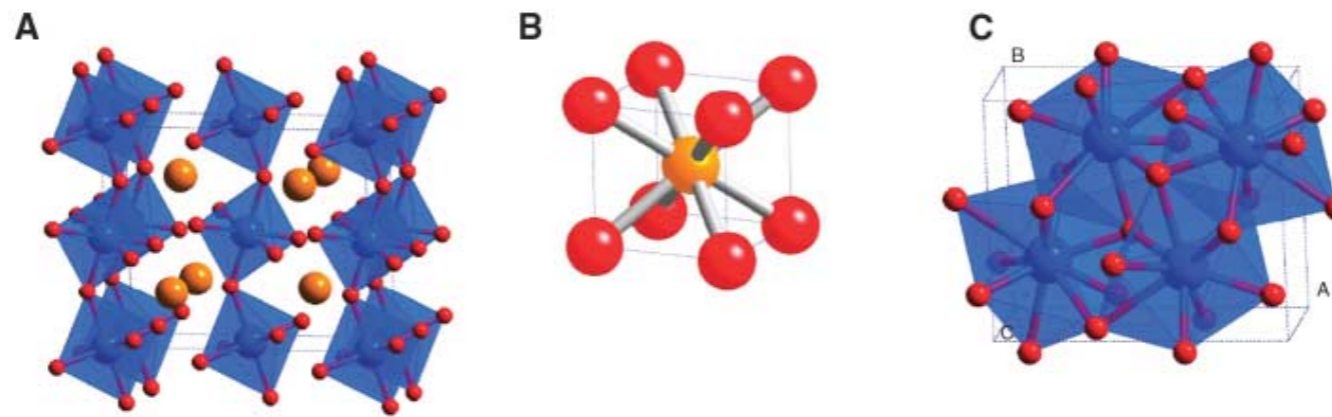
# a sampler of recent applications

## Dissociation of $\text{MgSiO}_3$ in the Cores of Gas Giants and Terrestrial Exoplanets

Koichiro Umemoto,<sup>1</sup> Renata M. Wentzcovitch,<sup>1\*</sup> Philip B. Allen<sup>2</sup>

www.sciencemag.org SCIENCE VOL 311 17 FEBRUARY 2006

983



# a sampler of recent applications

PRL 100, 257001 (2008)

PHYSICAL REVIEW LETTERS

week ending  
27 JUNE 2008



## *Ab Initio* Description of High-Temperature Superconductivity in Dense Molecular Hydrogen

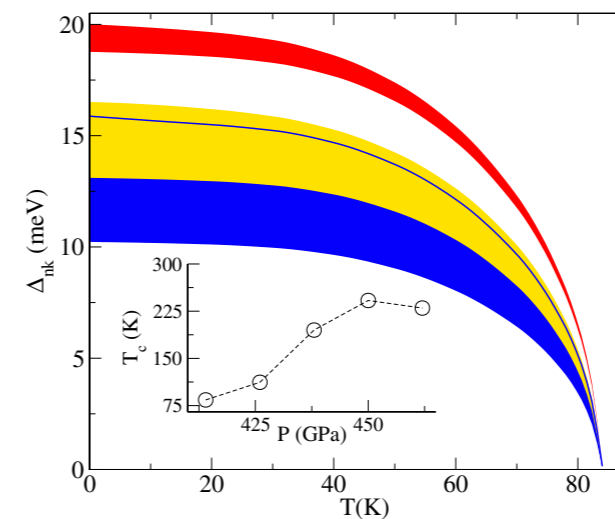
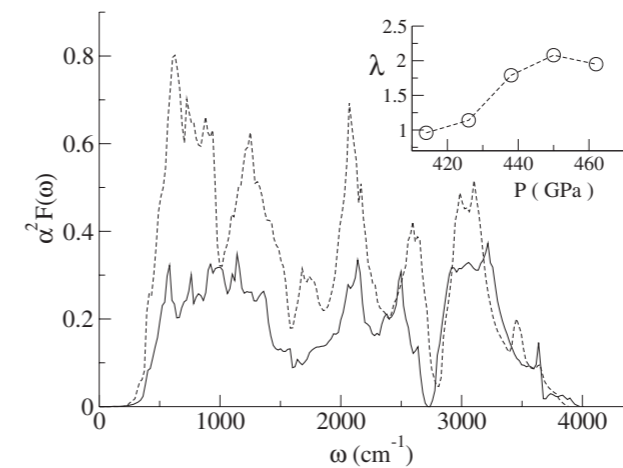
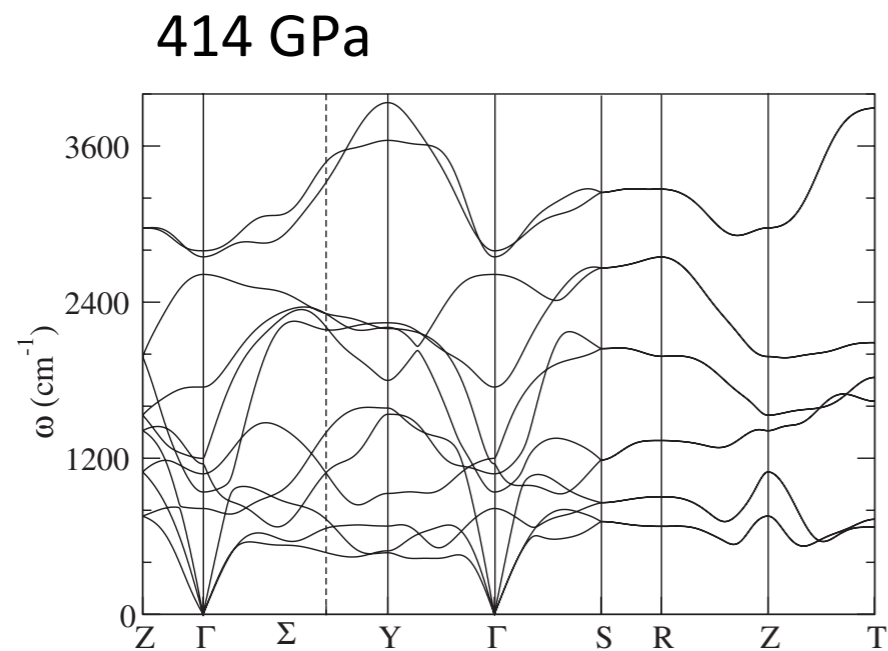
P. Cudazzo,<sup>1</sup> G. Profeta,<sup>1</sup> A. Sanna,<sup>2,3</sup> A. Floris,<sup>3</sup> A. Continenza,<sup>1</sup> S. Massidda,<sup>2</sup> and E. K. U. Gross<sup>3</sup>

<sup>1</sup>CNISM - Dipartimento di Fisica, Università degli Studi dell'Aquila, Via Vetoio 10, I-67010 Coppito (L'Aquila) Italy

<sup>2</sup>SLACS-INFN/CNR—Dipartimento di Fisica, Università degli Studi di Cagliari, I-09124 Monserrato (CA), Italy

<sup>3</sup>Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, D-14195 Berlin, Germany

(Received 7 December 2007; published 23 June 2008; corrected 27 June 2008)



# a sampler of recent applications

PRL 100, 257001 (2008)

PHYSICAL REVIEW LETTERS

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## *Ab Initio* Description of High-Temperature Superconductivity in Dense Molecular Hydrogen

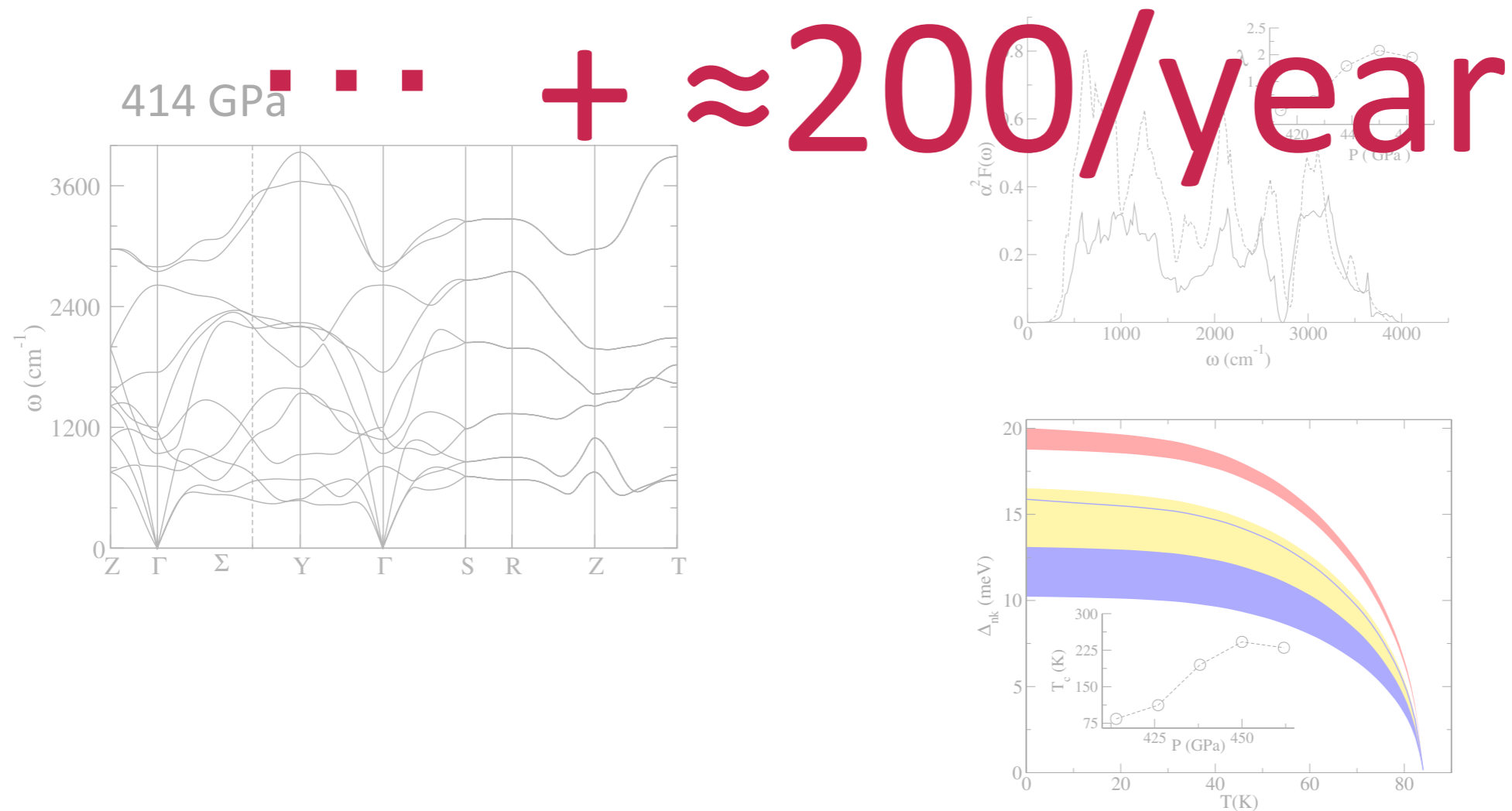
P. Cudazzo,<sup>1</sup> G. Profeta,<sup>1</sup> A. Sanna,<sup>2,3</sup> A. Floris,<sup>3</sup> A. Continenza,<sup>1</sup> S. Massidda,<sup>2</sup> and E. K. U. Gross<sup>3</sup>

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simulating electronic  
charge fluctuations ...

# optical spectra from TDDF (perturbation) T

$$i\frac{\partial\phi_v(\mathbf{r},t)}{\partial t} = (-\Delta + v_{KS}(\mathbf{r},t))\phi_v(\mathbf{r},t)$$

$$v_{KS}(\mathbf{r},t) = v(\mathbf{r},t) + \int \frac{n(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' + v_{xc}[n](\mathbf{r},t)$$

$$n(\mathbf{r},t) = \sum_v |\phi_v(\mathbf{r},t)|^2$$

E. Runge and E.K.U. Gross, Phys. Rev. Lett. **52**, 997 (1984)



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$$v(\mathbf{r},t) \rightarrow n(\mathbf{r},t)$$

$$v_0(\mathbf{r}) - e\mathbf{r} \cdot \mathbf{E}(t) \rightarrow n_0(\mathbf{r}) + n'(\mathbf{r},t)$$

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$$v_0(\mathbf{r}) - e\mathbf{r} \cdot \mathbf{E}(t) \rightarrow n_0(\mathbf{r}) + n'(\mathbf{r},t)$$

$$\mathbf{d}(t) = -e \int \mathbf{r}n'(\mathbf{r},t) d\mathbf{r}$$

$$\alpha(\omega) = \frac{\tilde{\mathbf{d}}(\omega)}{\tilde{\mathbf{E}}(\omega)}$$

# optical spectra from TDDF (perturbation) T

$$i\frac{\partial\phi_v(\mathbf{r},t)}{\partial t} = (-\Delta + v_{KS}(\mathbf{r},t))\phi_v(\mathbf{r},t)$$

$$v_{KS}(\mathbf{r},t) = v(\mathbf{r},t) + \int \frac{n(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' + v_{xc}[n](\mathbf{r},t)$$

$$n(\mathbf{r},t) = \sum_v |\phi_v(\mathbf{r},t)|^2$$

E. Runge and E.K.U. Gross, Phys. Rev. Lett. **52**, 997 (1984)

$$\rho(t) = \sum_v |\phi_v(t)\rangle\langle\phi_v(t)|$$

$$\langle A(t)\rangle = \text{Tr}(\rho(t)A)$$

$$i\dot{\rho}(t) = [H_{KS}(t), \rho(t)]$$

optical spectra from TDDF (perturbation) T

$$i\dot{\rho}(t) = [H_{KS}(t), \rho(t)]$$

# optical spectra from TDDF (perturbation) T

$$i\dot{\rho}(t) = [H_{KS}(t), \rho(t)]$$

$$\begin{aligned}\rho(t) &= \rho^\circ + \rho'(t) \\ H_{KS}(t) &= H^\circ + V'_{ext}(t) + V'_{HXC}(t)\end{aligned}$$

$$i \dot{\rho}' = [H^\circ, \rho'] + [V'_{HXC}, \rho^\circ] + [V'_{ext}, \rho^\circ] + \mathcal{O}(V'^2)$$

# optical spectra from TDDF (perturbation) T

$$i\dot{\rho}(t) = [H_{KS}(t), \rho(t)]$$

$$\begin{aligned}\rho(t) &= \rho^\circ + \rho'(t) \\ H_{KS}(t) &= H^\circ + V'_{ext}(t) + V'_{HXC}(t)\end{aligned}$$

$$i \dot{\rho}' = [H^\circ, \rho'] + [V'_{HXC}(\rho'), \rho^\circ] + [V'_{ext}, \rho^\circ]$$

$$i \dot{\rho}' = \mathcal{L} \rho' + [V'_{ext}, \rho^\circ]$$

# optical spectra from TDDF (perturbation) T

$$i\dot{\rho}(t) = [H_{KS}(t), \rho(t)]$$

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$$i \dot{\rho}' = [H^\circ, \rho'] + [V'_{HXC}(\rho'), \rho^\circ] + [V'_{ext}, \rho^\circ]$$

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\tilde{V}'_{ext}(\omega), \rho^\circ]$$

optical spectra from TDDF (perturbation) T

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\tilde{V}'_{ext}(\omega), \rho^{\circ}]$$



# optical spectra from TDDF (perturbation) T

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\cancel{V'_{ext}(\omega)}, \rho^{\circ}]$$

free oscillations

$$\mathcal{L} \tilde{\rho}' = \omega \tilde{\rho}'$$

excitation energies  
and oscillator strengths

# optical spectra from TDDF (perturbation) T

- 😊 ideally suited to chase after individual excitation energies / oscillator strengths
- 😞 inefficient for large systems and/or in the continuum of even small ones

$$\mathcal{L} \tilde{\rho}' = \omega \tilde{\rho}'$$

Casida's equation

excitation energies  
and oscillator strengths

optical spectra from TDDF (perturbation) T

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\tilde{V}'_{ext}(\omega), \rho^{\circ}]$$

optical spectra from TDDF (perturbation) T

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\tilde{V}'_{ext}(\omega), \rho^{\circ}]$$

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optical spectra from TDDF (perturbation) T

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\tilde{V}'_{ext}(\omega), \rho^{\circ}]$$

$$\begin{aligned}\alpha(\omega) &= \text{Tr}(\mathbf{d}\tilde{\rho}'(\omega)) \\ &= (\mathbf{d}, (\omega - \mathcal{L})^{-1} \cdot [\tilde{V}'_{ext}(\omega), \rho^{\circ}])\end{aligned}$$

# optical spectra from TDDF (perturbation) T

$$(\omega - \mathcal{L})\tilde{\rho}'(\omega) = [\tilde{V}'_{ext}(\omega), \rho^{\circ}]$$

$$\begin{aligned}\alpha(\omega) &= \text{Tr}(\mathbf{d}\tilde{\rho}'(\omega)) \\ &= (\mathbf{d}, (\omega - \mathcal{L})^{-1} \cdot [\tilde{V}'_{ext}(\omega), \rho^{\circ}]) \\ &\equiv (u, (\omega - \mathcal{L})^{-1} \cdot v)\end{aligned}$$

# the Lanczos connection

$$g(\omega) = \langle \phi_0 | (\omega - \mathcal{H})^{-1} | \phi_0 \rangle$$

# the Lanczos connection

$$g(\omega) = \langle \phi_0 | (\omega - \mathcal{H})^{-1} | \phi_0 \rangle$$

J. Phys. C: Solid State Phys., Vol. 5, 1972. Printed in Great Britain. © 1972

## **Electronic structure based on the local atomic environment for tight-binding bands**

R HAYDOCK, VOLKER HEINE and M J KELLY

Cavendish Laboratory, Cambridge, UK



# the Lanczos connection

$$g(\omega) = \langle \phi_0 | (\omega - \mathcal{H})^{-1} | \phi_0 \rangle$$

$$\phi_{-1} = 0$$

$$b_{n+1} \phi_{n+1} = (\mathcal{H} - a_n) \phi_n - b_n \phi_{n-1}$$

$$\langle \phi_{n+1} | \phi_{n+1} \rangle = 1$$

$$a_n = \langle \phi_n | \mathcal{H} | \phi_n \rangle$$

# the Lanczos connection

$$g(\omega) = \langle \phi_0 | (\omega - \mathcal{H})^{-1} | \phi_0 \rangle$$

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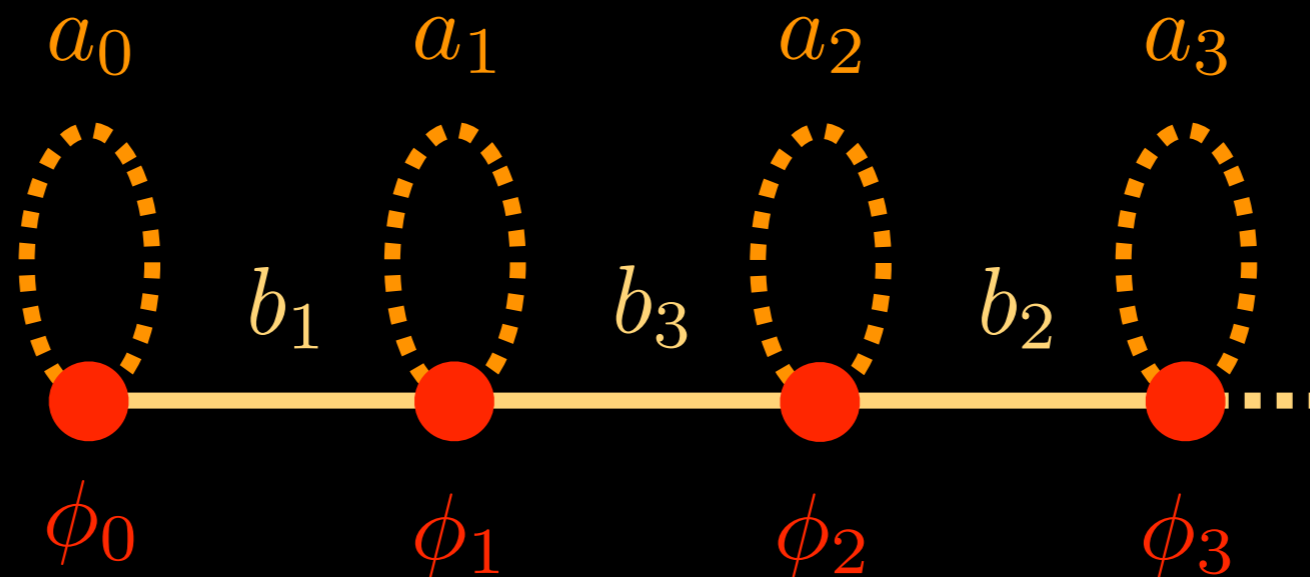
$$\langle \phi_{n+1} | \phi_{n+1} \rangle = 1$$

$$a_n = \langle \phi_n | \mathcal{H} | \phi_n \rangle$$

$$\mathcal{H} = \begin{pmatrix} a_0 & b_1 & 0 & \cdots & 0 \\ b_1 & a_1 & b_2 & 0 & \vdots \\ 0 & b_2 & a_2 & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & b_n \\ 0 & \cdots & 0 & b_n & a_n \end{pmatrix}$$

# the Lanczos connection

$$g(\omega) = \langle \phi_0 | (\omega - \mathcal{H})^{-1} | \phi_0 \rangle$$

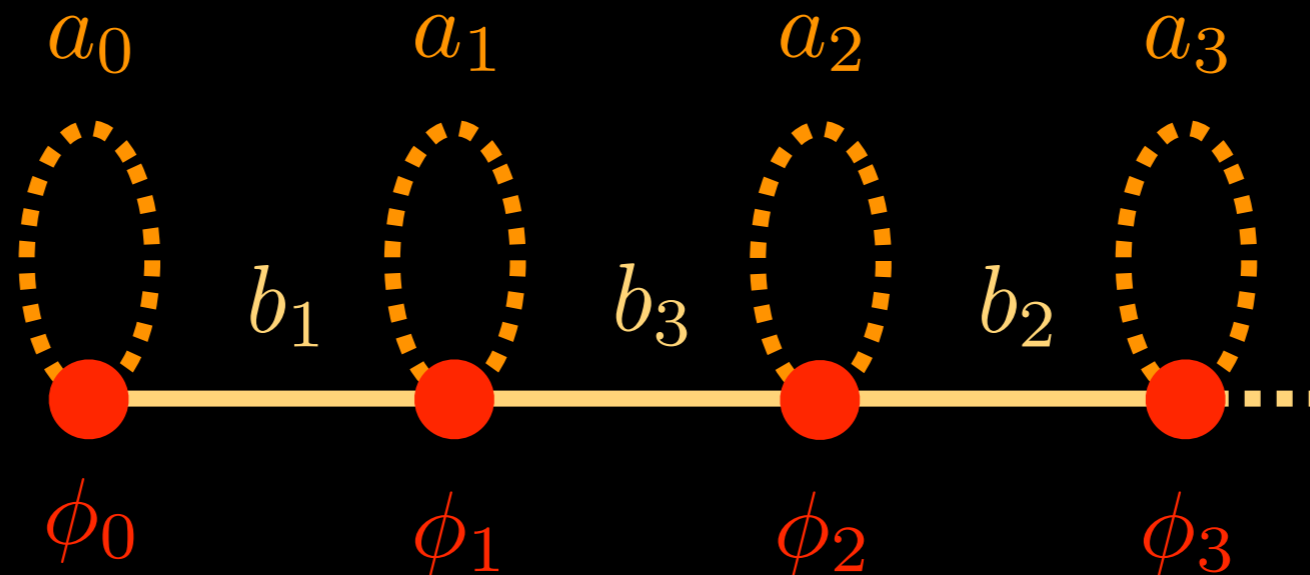


$$\mathcal{H} = \begin{pmatrix} a_0 & b_1 & 0 & \dots & 0 \\ b_1 & a_1 & b_2 & 0 & \vdots \\ 0 & b_2 & a_2 & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & b_n \\ 0 & \dots & 0 & b_n & a_n \end{pmatrix}$$

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$$g(\omega) = \langle \phi_0 | (\omega - \mathcal{H})^{-1} | \phi_0 \rangle$$

$$\mathcal{H} = \begin{pmatrix} a_0 & b_1 & 0 & \cdots & 0 \\ b_1 & a_1 & b_2 & 0 & \vdots \\ 0 & b_2 & a_2 & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & b_n \\ 0 & \cdots & 0 & b_n & a_n \end{pmatrix}$$



$$g(\omega) = \frac{1}{\omega - a_0 + \frac{b_1^2}{\omega - a_1 + \frac{b_2^2}{\omega - a_2 + \cdots}}}$$

the DFPT representation

$$\tilde{\rho}'(\omega) = \begin{pmatrix} 0 & Y^\dagger \\ X & 0 \end{pmatrix} \begin{matrix} \mathbf{v} \\ \mathbf{c} \end{matrix}$$

# the DFPT representation

$$\tilde{\rho}'(\omega) = \sum_{cv} \left( X_{cv}(\omega) |\varphi_c^\circ\rangle \langle \varphi_v^\circ| + Y_{cv}(\omega) |\varphi_v^\circ\rangle \langle \varphi_c^\circ| \right)$$

# the DFPT representation

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$$|\{x_v(\mathbf{r})\}, \{y_v(\mathbf{r})\}\rangle$$

$$P_v x_v = P_v y_v = 0$$



# the DFPT representation

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$$\begin{array}{l} \mathcal{L} \tilde{\rho}' \\ \mathcal{L}^\top \tilde{\rho}' \end{array} \Rightarrow \begin{array}{l} \{H^\circ x_v(\mathbf{r})\} \\ \{H^\circ y_v(\mathbf{r})\} \end{array} \quad \& \quad \{V'_{ee}(\mathbf{r}) \varphi_v^\circ(\mathbf{r})\}$$

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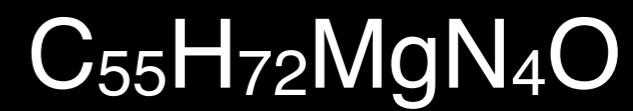
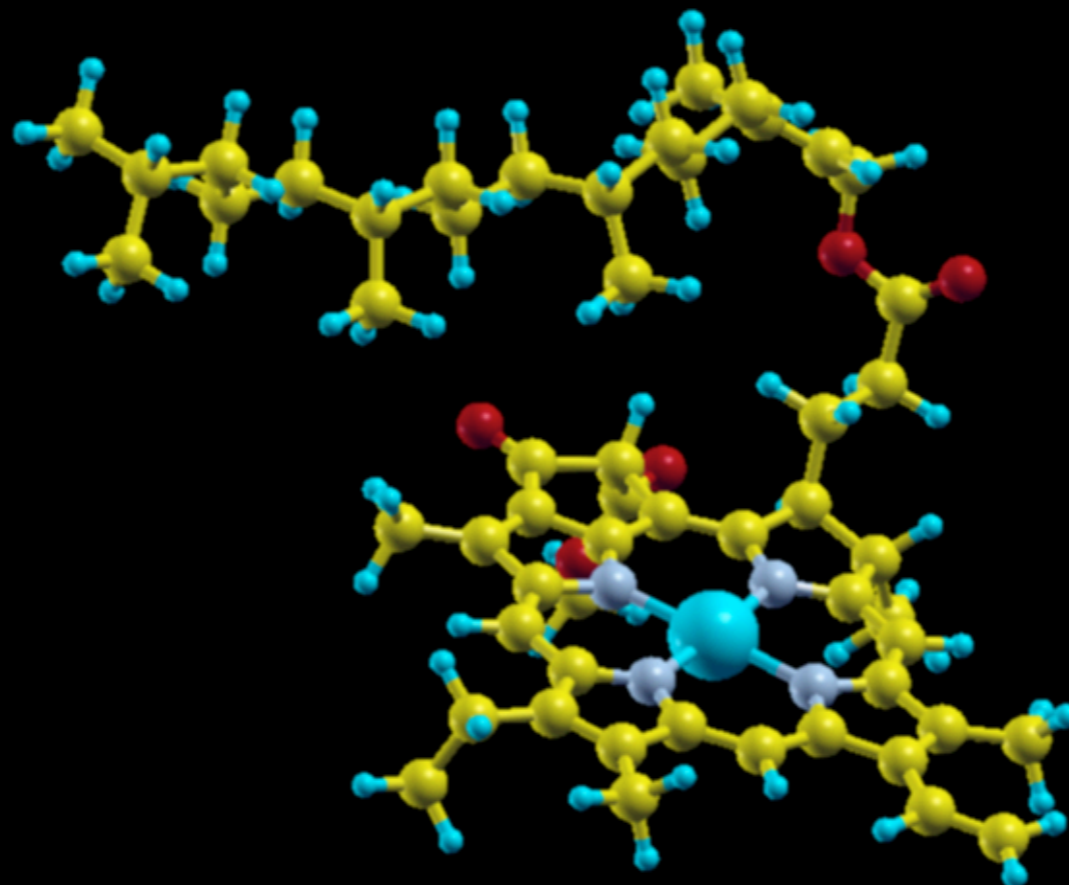
$$|\{x_v(\mathbf{r})\}, \{y_v(\mathbf{r})\}\rangle$$

$$P_v x_v = P_v y_v = 0$$

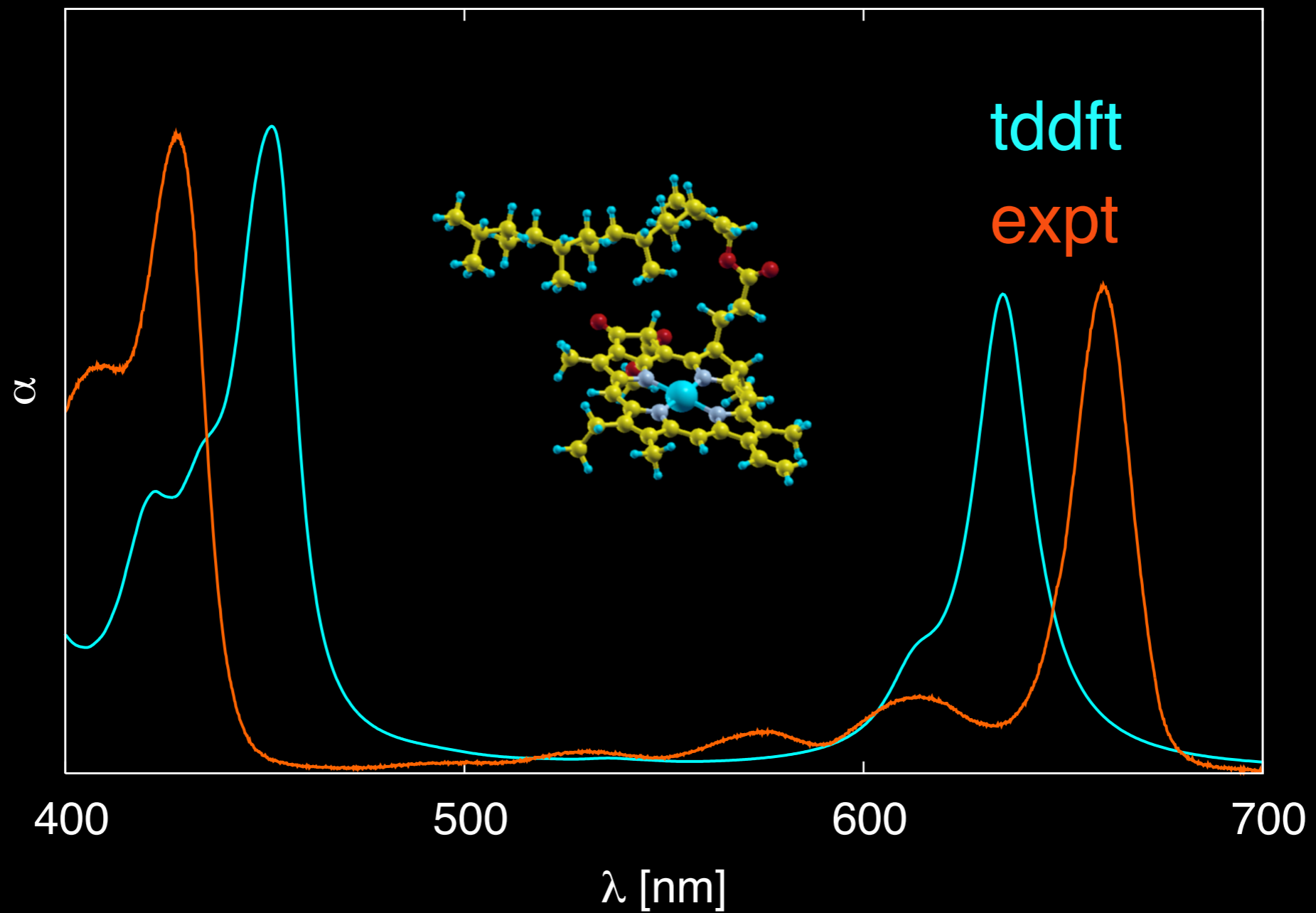
$$\begin{array}{l} \mathcal{L} \tilde{\rho}' \\ \mathcal{L}^\top \tilde{\rho}' \end{array} \Rightarrow \begin{array}{l} \{H^\circ x_v(\mathbf{r})\} \\ \{H^\circ y_v(\mathbf{r})\} \end{array} \quad \& \quad \{V'_{ee}(\mathbf{r}) \varphi_v^\circ(\mathbf{r})\}$$

$$n'(\mathbf{r}) = \frac{1}{2} \sum_v (x_v(\mathbf{r}) + y_v(\mathbf{r})) \varphi_v^\circ(\mathbf{r})$$

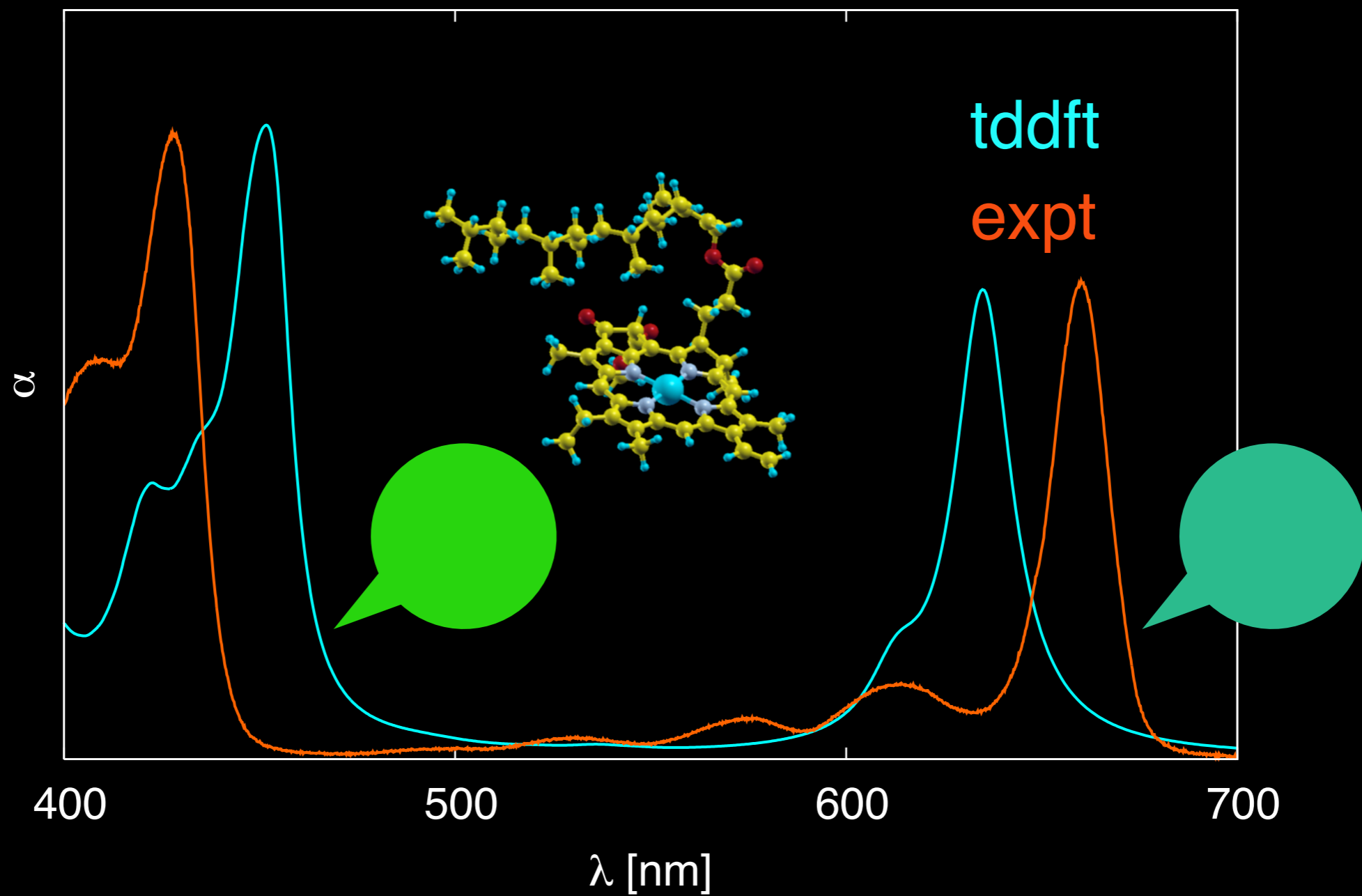
# chlorofyll a



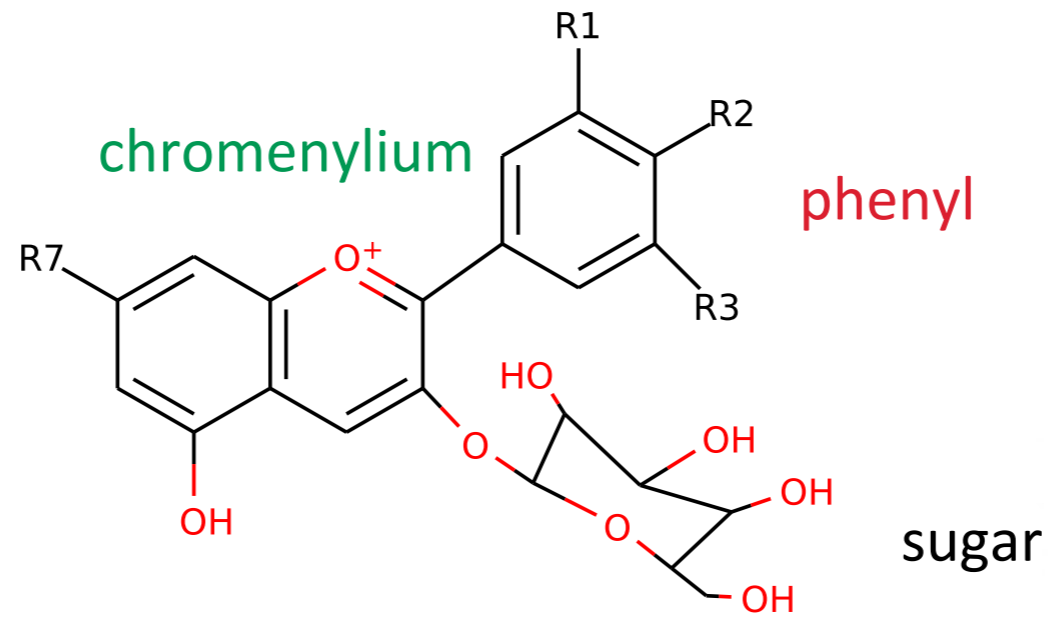
# chlorofyll a

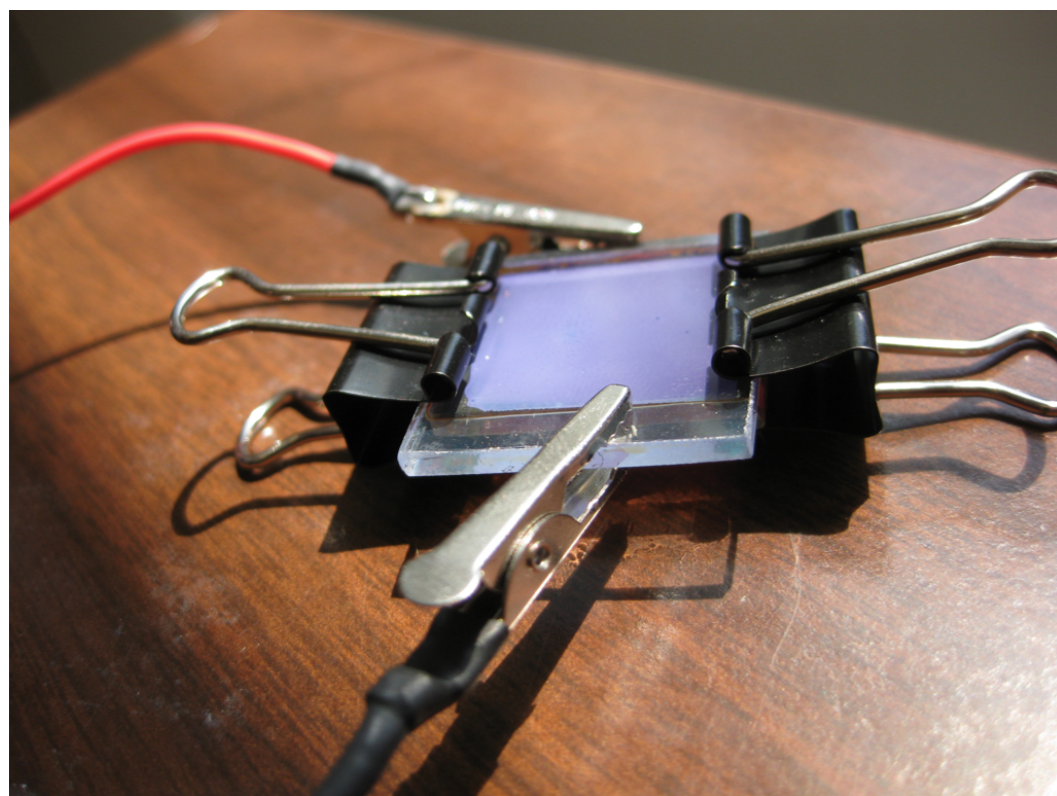


# chlorofyll a



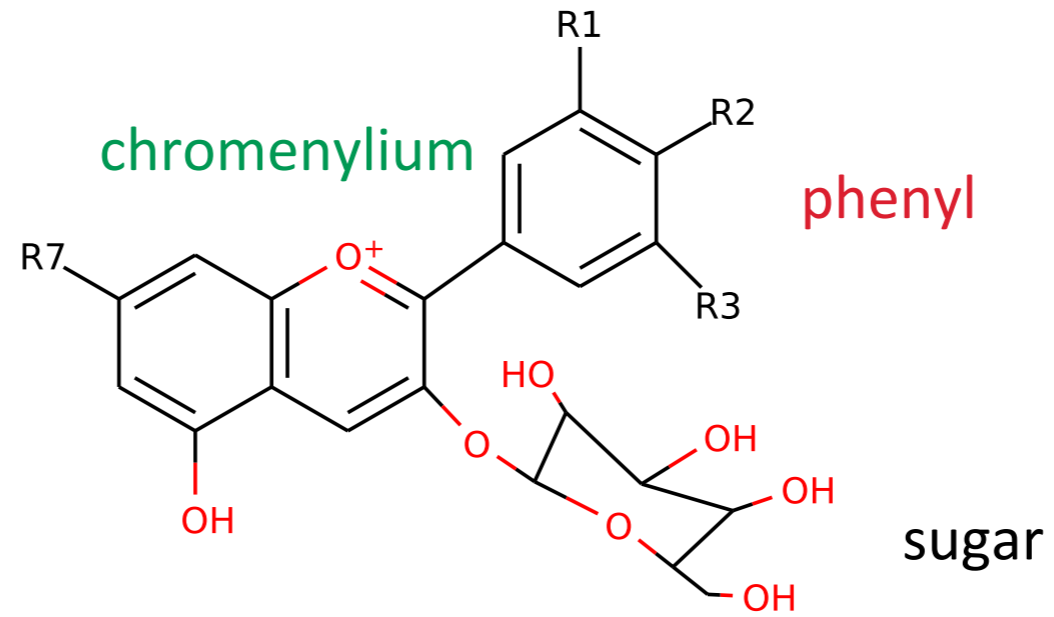
# anthocyanins





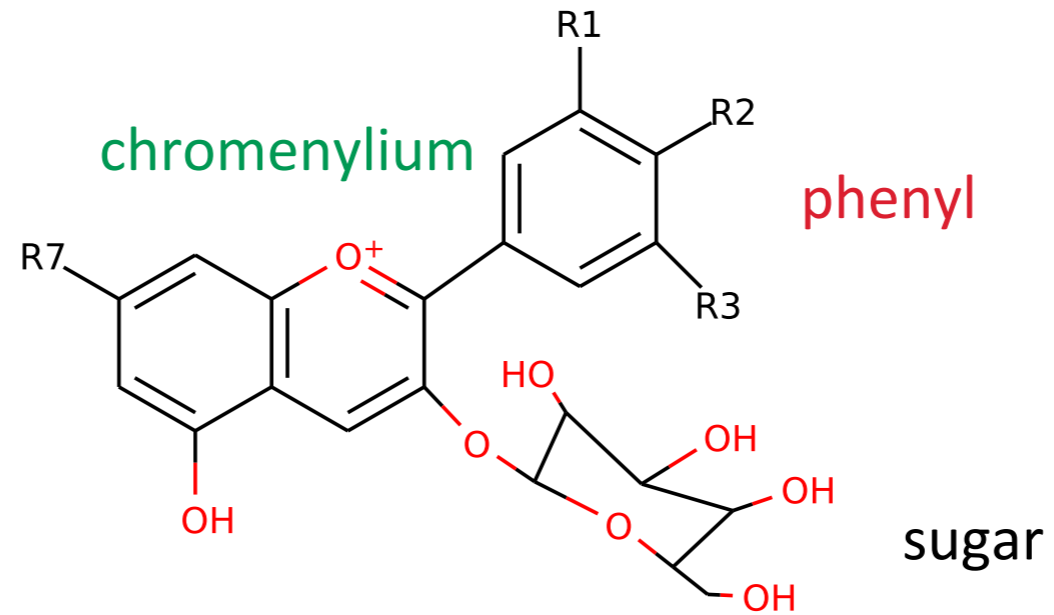
N.J. Cherepy, G.P Smestad, M. Grätzel, and J.Z Zhang, J. Phys. Chem. B **101**, 9342 (1997)

# anthocyanins





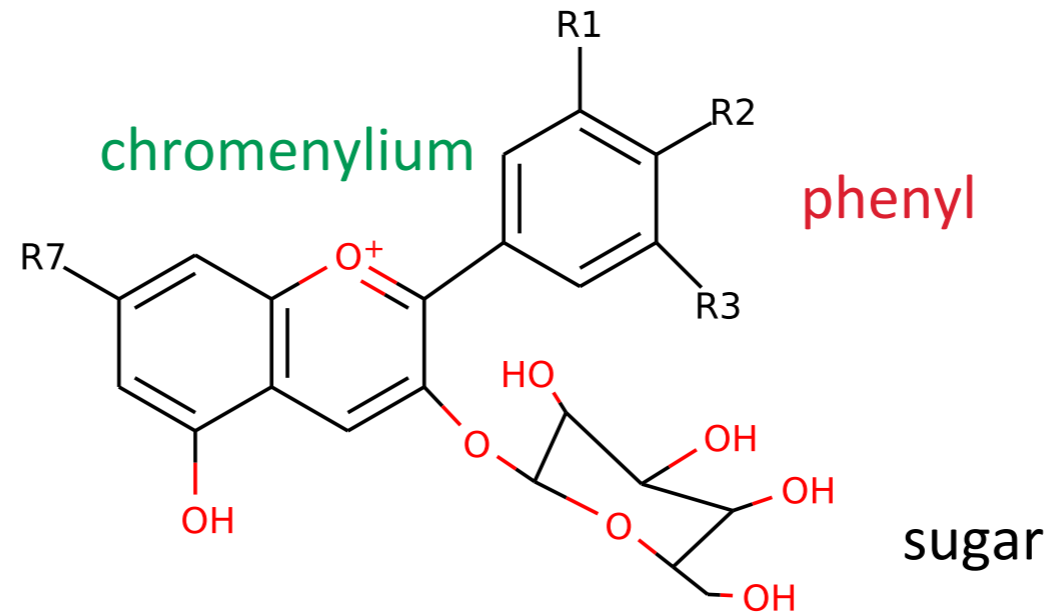
# anthocyanins



| anthocyanin | R1  | R2  | R3 | R7  |
|-------------|-----|-----|----|-----|
| cyanin      | -OH | -OH | -H | -OH |



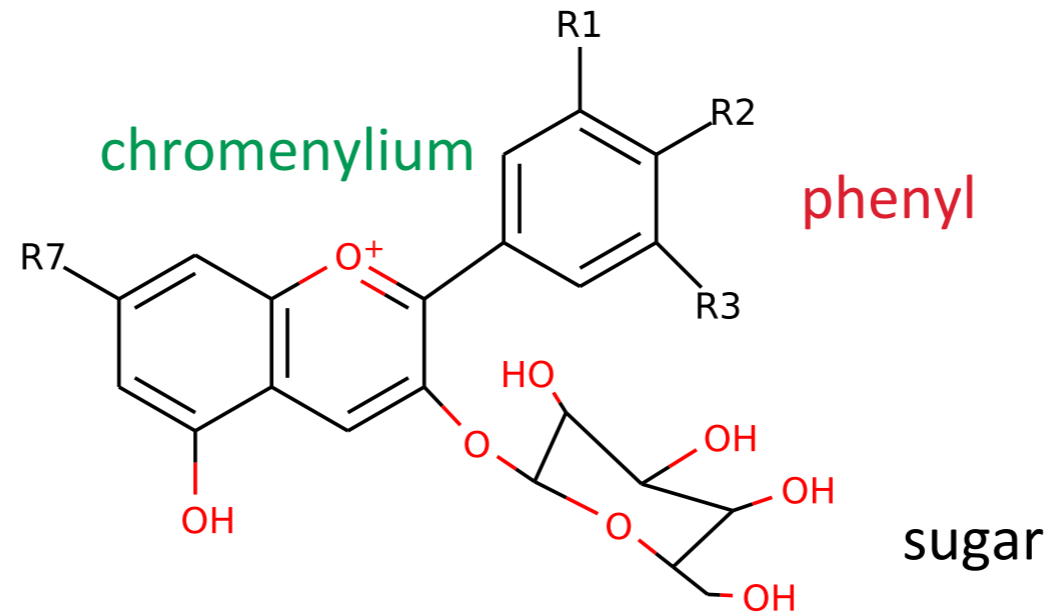
# anthocyanins



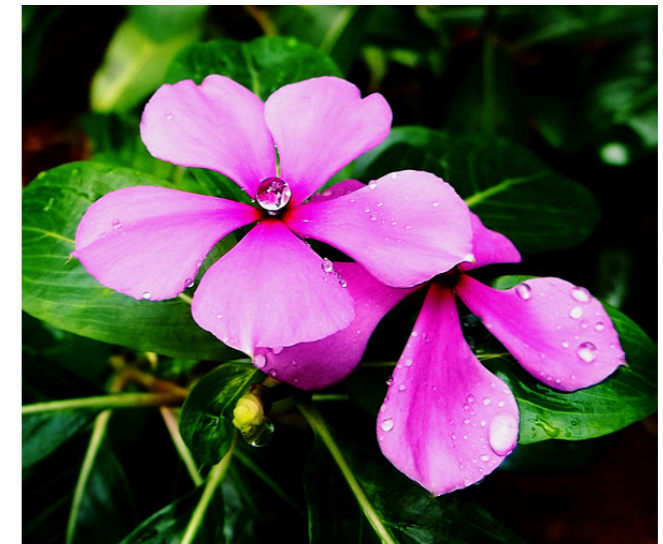
| anthocyanin | R1                | R2  | R3 | R7  |
|-------------|-------------------|-----|----|-----|
| cyanin      | -OH               | -OH | -H | -OH |
| peonin      | -OCH <sub>3</sub> | -OH | -H | -OH |



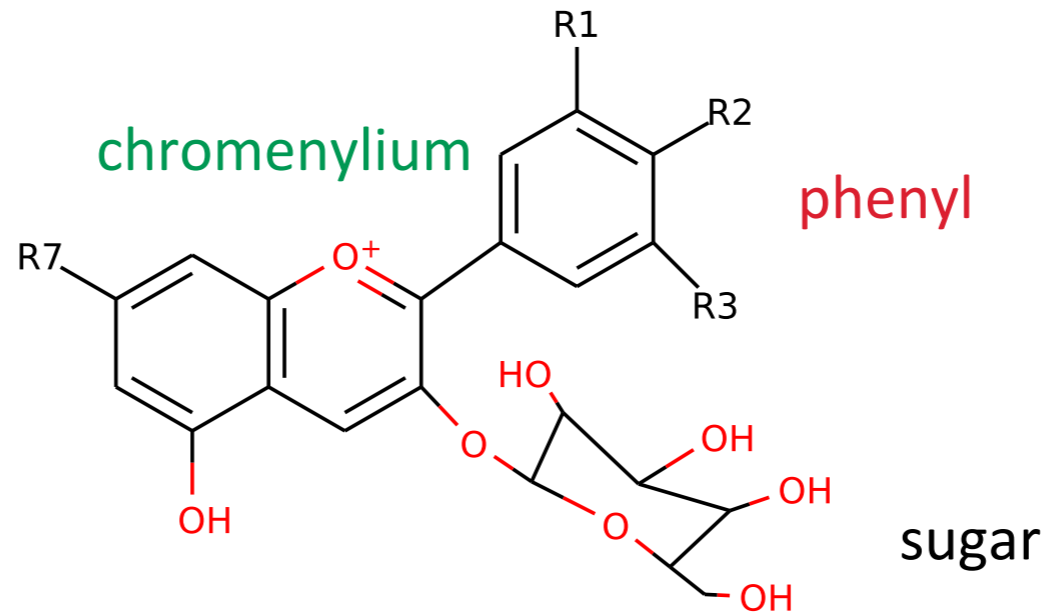
# anthocyanins



| anthocyanin | R1                | R2  | R3 | R7                |
|-------------|-------------------|-----|----|-------------------|
| cyanin      | -OH               | -OH | -H | -OH               |
| peonin      | -OCH <sub>3</sub> | -OH | -H | -OH               |
| rosinin     | -OH               | -OH | -H | -OCH <sub>3</sub> |



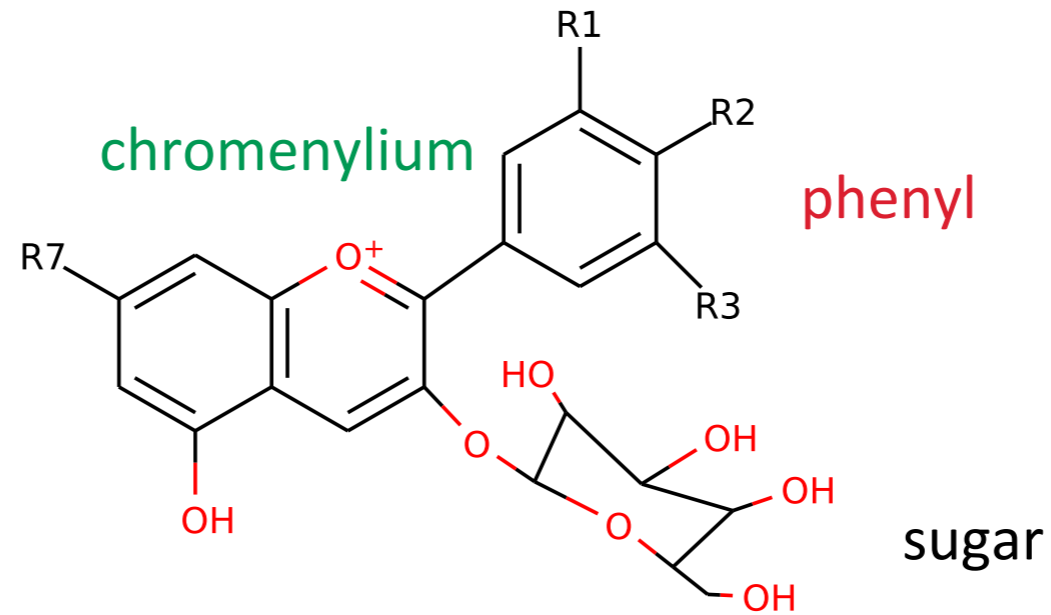
# anthocyanins



| anthocyanin | R1                | R2  | R3                | R7                |
|-------------|-------------------|-----|-------------------|-------------------|
| cyanin      | -OH               | -OH | -H                | -OH               |
| peonin      | -OCH <sub>3</sub> | -OH | -H                | -OH               |
| rosinin     | -OH               | -OH | -H                | -OCH <sub>3</sub> |
| malvin      | -OCH <sub>3</sub> | -OH | -OCH <sub>3</sub> | -OH               |



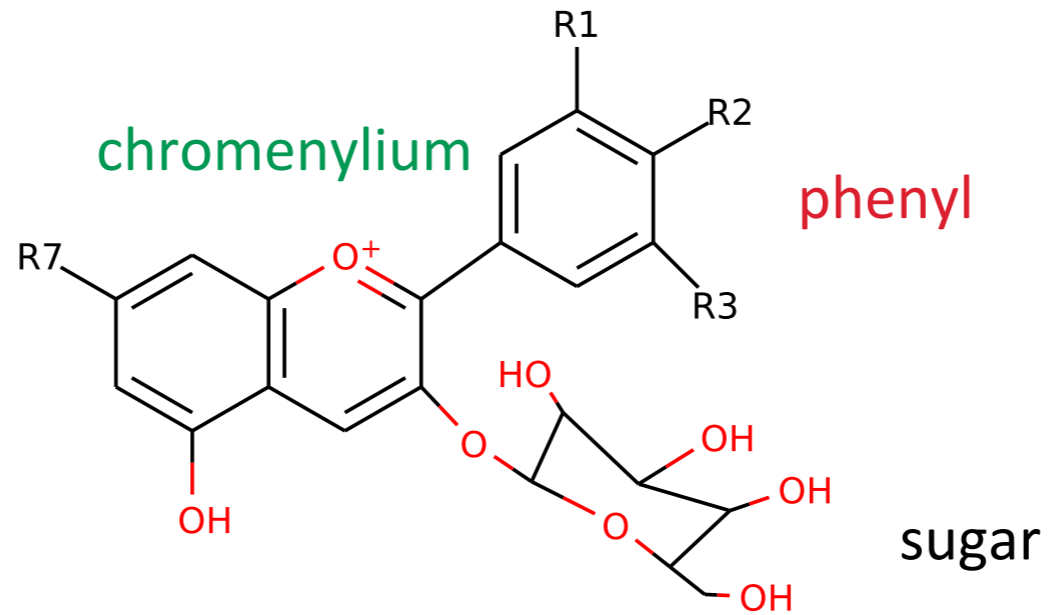
# anthocyanins



| anthocyanin | R1                | R2  | R3                | R7                |
|-------------|-------------------|-----|-------------------|-------------------|
| cyanin      | -OH               | -OH | -H                | -OH               |
| peonin      | -OCH <sub>3</sub> | -OH | -H                | -OH               |
| rosinin     | -OH               | -OH | -H                | -OCH <sub>3</sub> |
| malvin      | -OCH <sub>3</sub> | -OH | -OCH <sub>3</sub> | -OH               |
| delphinin   | -OH               | -OH | -OCH <sub>3</sub> | -OH               |



# anthocyanins



| anthocyanin | R1                | R2  | R3                | R7                |
|-------------|-------------------|-----|-------------------|-------------------|
| cyanin      | -OH               | -OH | -H                | -OH               |
| peonin      | -OCH <sub>3</sub> | -OH | -H                | -OH               |
| rosinin     | -OH               | -OH | -H                | -OCH <sub>3</sub> |
| malvin      | -OCH <sub>3</sub> | -OH | -OCH <sub>3</sub> | -OH               |
| delphinin   | -OH               | -OH | -OCH <sub>3</sub> | -OH               |
| pelargonin  | -H                | -OH | -OH               | -OH               |



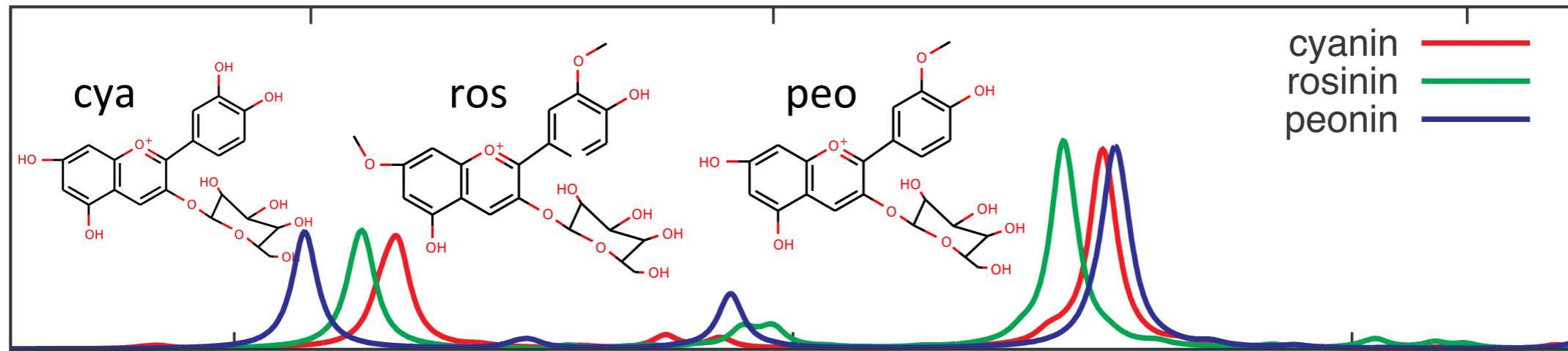
# spectrum of anthocyanins

Wavelength (nm)

600

500

400



Cya

Ros

Peo

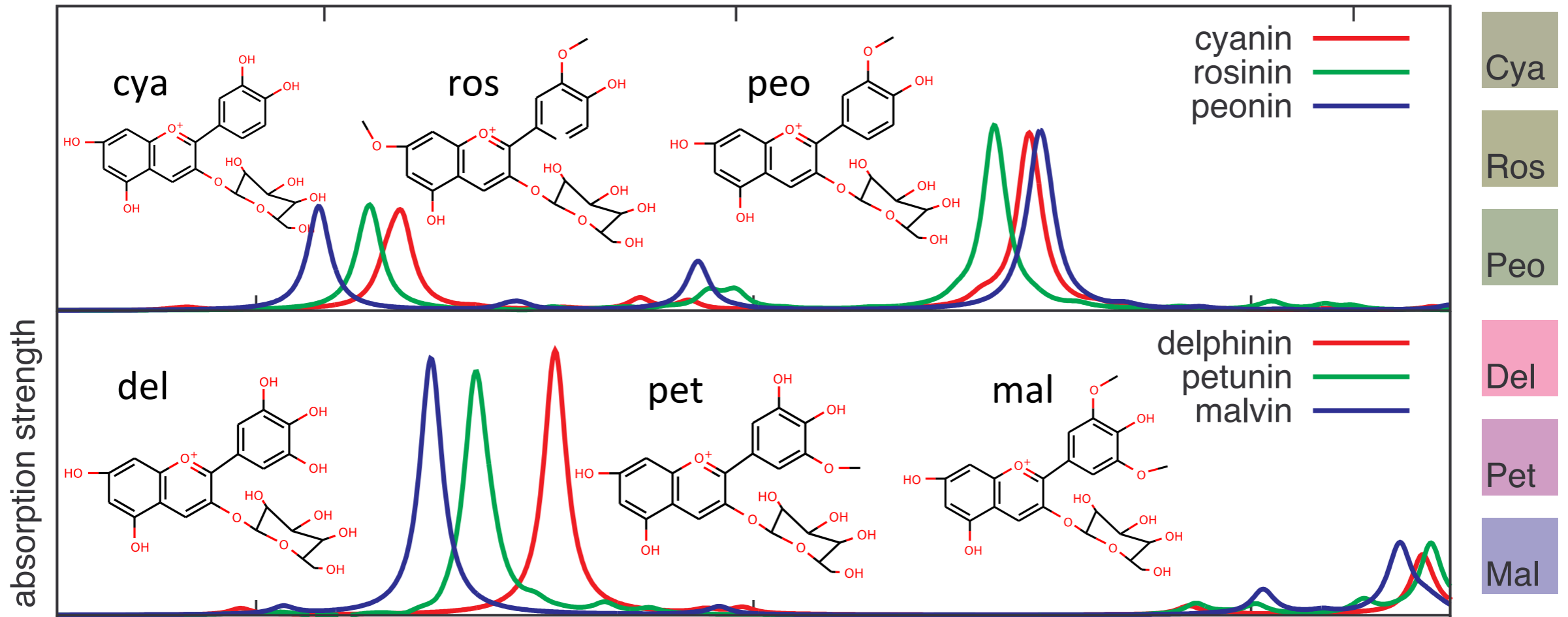
# spectrum of anthocyanins

Wavelength (nm)

600

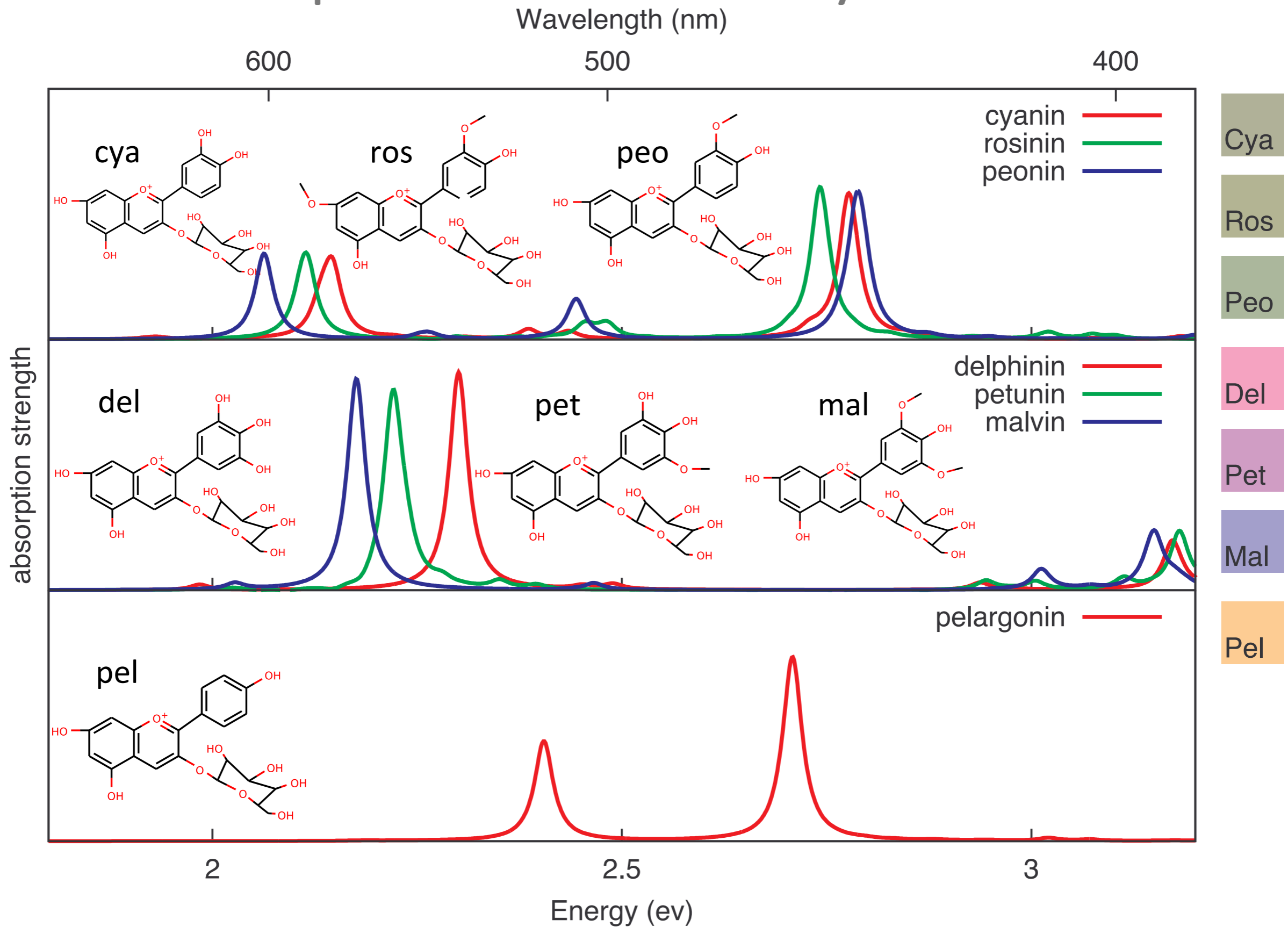
500

400





# spectrum of anthocyanins



X. Ge, S. Binnie, A. Calzolari, and SB, in preparation

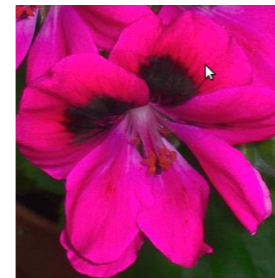
# spectrum of anthocyanins

delphinin



delphinium

pelargonin



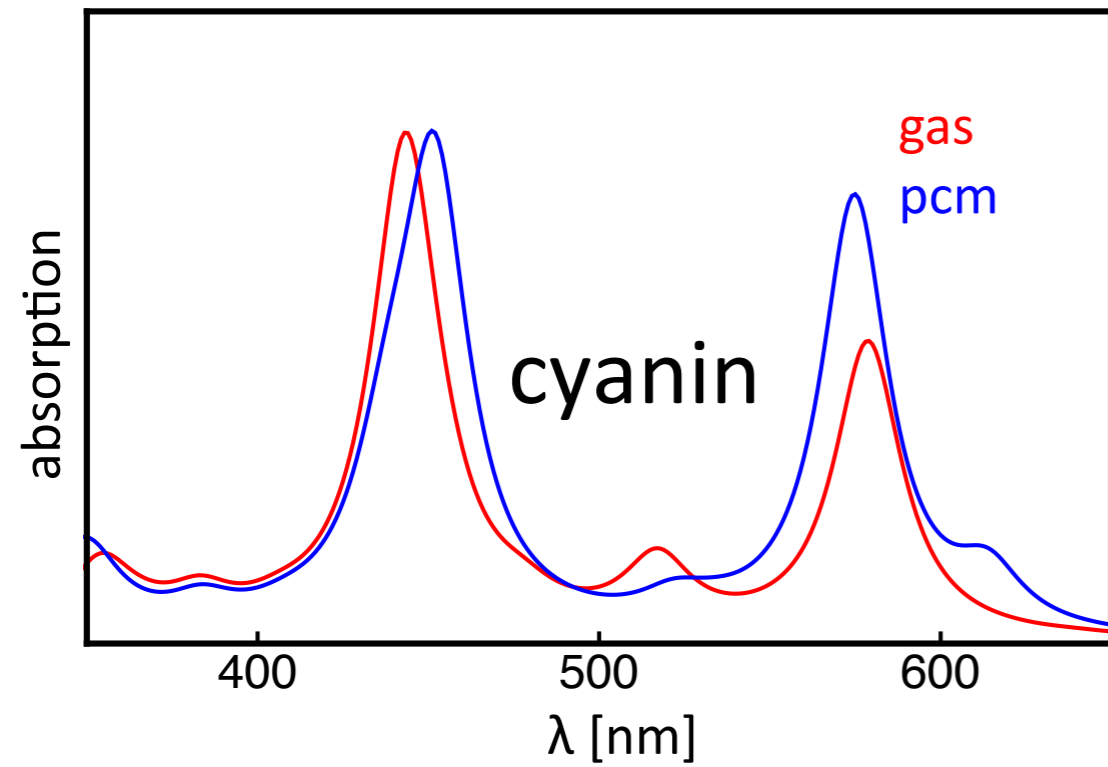
pelargonium

cyanin

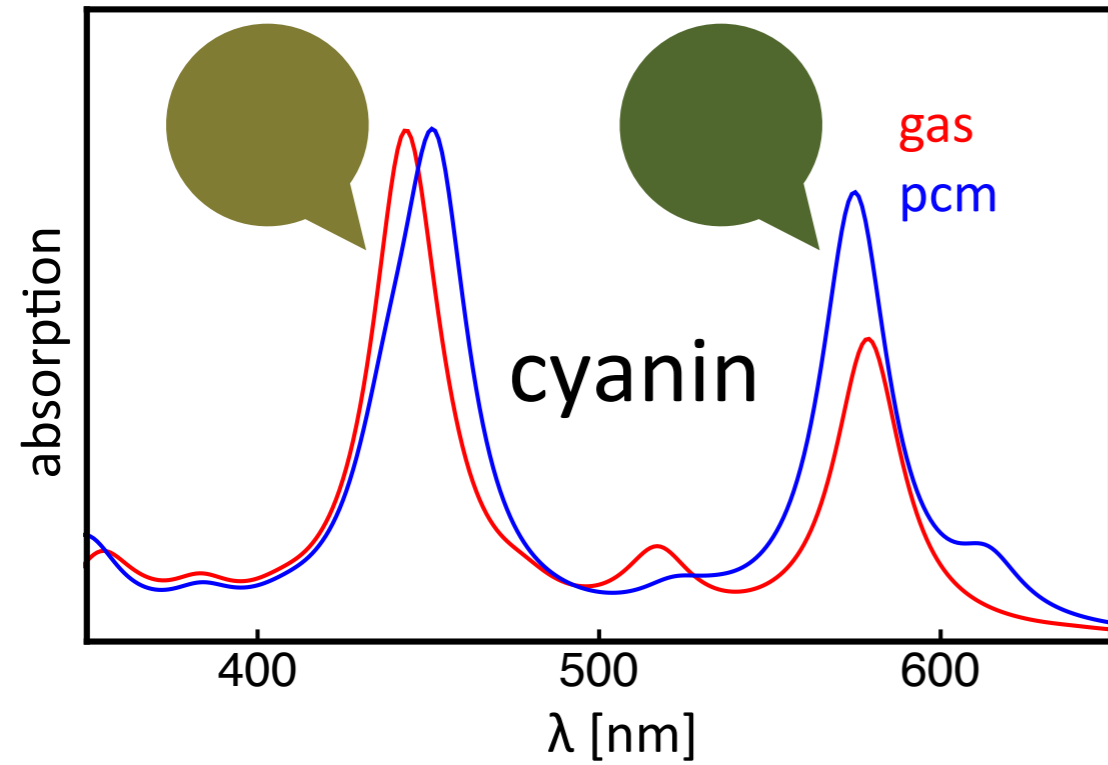


blueberry

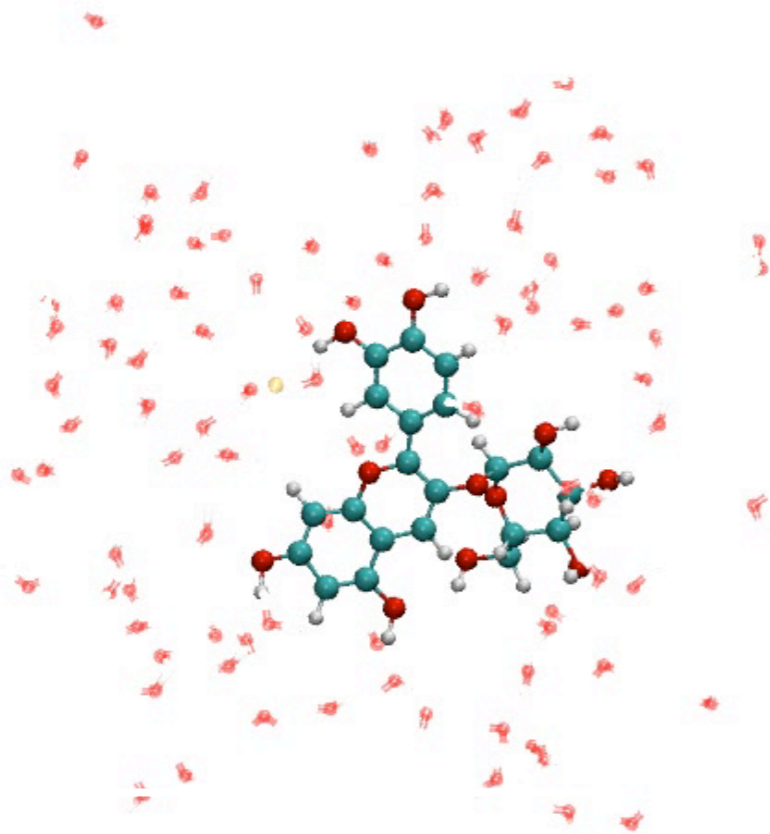
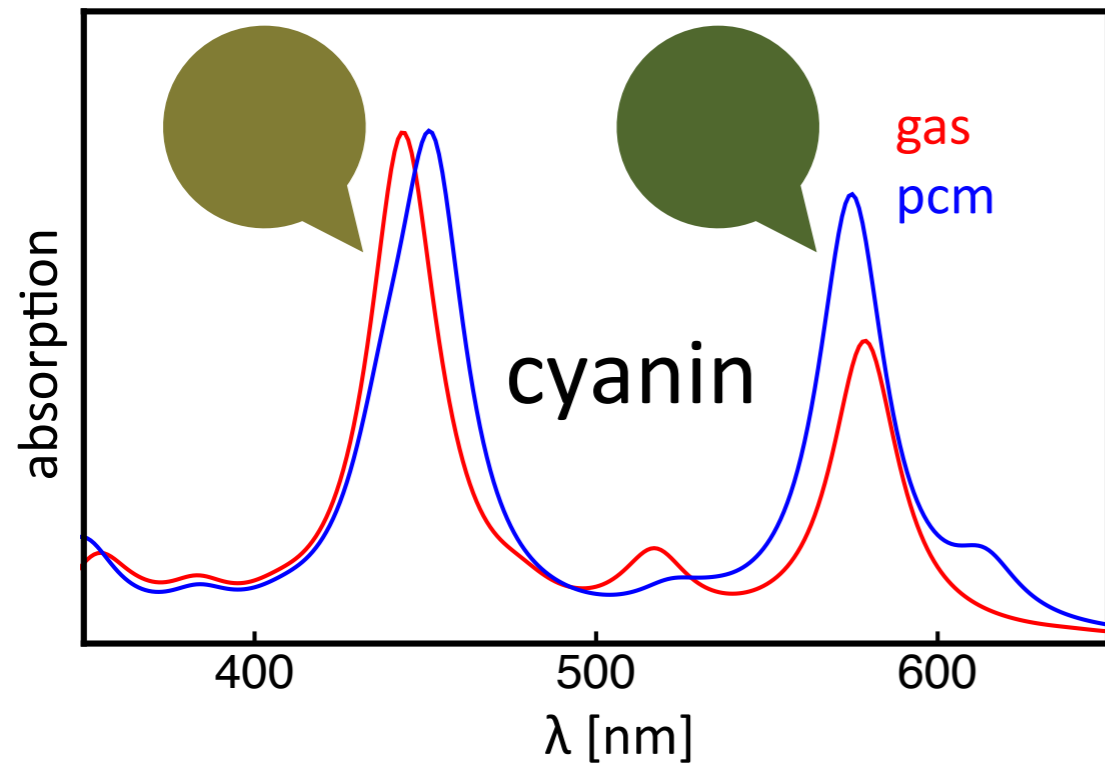
# optical effect of the solvent



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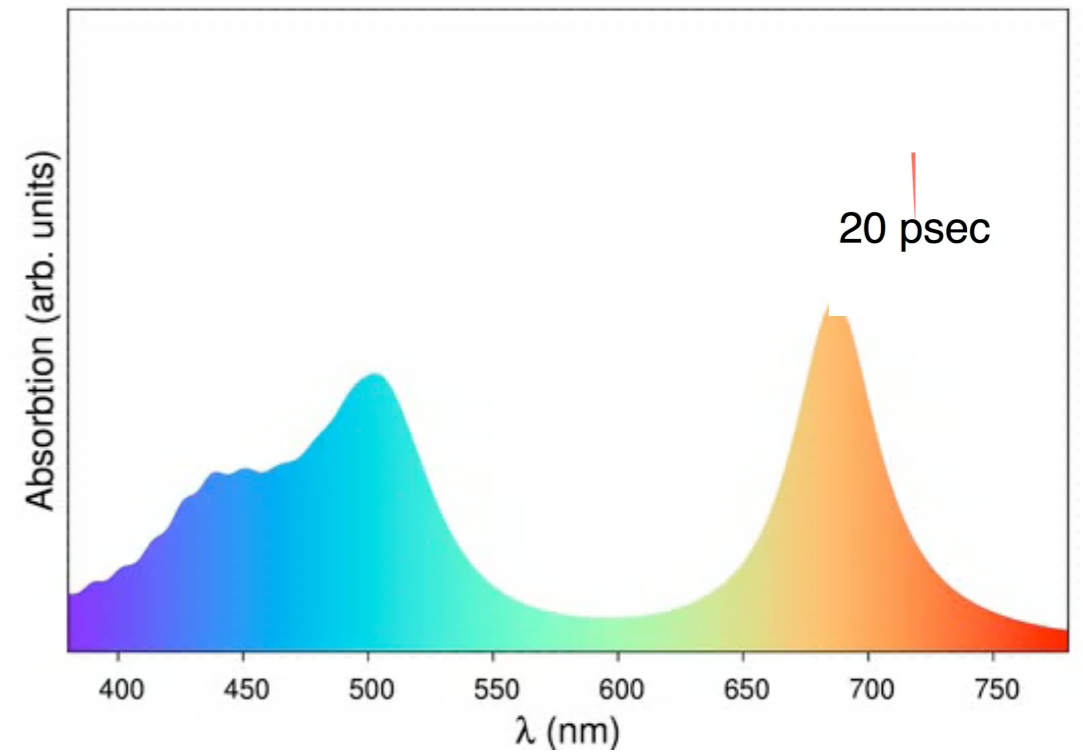
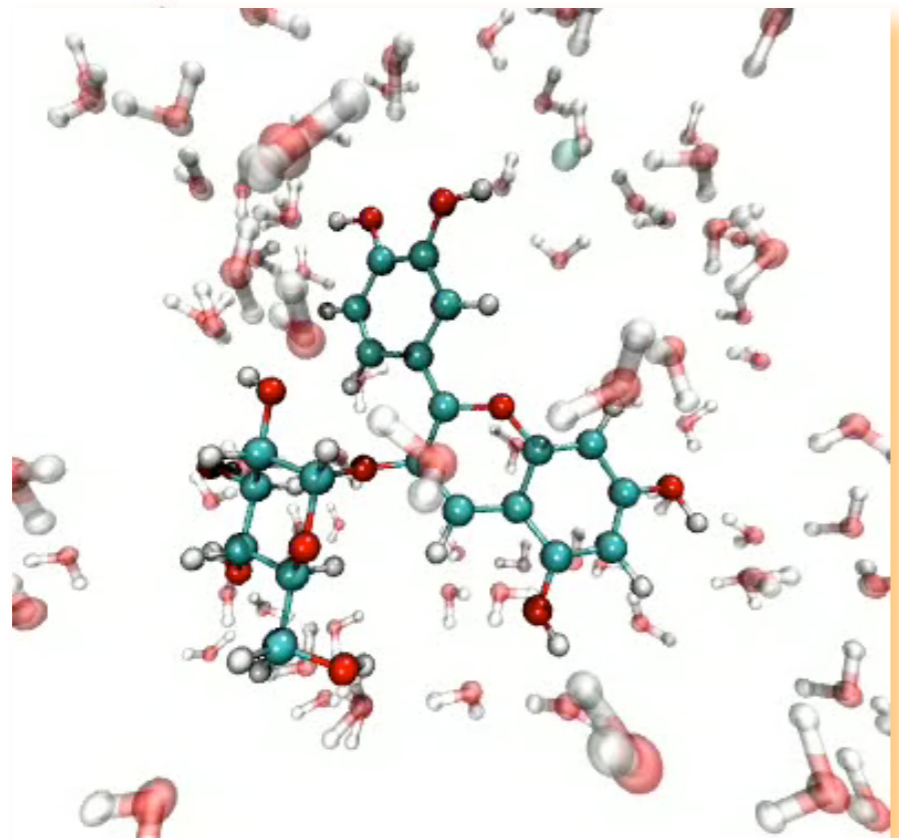
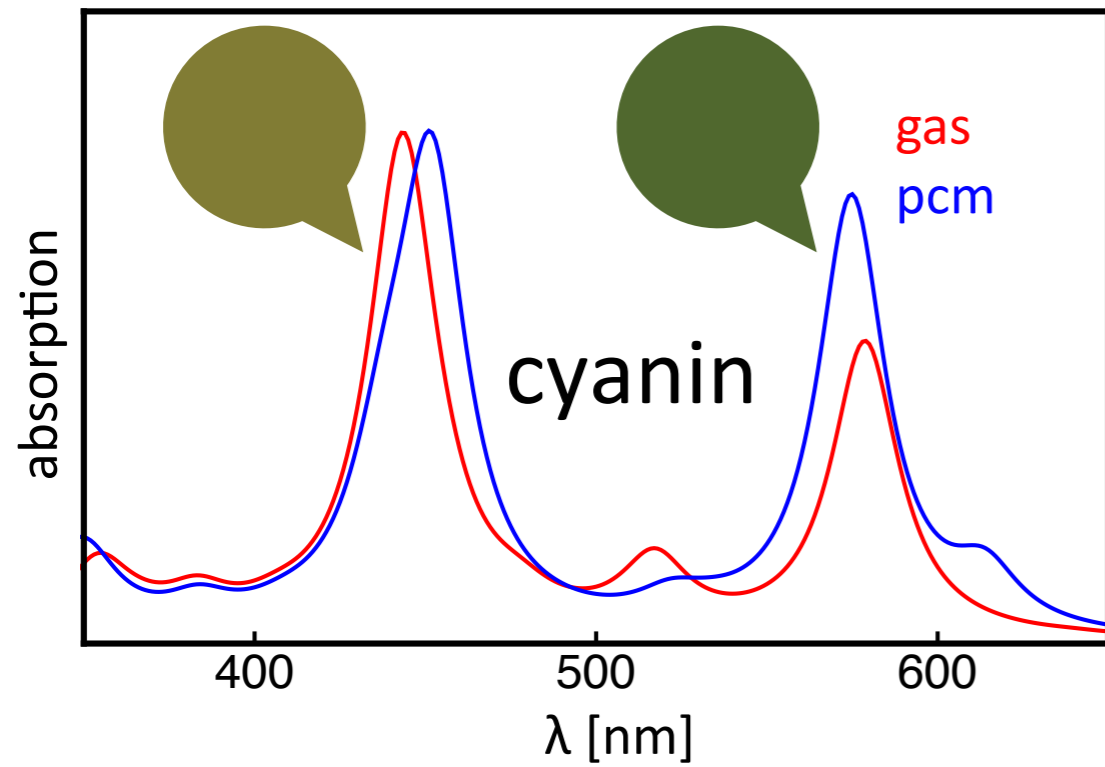


$C_{21}H_{21}O_{11}Cl@(H_2O)_{95}$

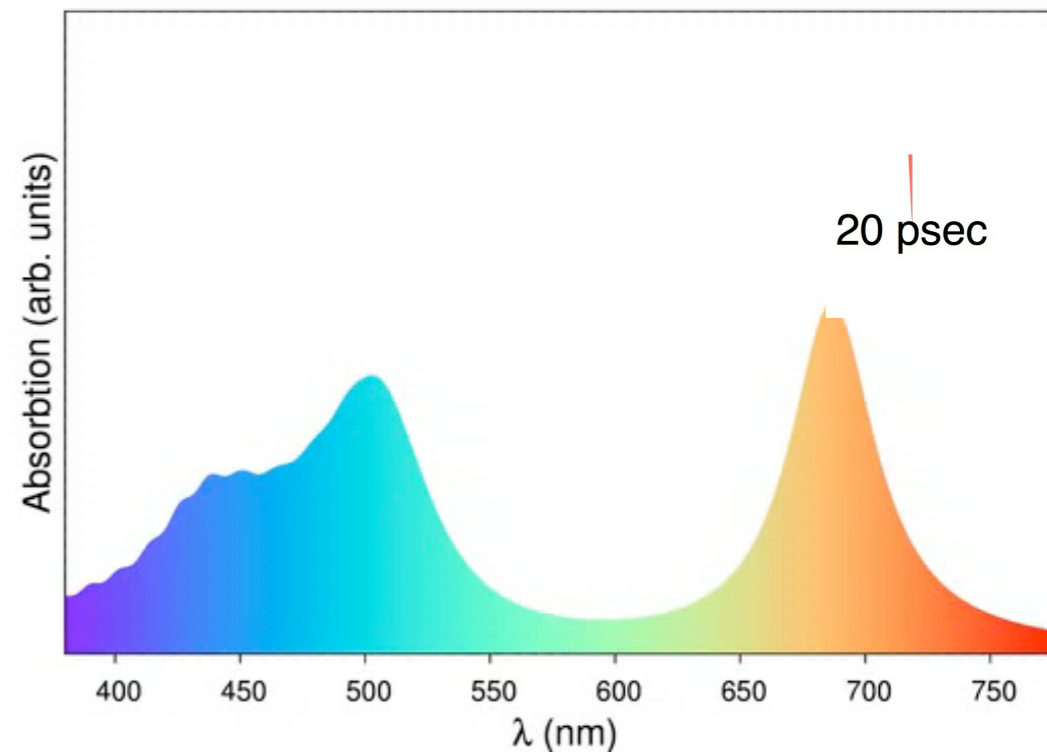
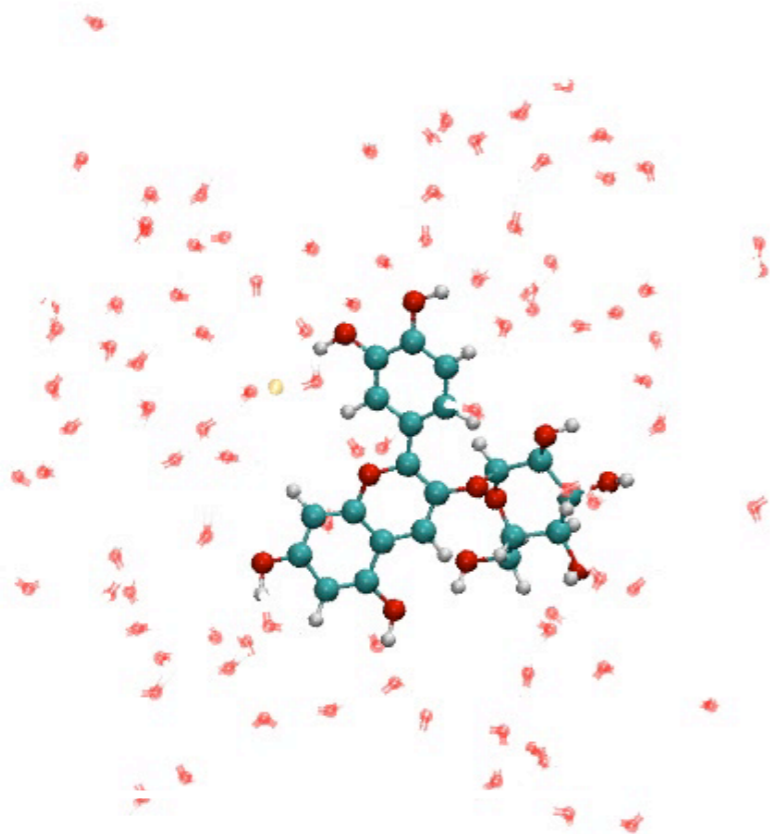
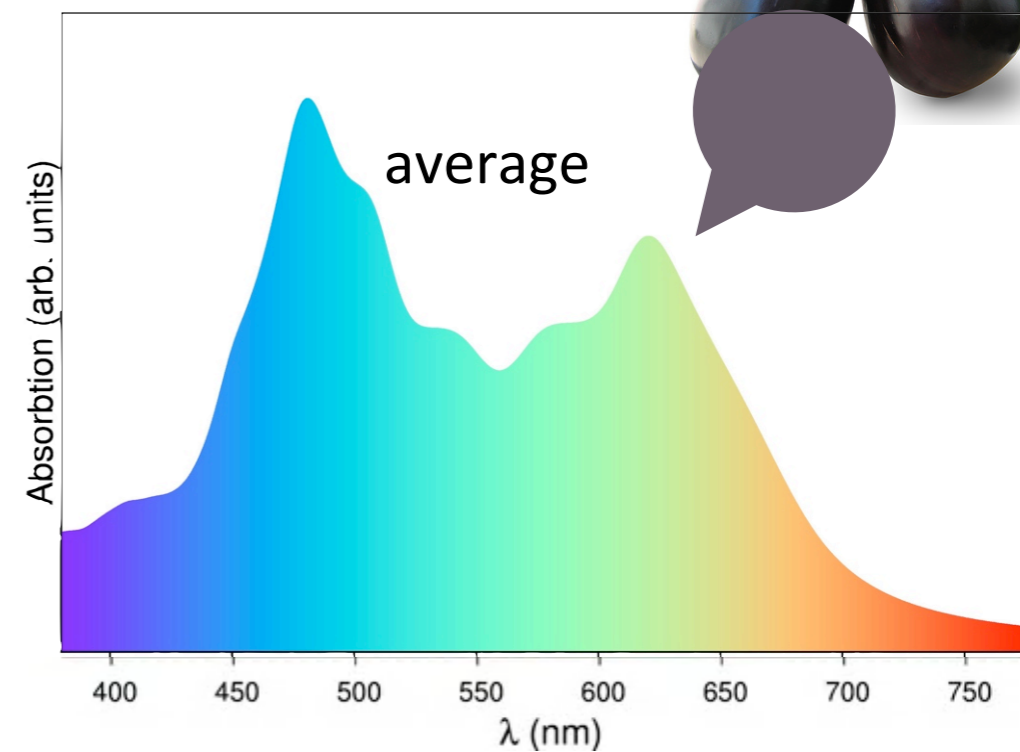
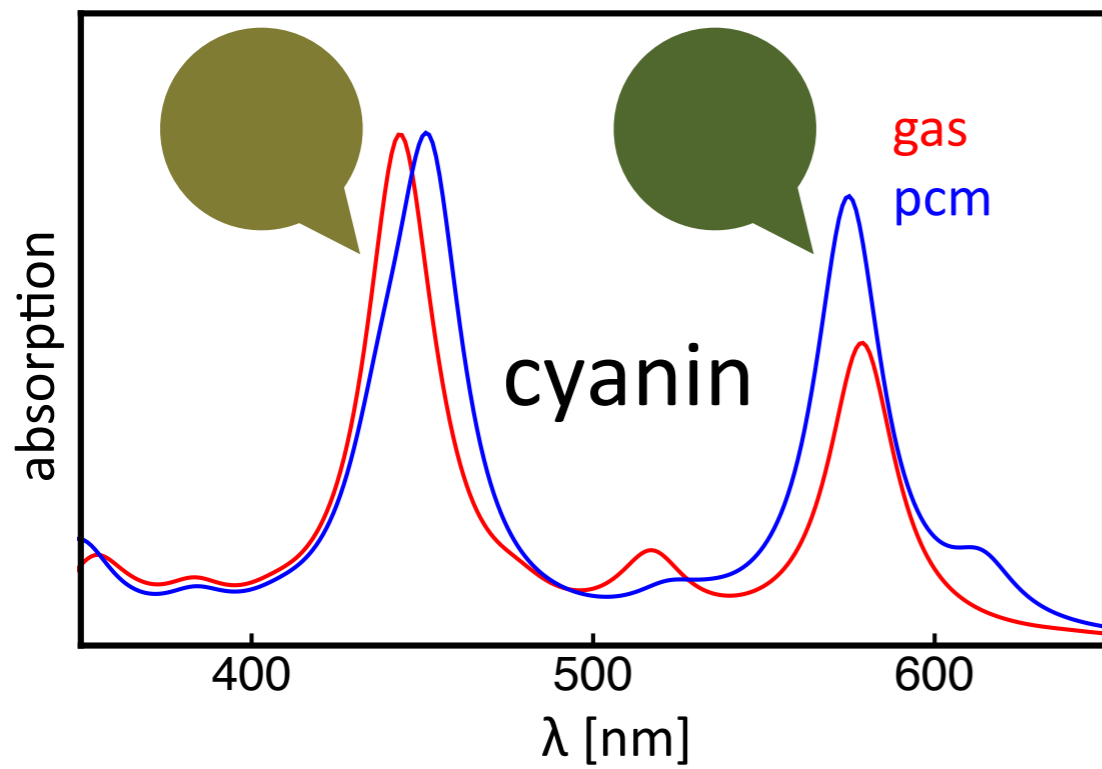
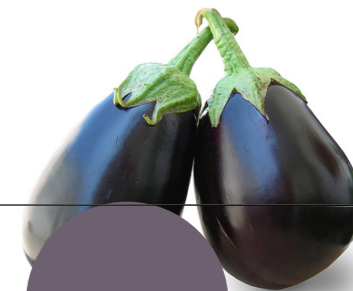
339 atoms

938 electrons

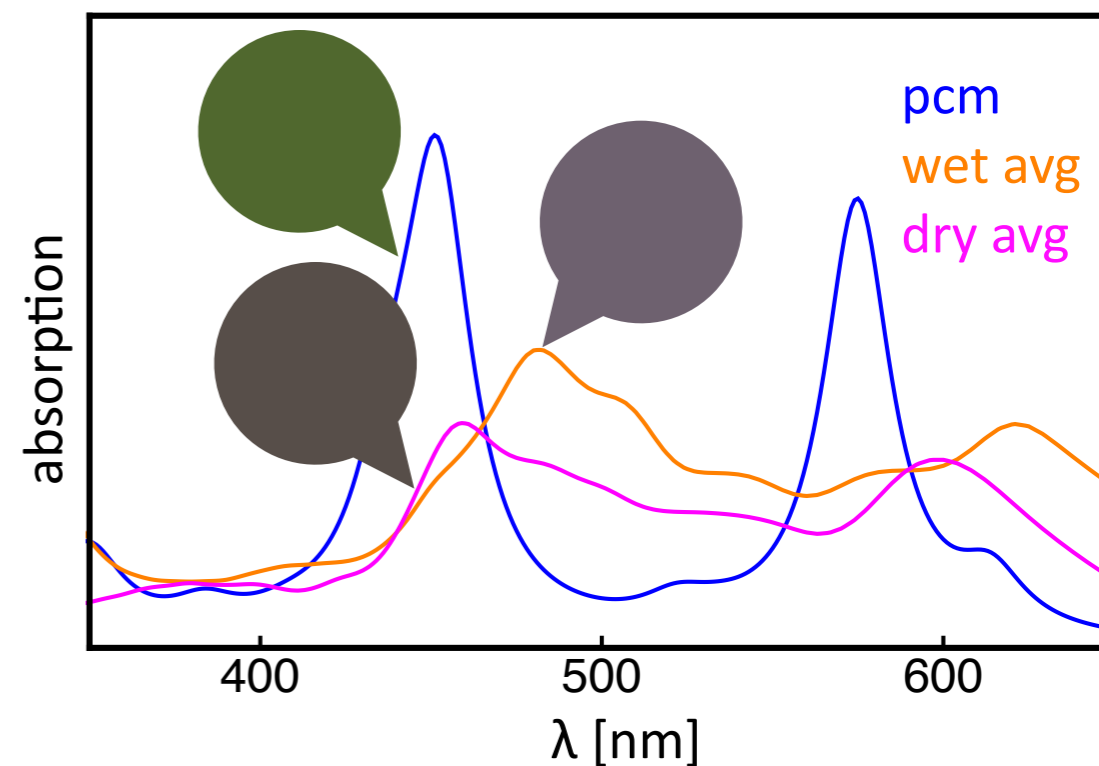
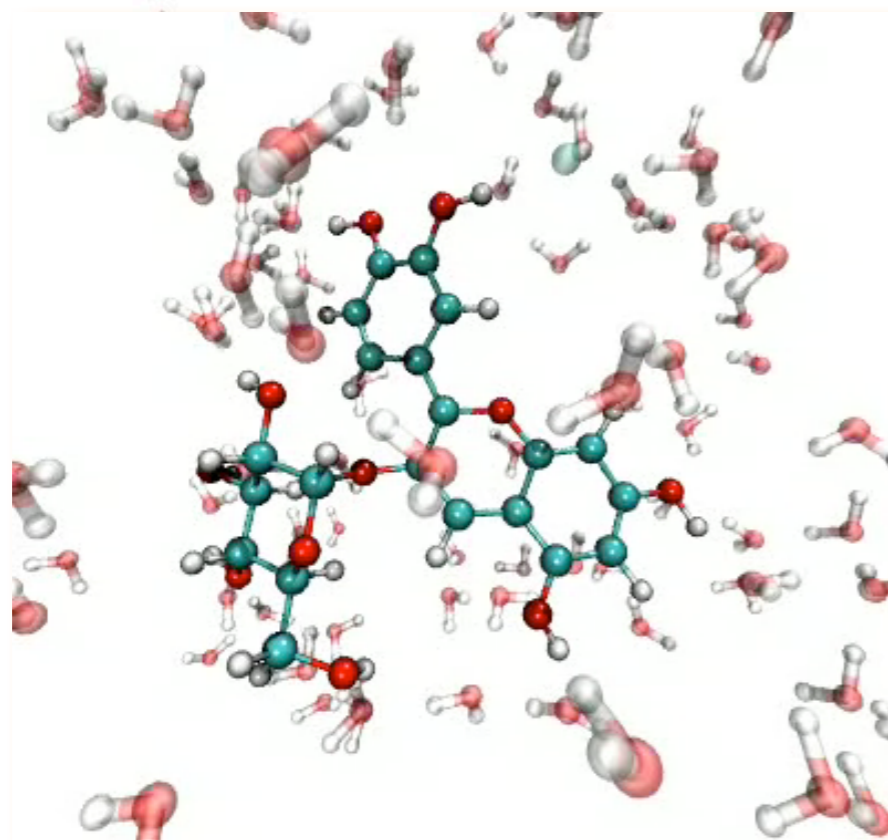
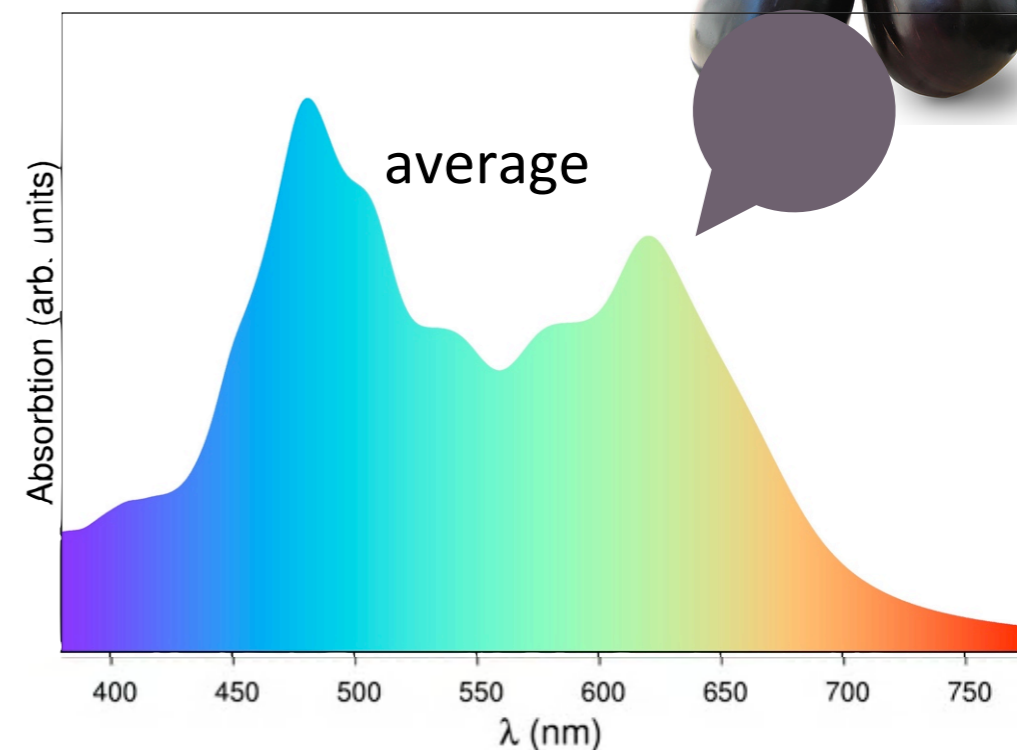
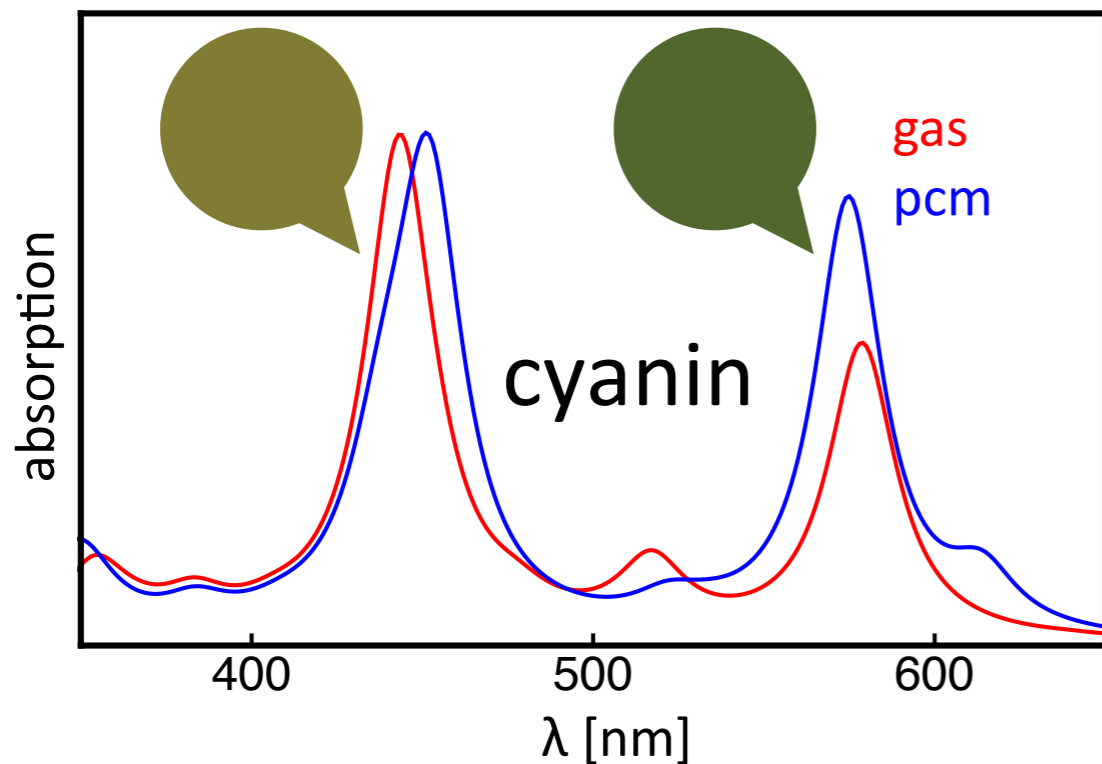
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# optical effect of the solvent





# optical effect of the solvent

delphinin



pelargonin



cyanin



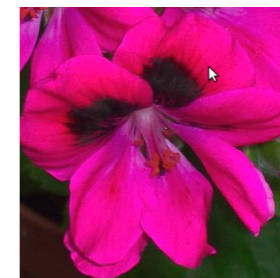
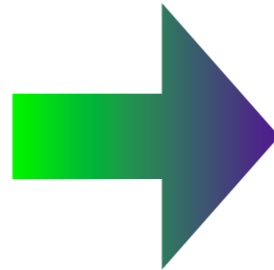
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delphinin



delphinium

pelargonin



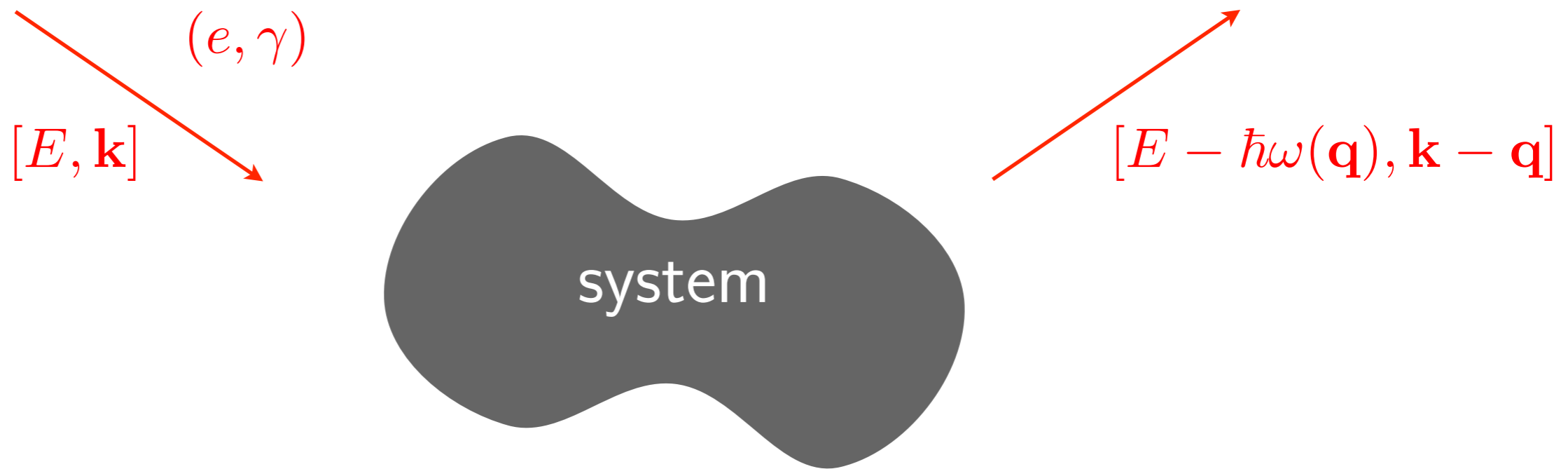
pelargonium

cyanin



blueberry

electron energy loss &  
inelastic X-ray scattering  
spectroscopies



$$\frac{d^2\sigma}{d\Omega_{\mathbf{q}}d\omega} \propto -\frac{4\pi e^2}{|\mathbf{q}|^2} \text{Im}\chi(\mathbf{q}, \mathbf{q}; \omega)$$

# traditional dielectric-matrix approach

$$\chi = \chi_0 + \chi_0 \cdot \kappa \cdot \chi$$

# traditional dielectric-matrix approach

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} \delta(t - t') + \frac{\delta\mu_{xc}(\mathbf{r}, t)}{\delta n(\mathbf{r}', t')}$$

$$\chi = \chi_0 + \chi_0 \cdot \kappa \cdot \chi$$

# traditional dielectric-matrix approach

$$\begin{aligned}\chi &= \chi_0 + \chi_0 \cdot \kappa \cdot \chi \\ &= (1 - \chi_0 \kappa)^{-1} \cdot \chi_0\end{aligned}$$

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$$\chi_0(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{cv} \left[ \frac{\phi_c^*(\mathbf{r}) \phi_v(\mathbf{r}) \phi_v^*(\mathbf{r}') \phi_c(\mathbf{r}')}{\omega - \epsilon_c + \epsilon_v + i\delta} - \frac{\phi_c^*(\mathbf{r}') \phi_v(\mathbf{r}') \phi_v^*(\mathbf{r}) \phi_c(\mathbf{r})}{\omega + \epsilon_c - \epsilon_v + i\delta} \right]$$



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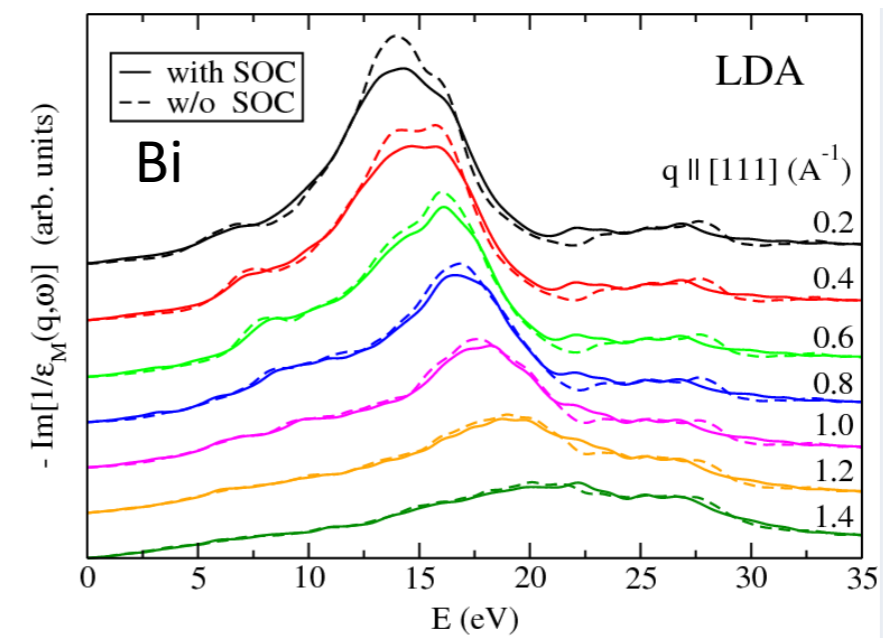
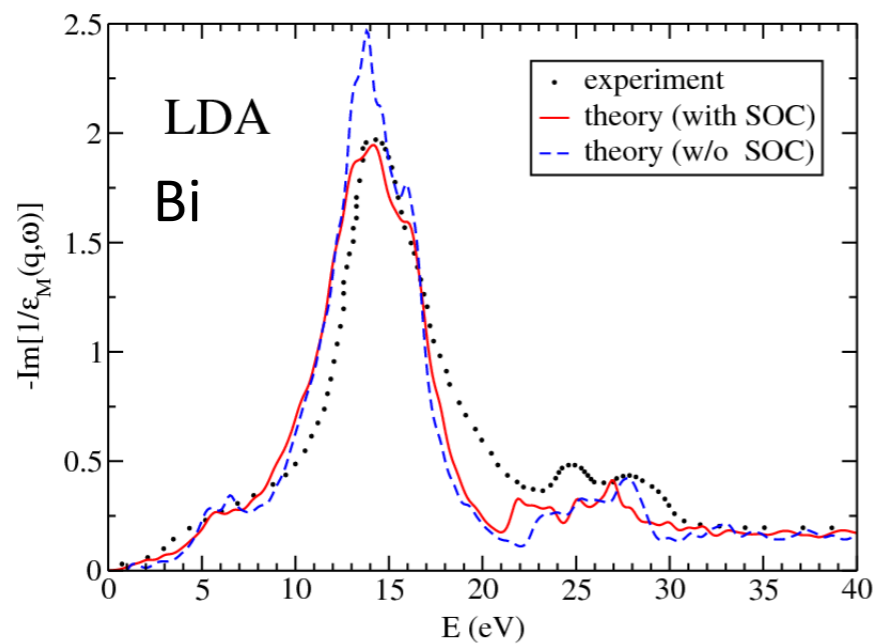
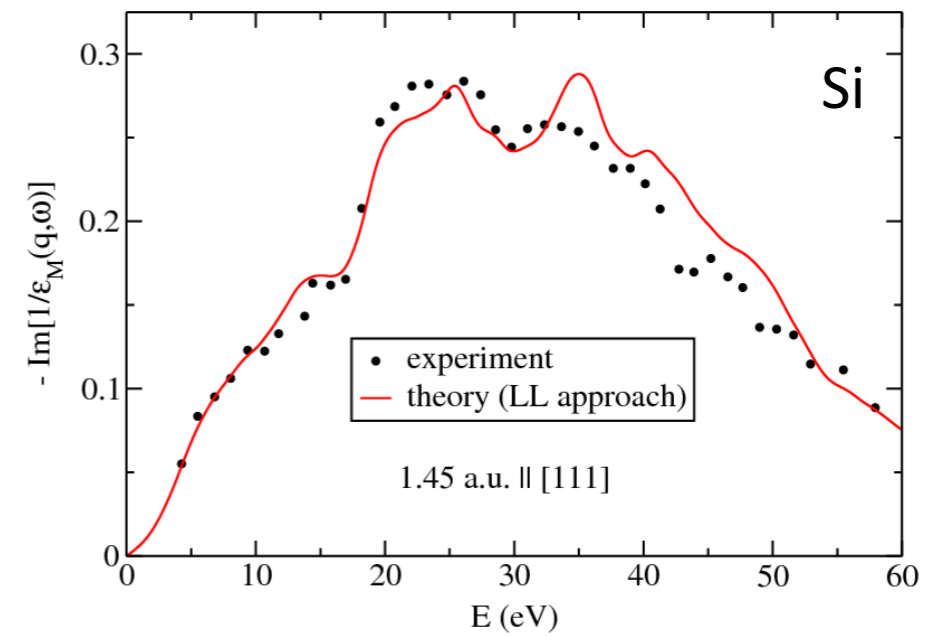
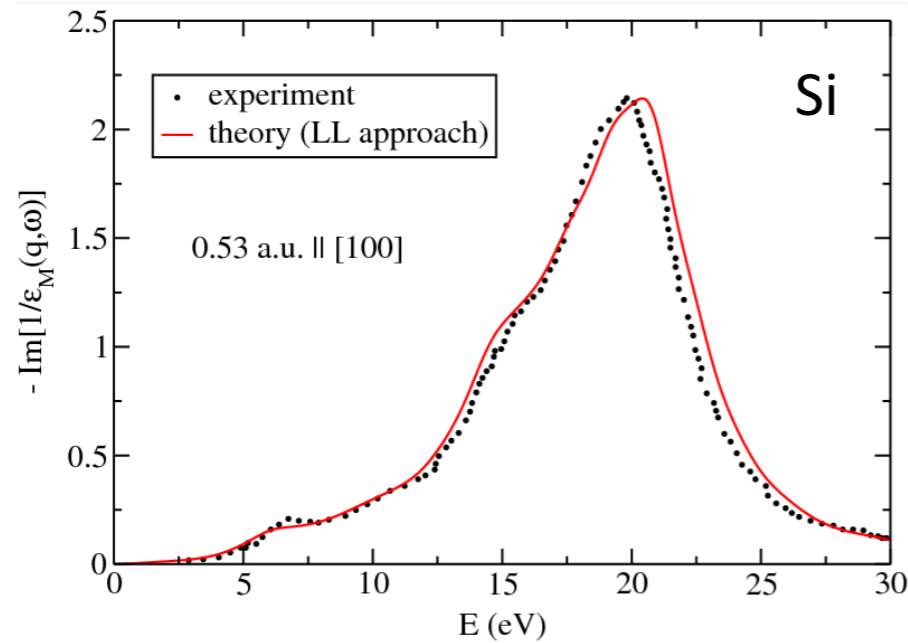
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- ☹️ Time consuming matrix operations [products, inversions:  $O(N^3)$ ] need to be repeated for many different frequencies
- ☹️ Worst of all, most of the information thus computed is wasted

do Lanczos!

# EELS & IXS spectra from Liouville-Lanczos TDDF(P)T

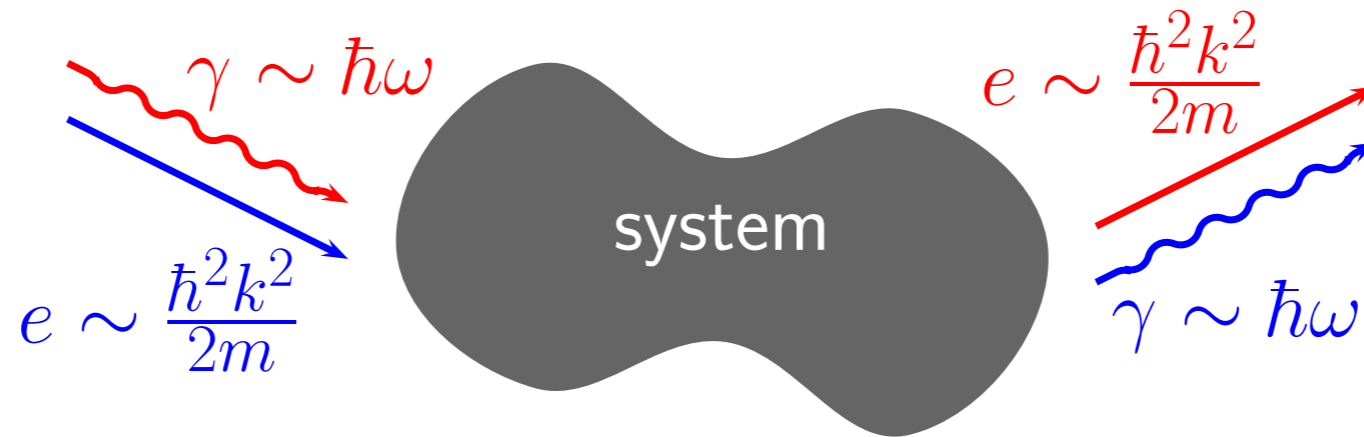


I. Timrov, N. Vast, R. Gebauer, & SB, PRB **88**, 64301 (2013)

photoemission spectroscopy ...



# photoemission spectroscopy



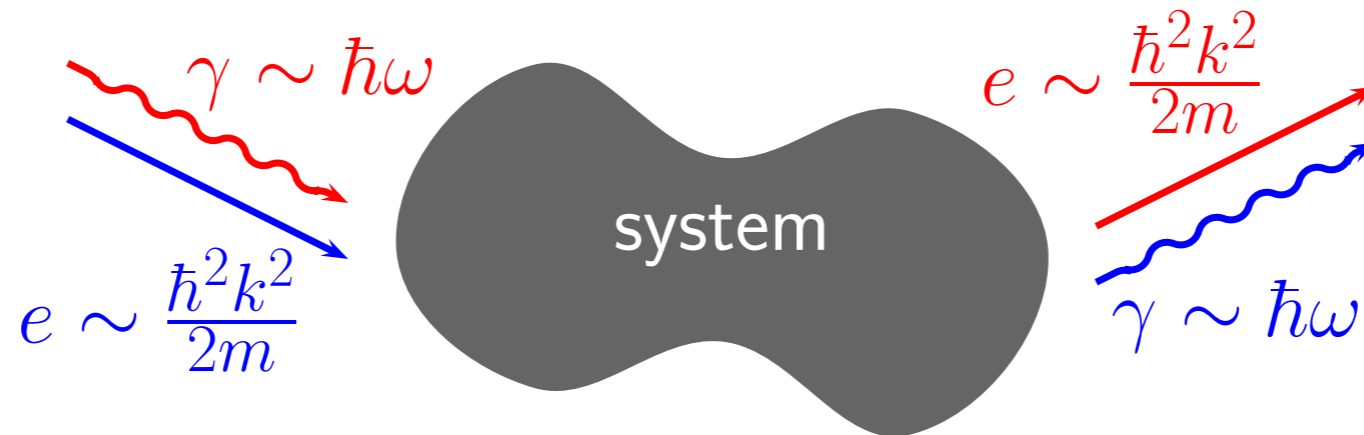
$$E_N + \hbar\omega = E_{N-1}^* + \frac{\hbar^2 k^2}{2m}$$

photoemission

$$E_N + \frac{\hbar^2 k^2}{2m} = E_{N+1}^* + \hbar\omega$$

inverse photoemission

# photoemission spectroscopy



$$E_N + \hbar\omega = E_{N-1}^* + \frac{\hbar^2 k^2}{2m} \quad \text{photoemission}$$

$$E_N + \frac{\hbar^2 k^2}{2m} = E_{N+1}^* + \hbar\omega \quad \text{inverse photoemission}$$

$$\left( -\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r}) + V_H(\mathbf{r}) \right) \psi_n(\mathbf{r}) + \int \Sigma(\mathbf{r}, \mathbf{r}'; \epsilon_n) \psi_n(\mathbf{r}') d\mathbf{r}' = \epsilon_n \psi_n(\mathbf{r})$$

$$\epsilon_n = \begin{cases} E_N - E_{N-1}^* & \text{if } \epsilon_n < \mu \\ E_{N+1}^* - E_N & \text{if } \epsilon_n > \mu \end{cases}$$

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$$W = \frac{1}{|\mathbf{r} - \mathbf{r}'|} \delta(t - t') + \int \frac{1}{|\mathbf{r} - \mathbf{r}''|} \Pi(\mathbf{r}'', \mathbf{r}'''; t - t') \frac{1}{|\mathbf{r}''' - \mathbf{r}'|} d\mathbf{r}'' d\mathbf{r}'''$$

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$\Sigma_X \mapsto$  Hartree-Fock

$\Sigma_C \mapsto$  GW

50 years after GW was invented by Hedin and  
30 years after it was first applied by Hybertsen and Louie,  
applications to systems of a few tens of atoms  
are still considered challenging

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how is that?

- ☹️ the calculation of the screened interaction requires the manipulation of huge matrices
- ☹️ the calculation of the electronic polarizability and electron self-energy requires extensive sums over virtual states

# choosing a (reduced) basis for the polarizability

$$\begin{aligned}\Pi(\mathbf{r}, \mathbf{r}', t - t') &= \frac{\delta n(\mathbf{r}, t)}{\delta V_{ext}(\mathbf{r}', t')} \\ \tilde{\Pi}(\mathbf{r}, \mathbf{r}', \omega) &= \tilde{P} \cdot (1 - v \cdot \tilde{P})^{-1}\end{aligned}$$

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- most important eigenvectors of  $\tilde{P}(\omega = 0)$  (Gygi and Galli, 2008)
- most important eigenvectors of  $P(t = 0)$  (Umari, 2010)
- a decent model would also do ...

avoiding the sum over virtual states

$$\begin{aligned} P(\mathbf{r}, \mathbf{r}'; i\omega) &= 2\text{Re} \sum_{cv} \psi_c(\mathbf{r})\psi_v(\mathbf{r})\psi_c(\mathbf{r}')\psi_v(\mathbf{r}') \frac{1}{i\omega - \epsilon_c + \epsilon_v} \\ &\doteq \sum_{\alpha\beta} \Phi_\alpha(\mathbf{r})\Phi_\beta(\mathbf{r}')P_{\alpha\beta}(i\omega) \end{aligned}$$

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**do Lanczos!**

(a similar trick can be used to calculate matrix elements of  $\Sigma_c$ )

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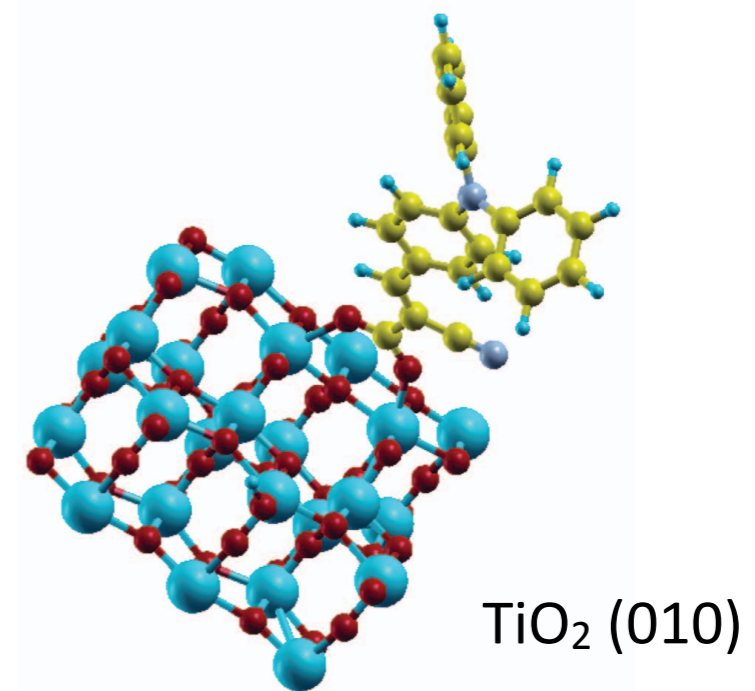
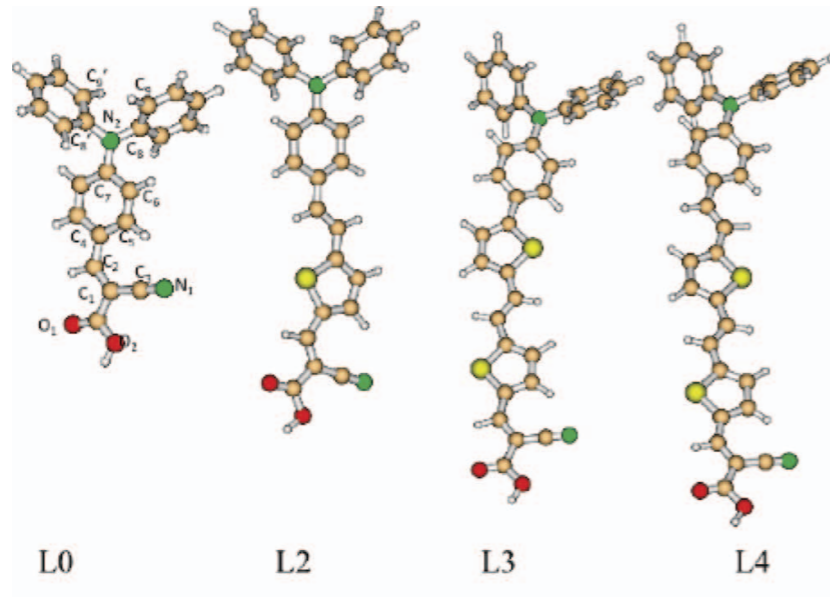
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(a similar trick can be used to calculate matrix elements of  $\Sigma_c$ )

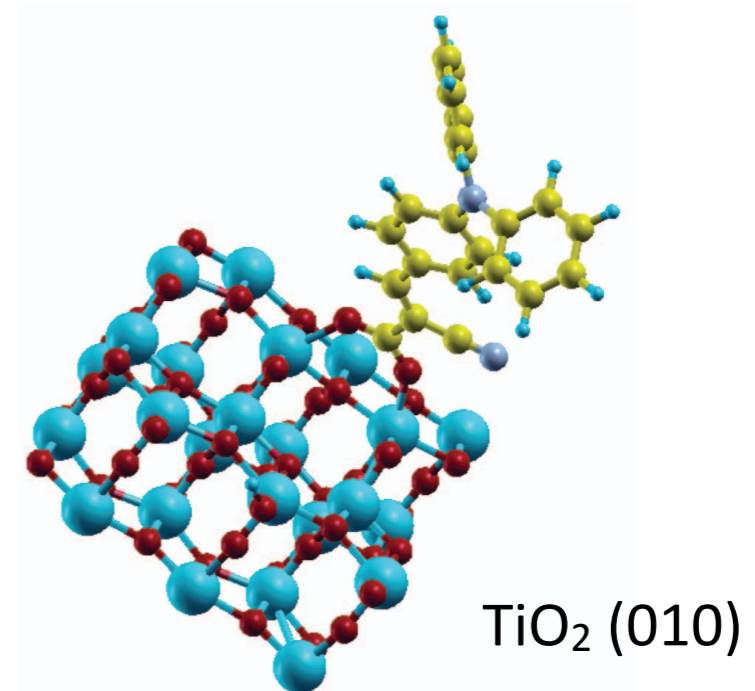
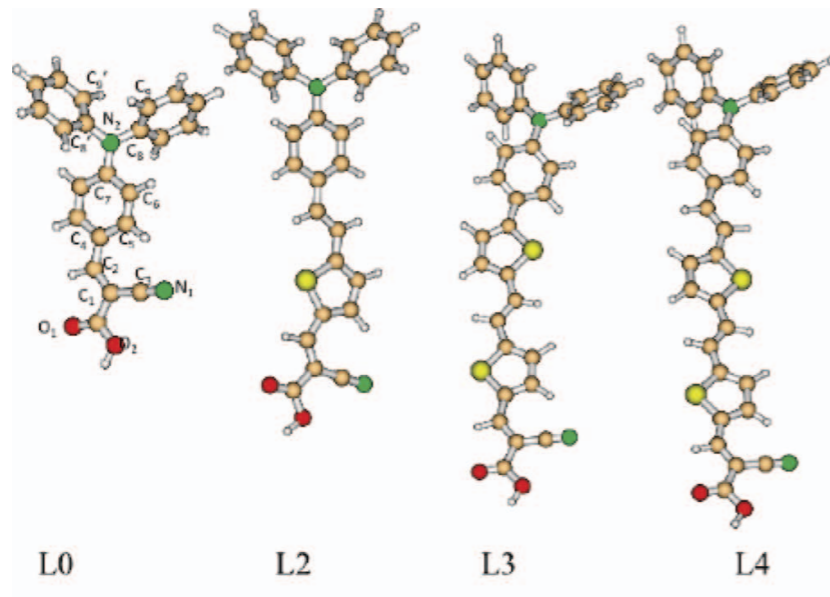
# energy levels in all-organic DSSCs



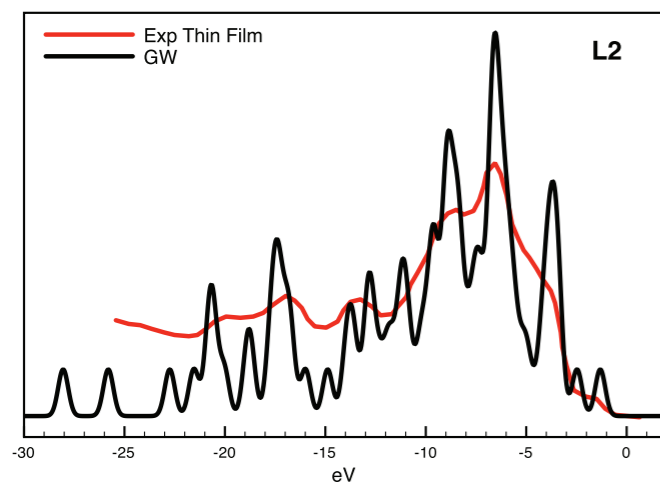
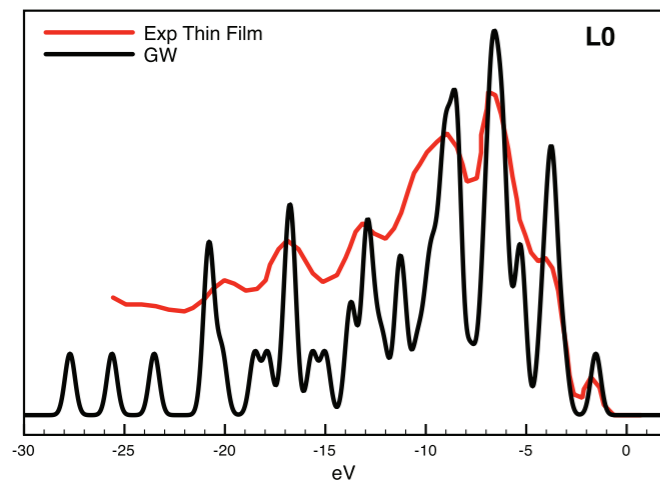
triphenylamine (TPA)- based sensitizers



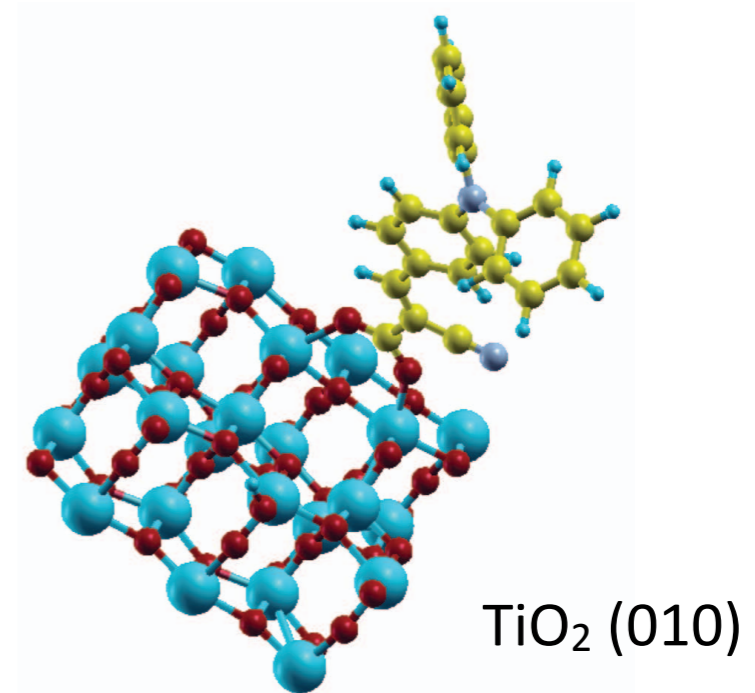
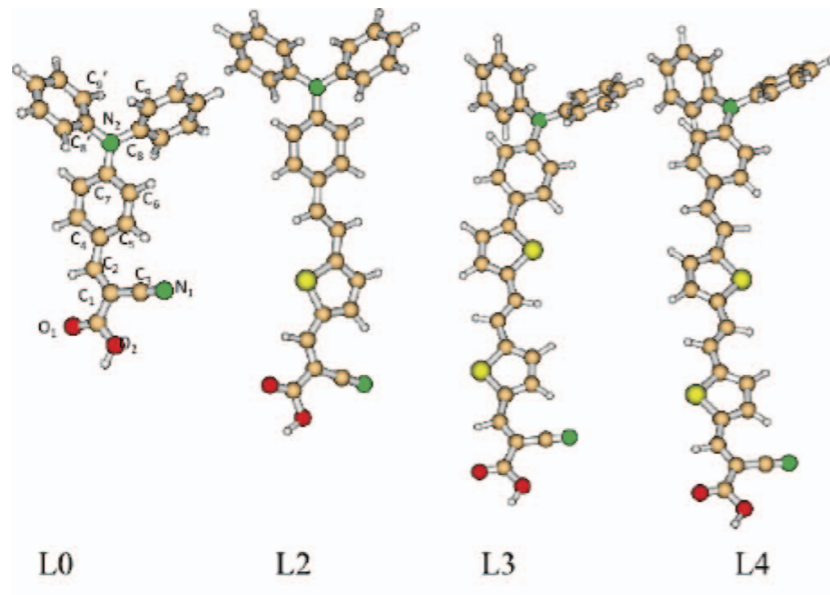
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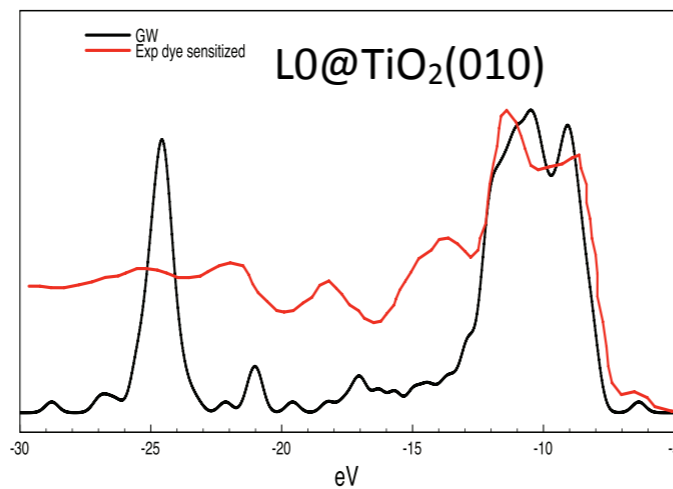
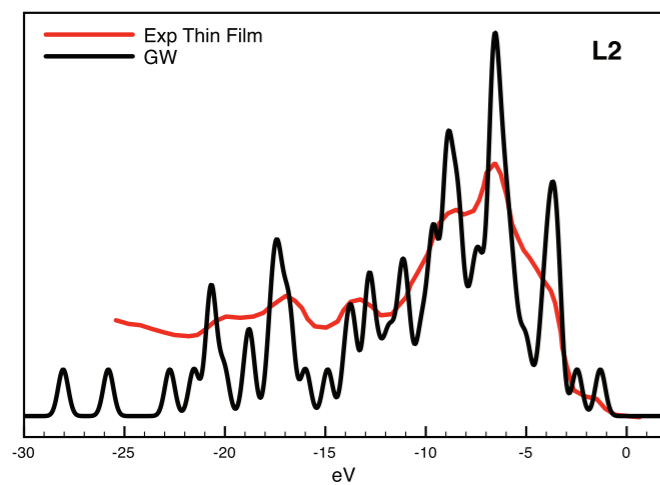
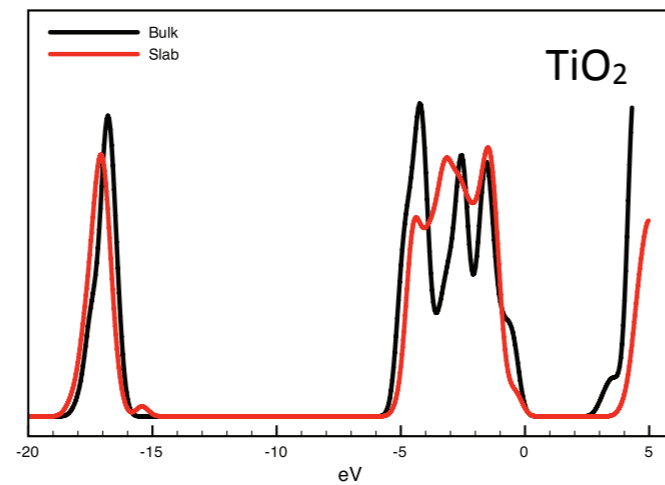
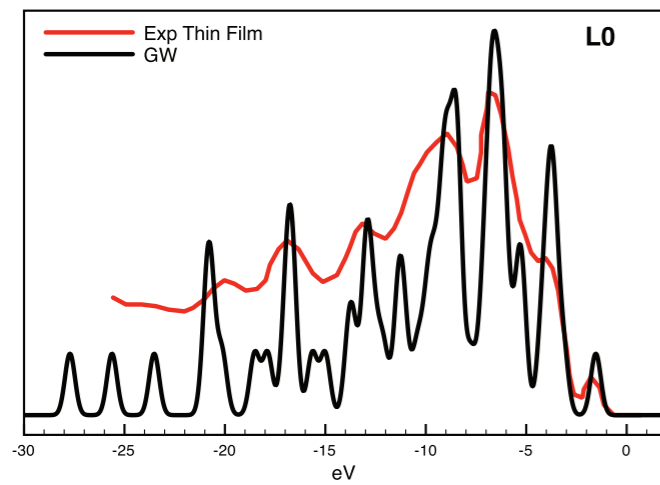
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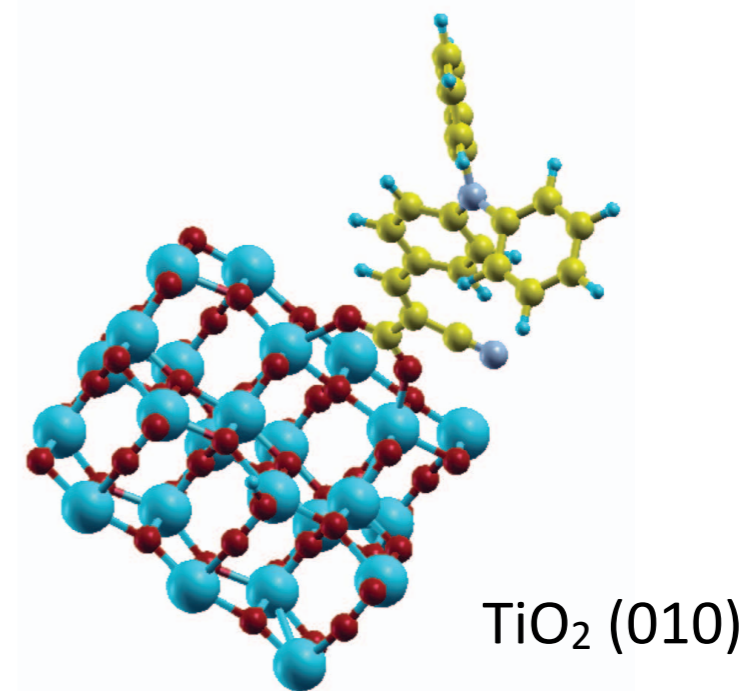
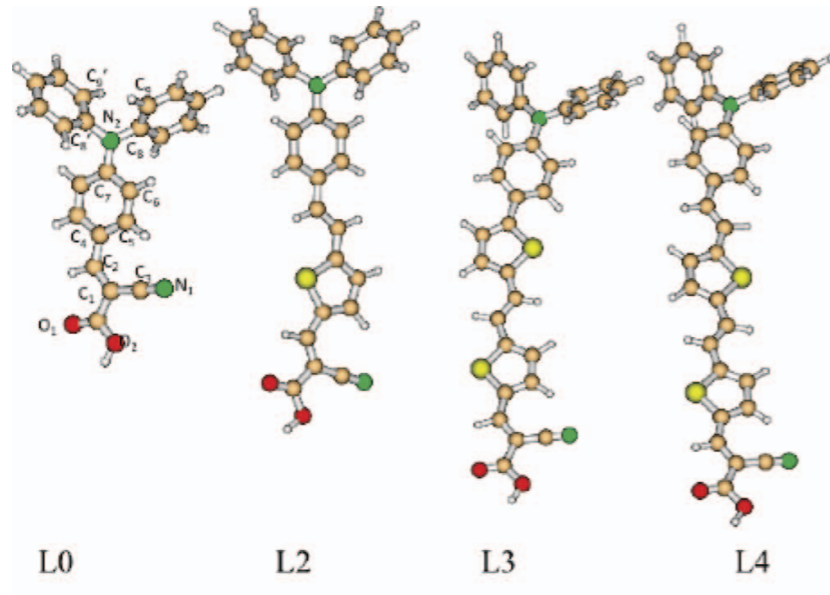
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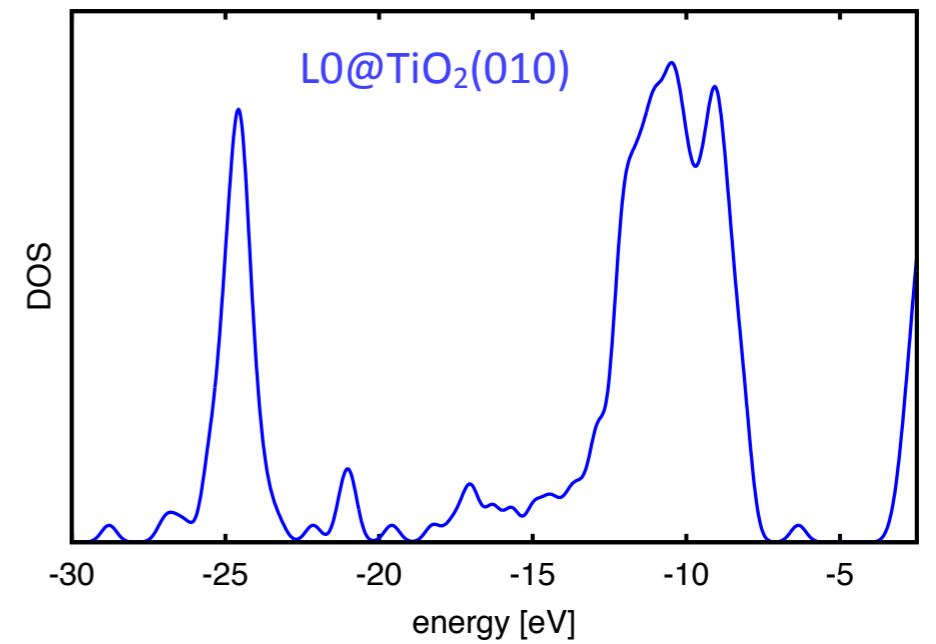
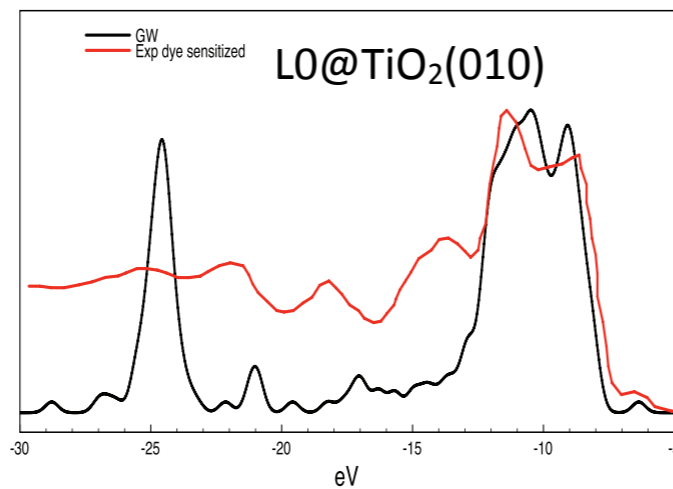
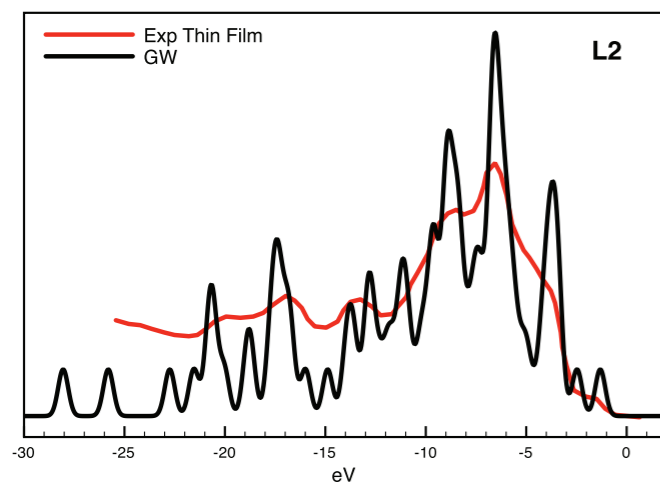
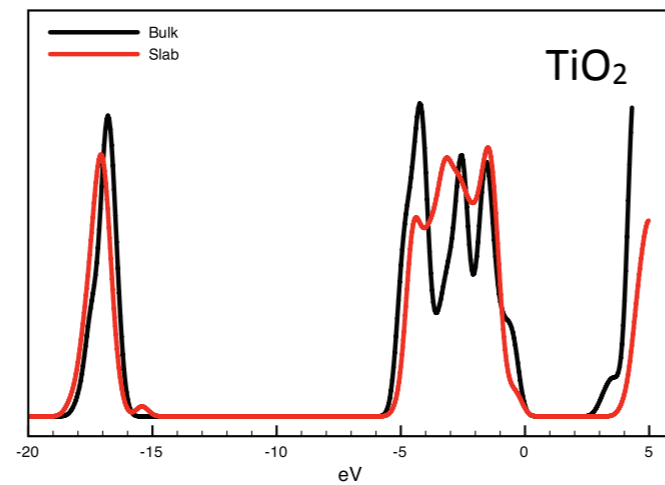
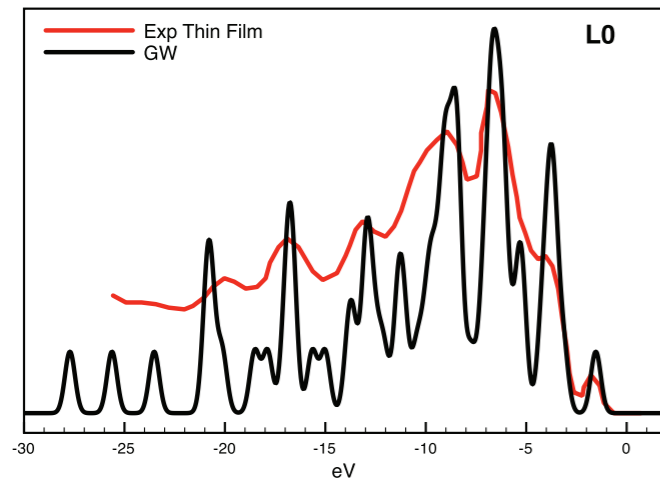
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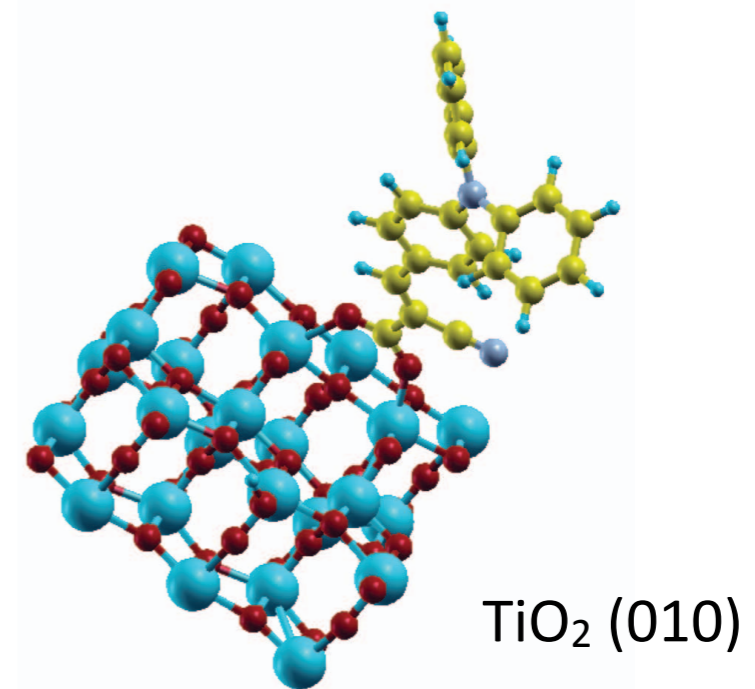
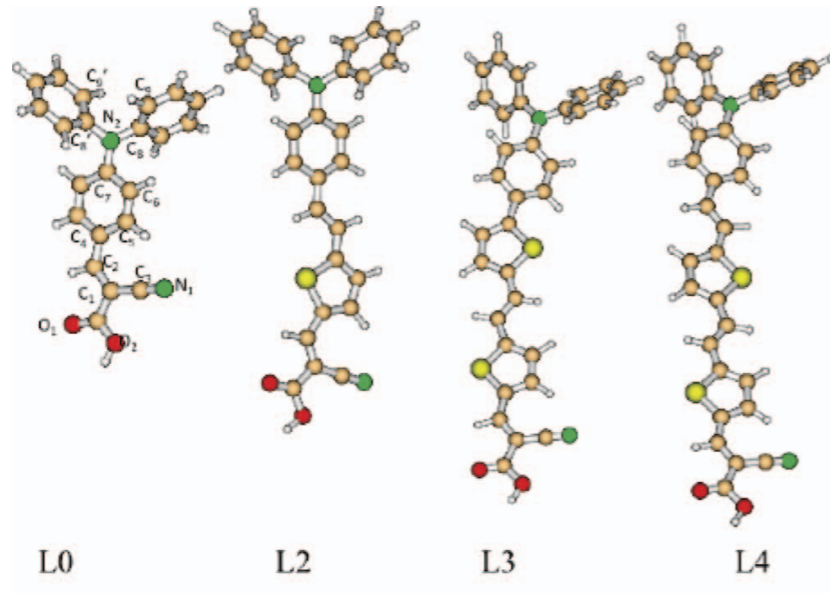
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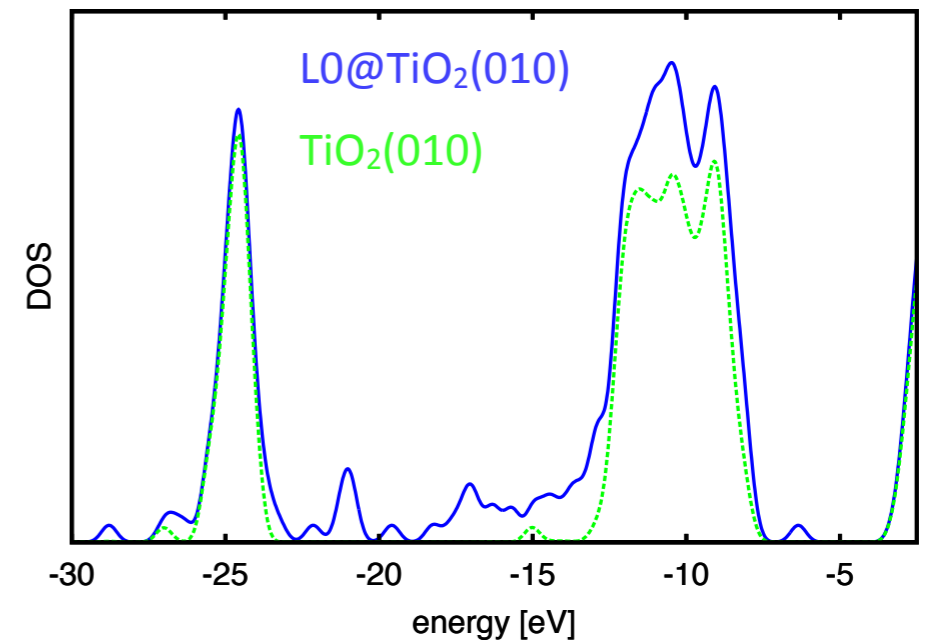
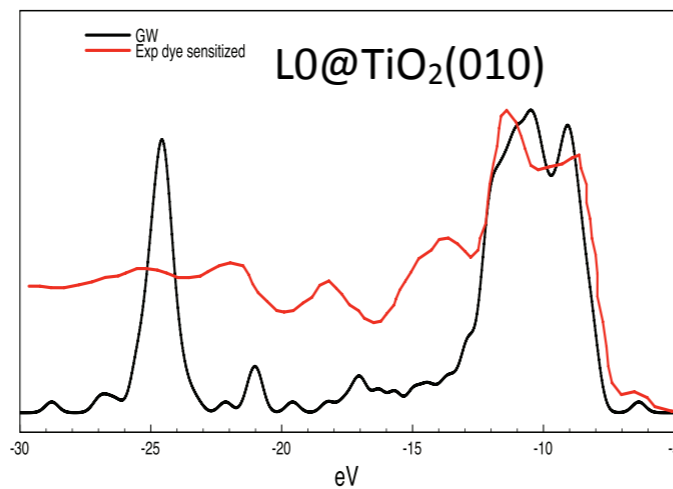
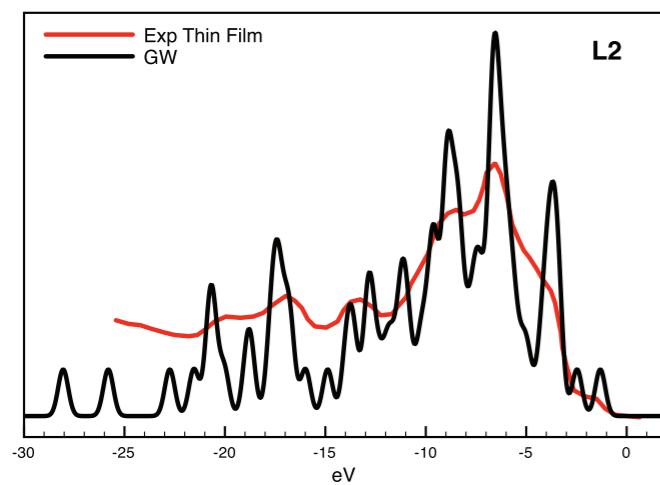
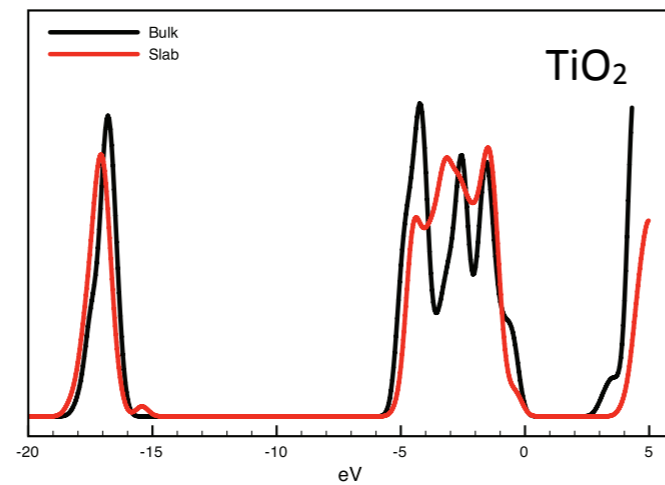
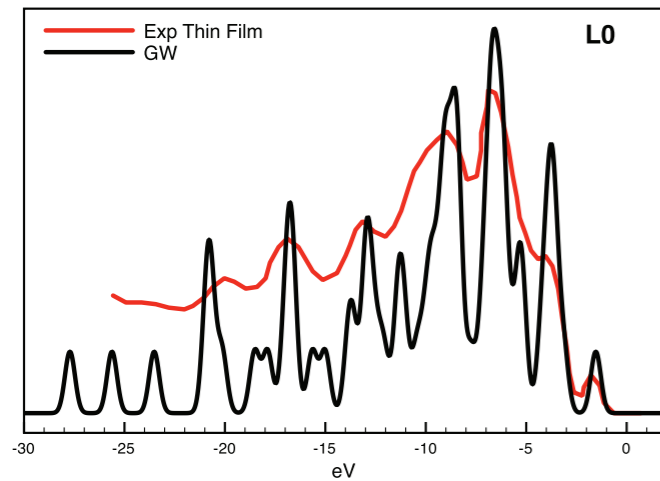
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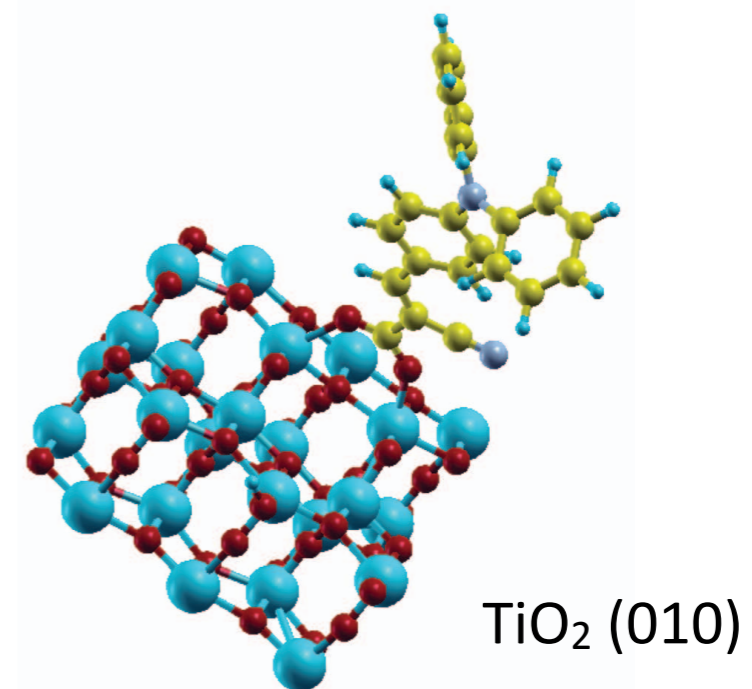
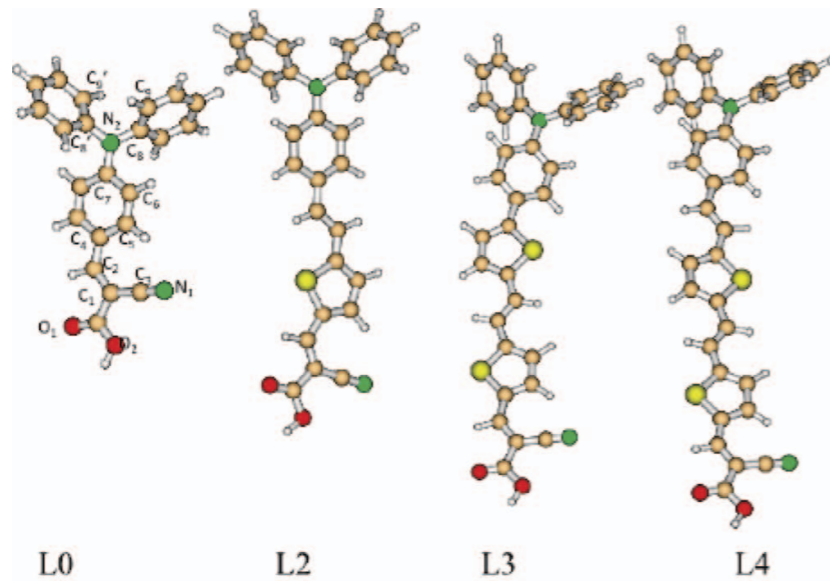
# energy levels in all-organic DSSCs



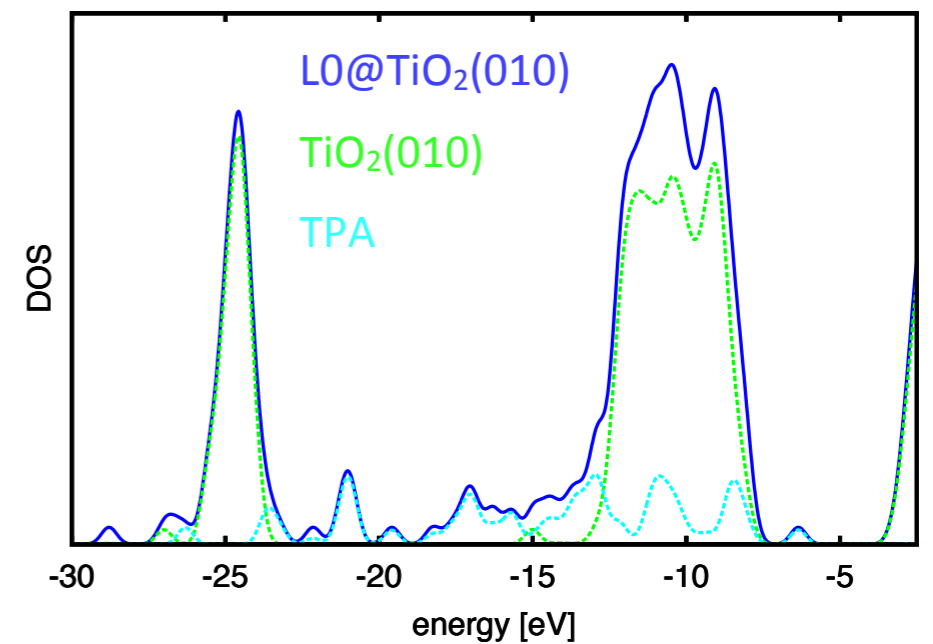
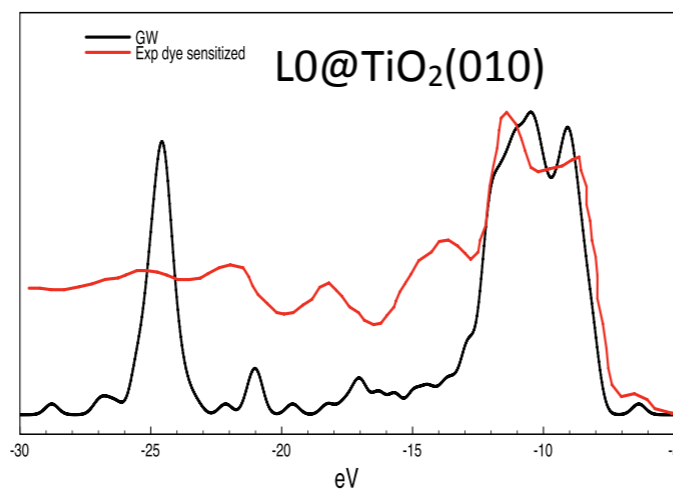
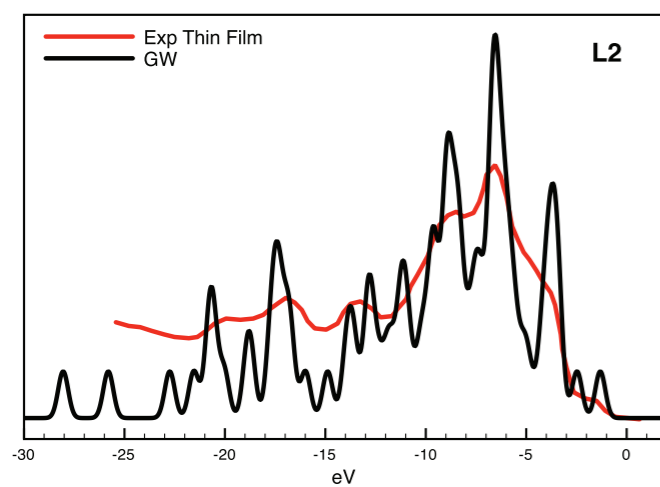
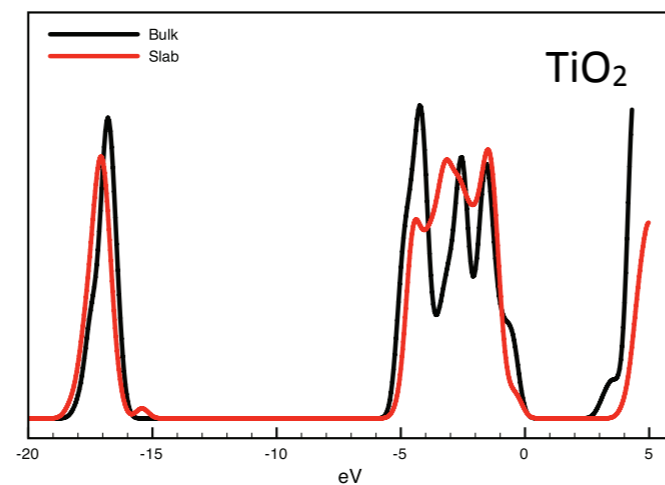
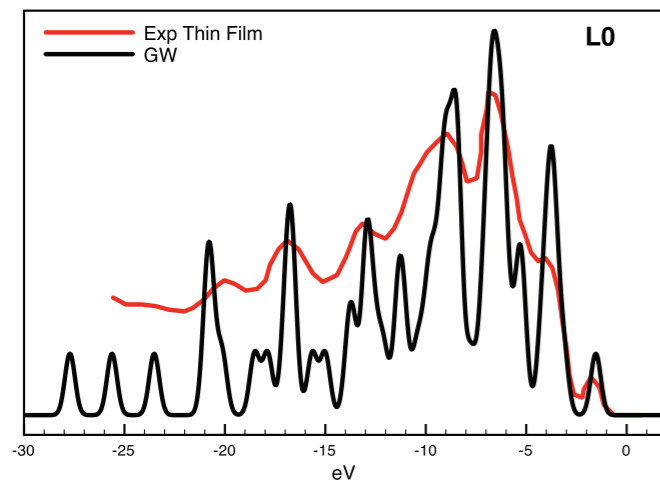
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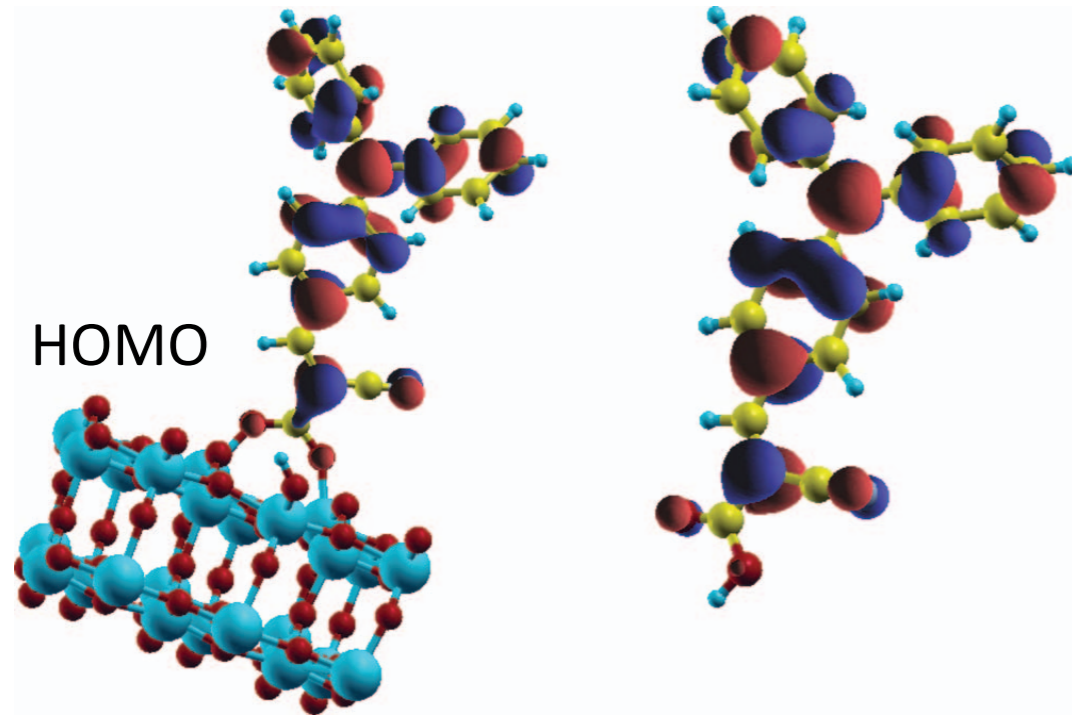
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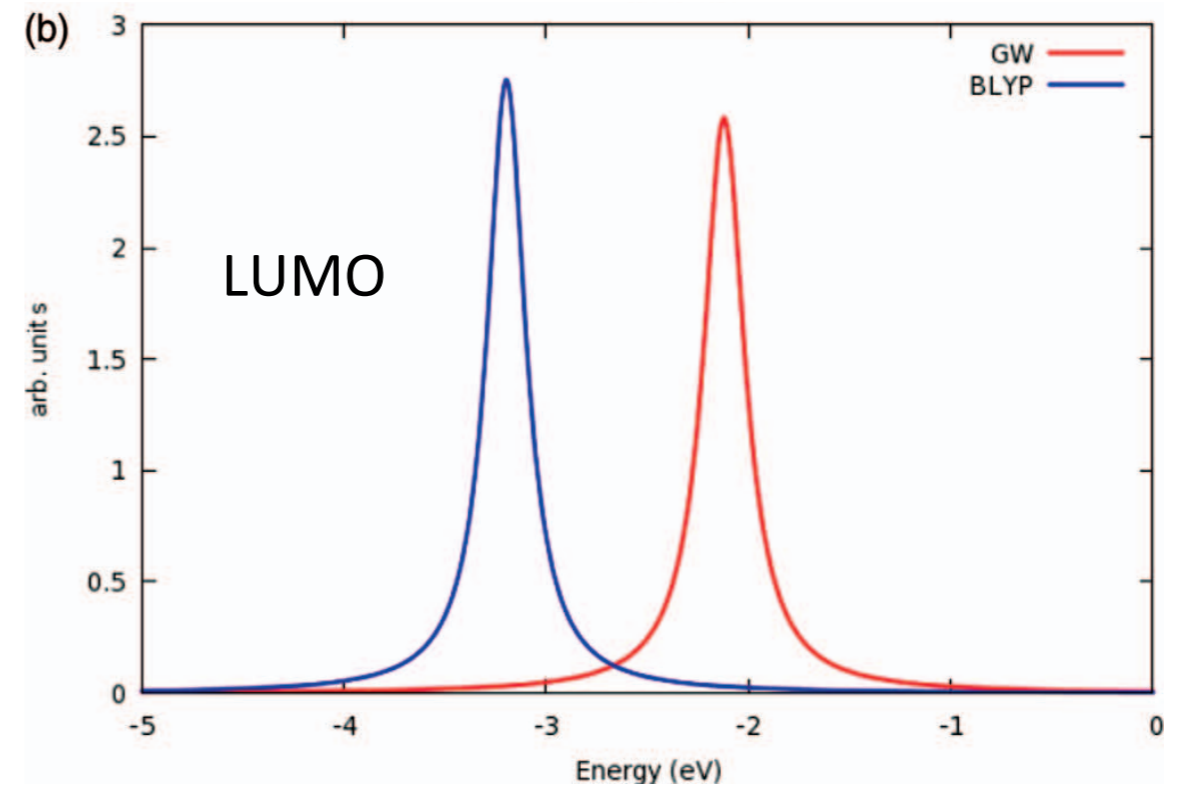
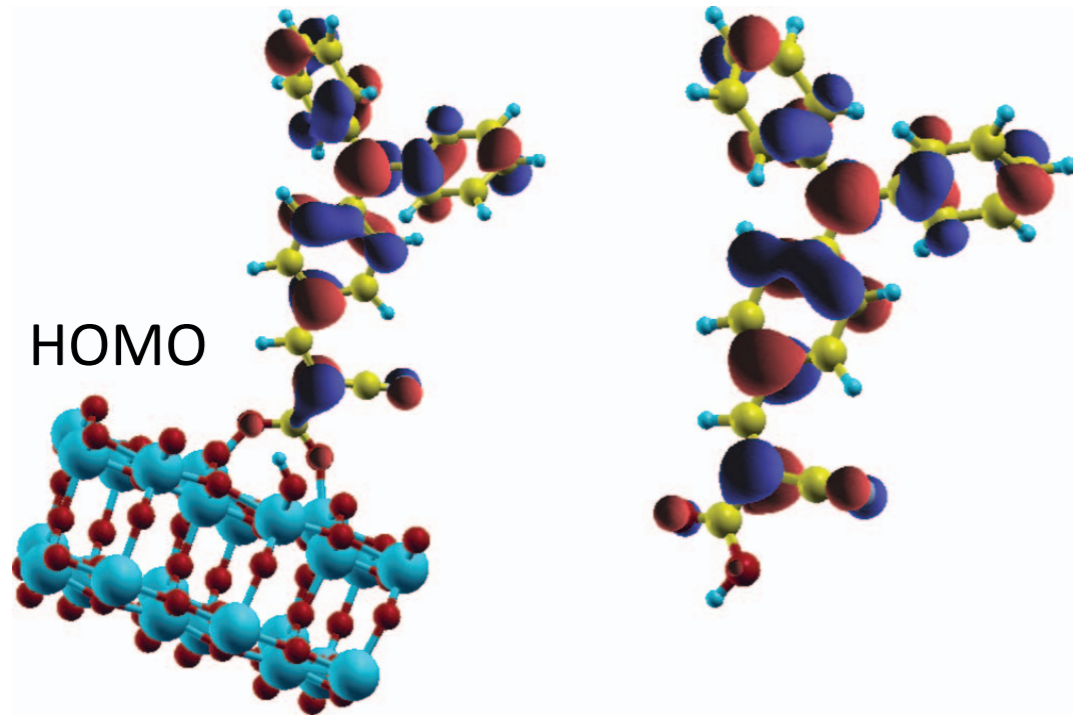
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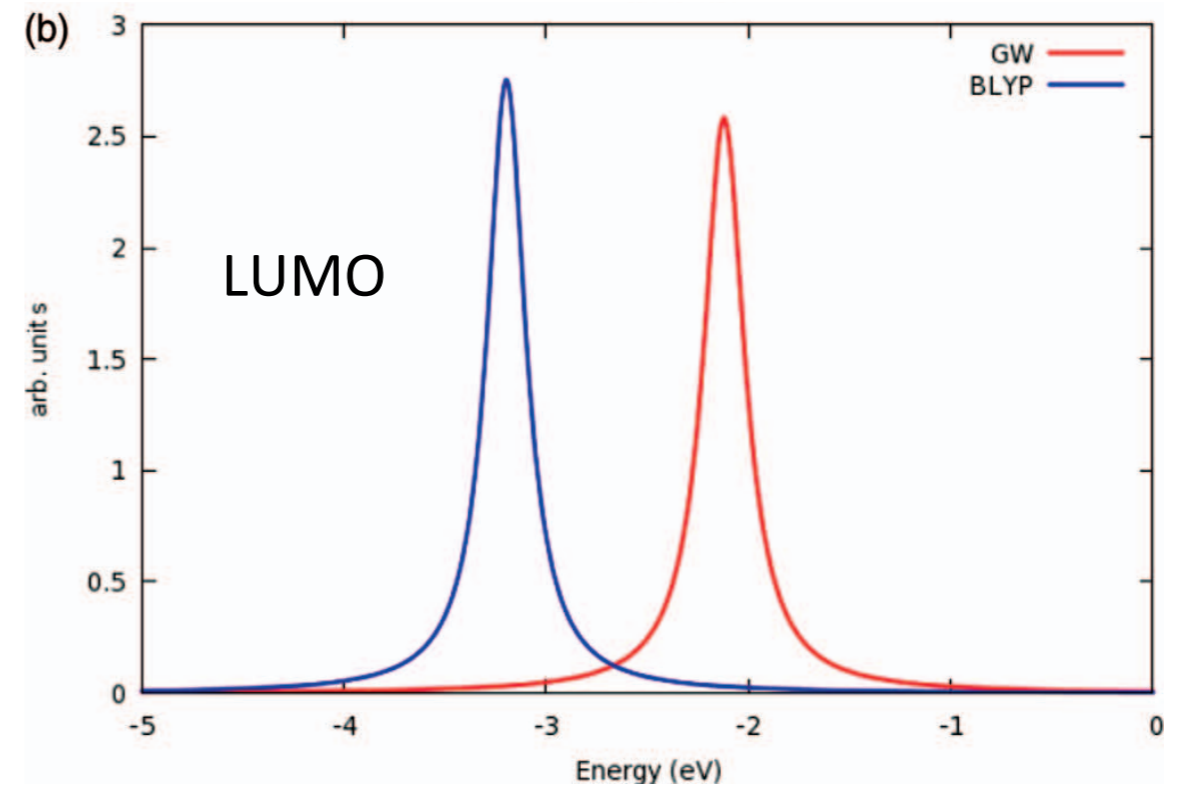
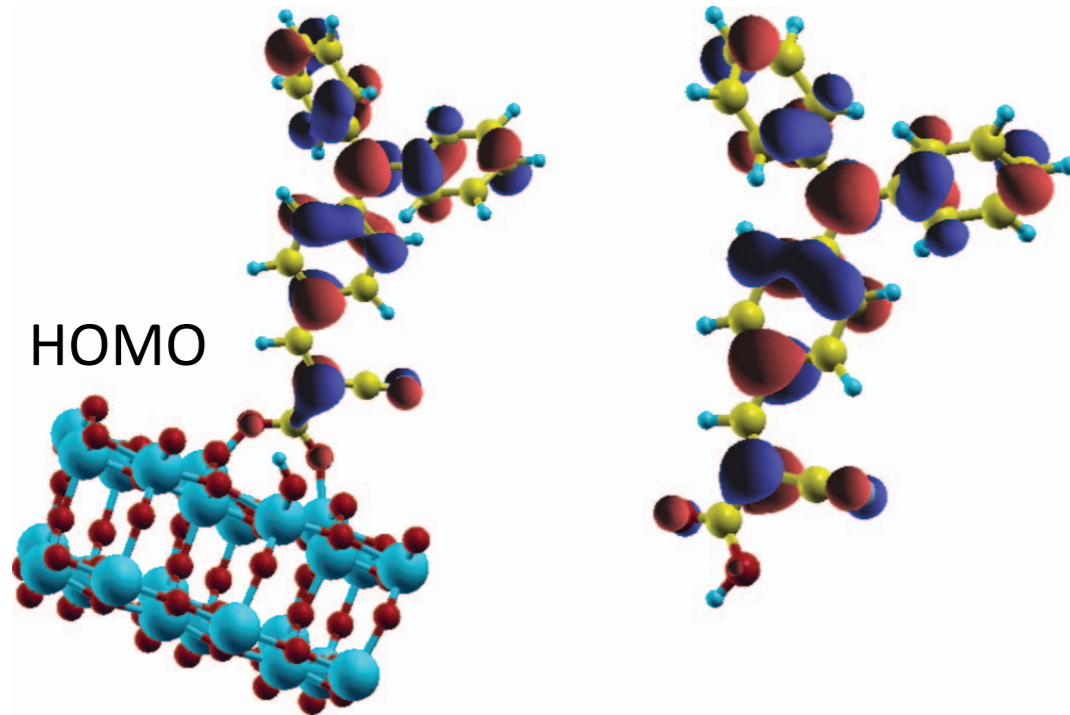


# energy levels in all-organic DSSCs



$$g_{LUMO}(\epsilon) = \sum_n |\langle \psi_n | \phi_{LUMO} \rangle|^2 \delta(\epsilon - \epsilon_n)$$

# energy levels in all-organic DSSCs

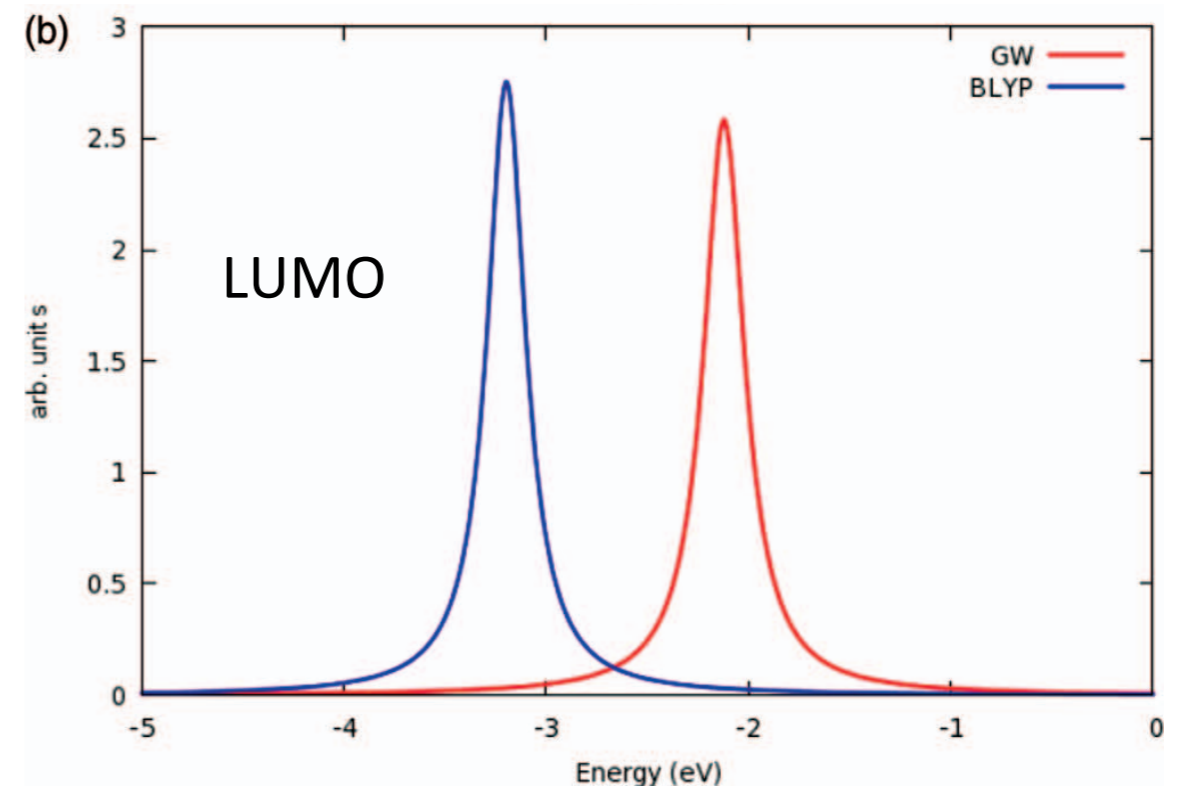
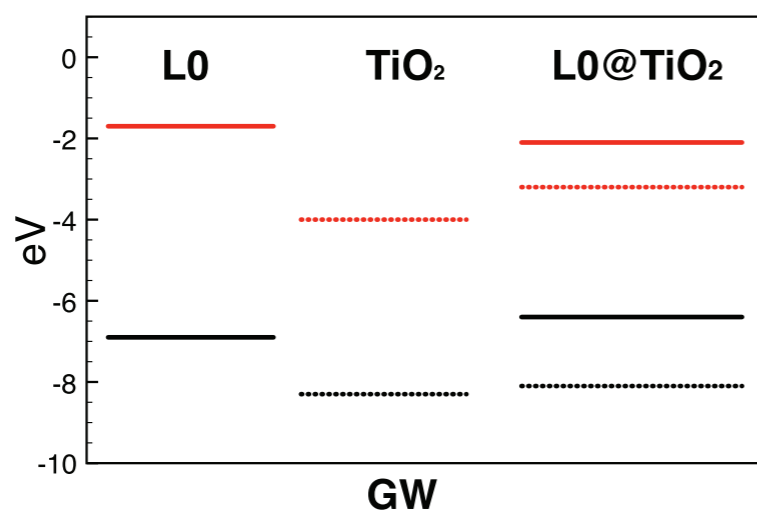
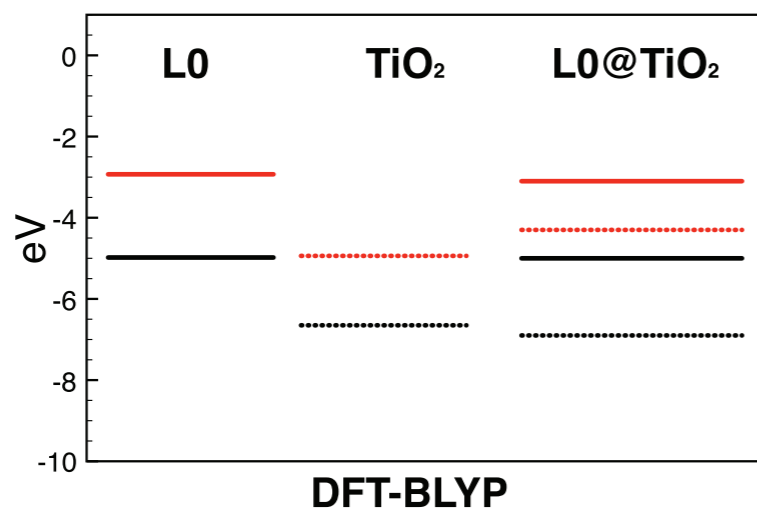
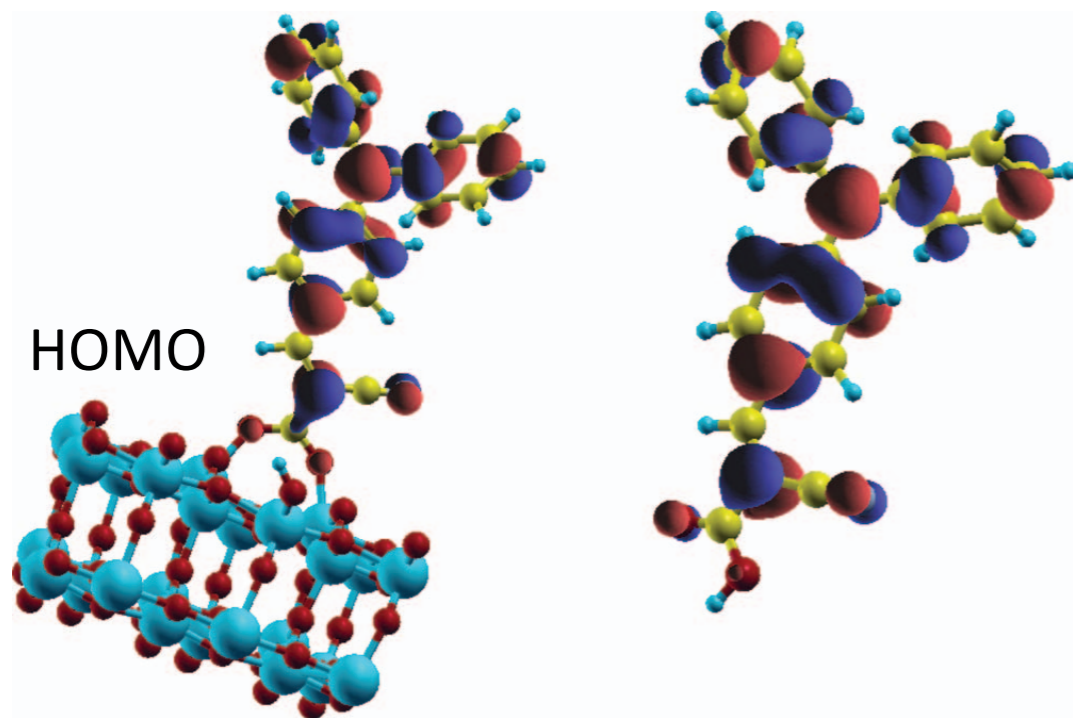


$$g_{LUMO}(\epsilon) = \sum_n |\langle \psi_n | \phi_{LUMO} \rangle|^2 \delta(\epsilon - \epsilon_n)$$

$$\tau \text{ [fs]} = \frac{658}{\Gamma \text{ [meV]}}$$
$$\approx 2.7 \div 2.8 \text{ [fs]}$$



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P. Umari et al. JCP **139**, 014709 (2013)



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Forum

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31.10.12

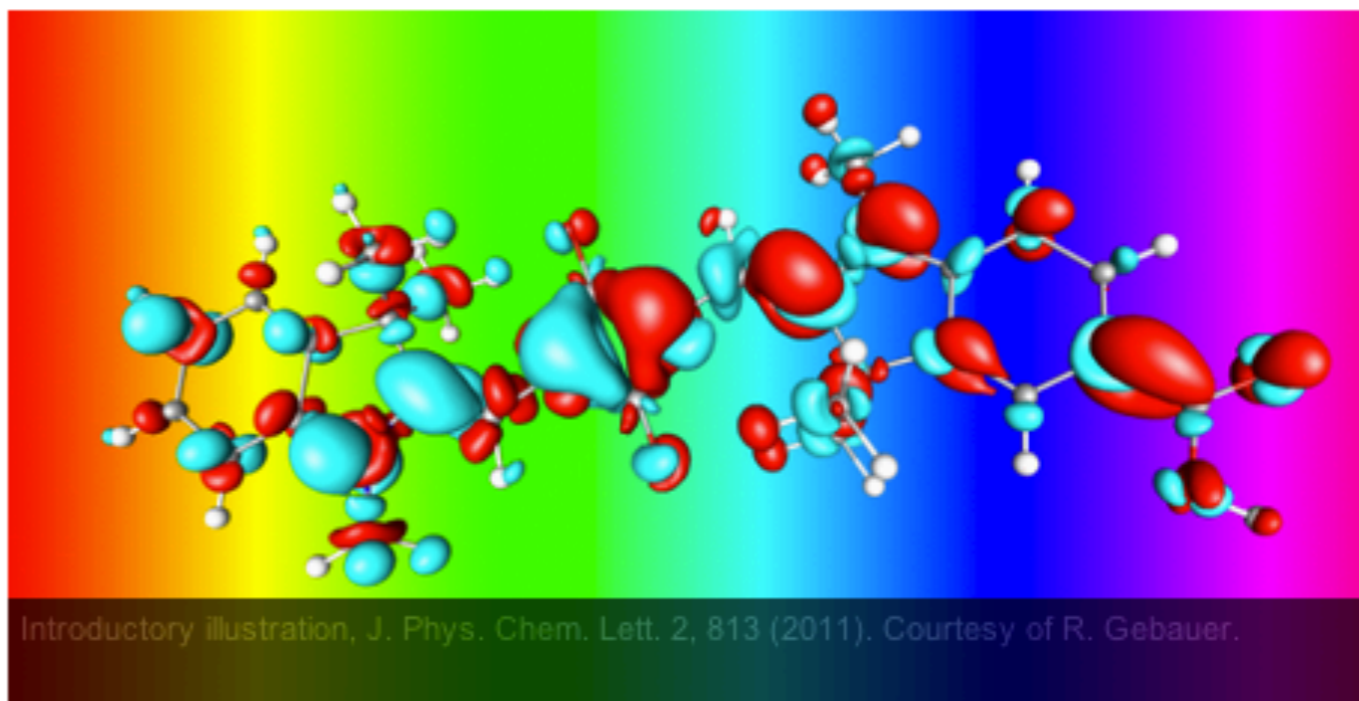
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02.10.12

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All users and developers are warmly encouraged to share their own developments



Introductory illustration, *J. Phys. Chem. Lett.* 2, 813 (2011). Courtesy of R. Gebauer.

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Thanks to

Alessandro Biancardi

Simon Binnie

Arrigo Calzolari

Xiaochuan Ge

Ralph Gebauer

Baris Malcioğlu

Dario Rocca

Iurii Timrov

Paolo Umari

Brent Walker

Thanks to

Alessandro Biancardi

Simon Binnie

Arrigo Calzolari

Xiaochuan Ge

Ralph Gebauer

Baris Malcioğlu

Dario Rocca

Iurii Timrov

Paolo Umari

Brent Walker

and to the QUANTUM ESPRESSO  
development & maintenance team

Dario Alfè, Francesco Antoniella, Gerardo Ballabio, Stefano Baroni, Simon Binnie, Mauro Boero, Claudia Bungaro, Giovanni Bussi, Matteo Calandra, Roberto Car, Carlo Cavazzoni, Paolo Cazzato, Davide Ceresoli, Gabriele Cipriani, Matteo Cococcioni, Andrea Dal Corso, Alberto Debernardi, Gernot Deinzer, Oswaldo Dieguez, Stefano Fabris, Guido Fratesi, Xiaochuan Ge, Ralph Gebauer, Paolo Giannozzi, Stefano de Gironcoli, Martin Hilgeman, Yosuke Kanai, Anton Kokalj, Axel Kohlmeyer, Konstantin Kudin, Michele Lazzeri, Baris Malcioglu, Francesco Mauri, Kurt Mäder, O. Barış Malcioğlu, Layla Martin Samos Colomer, Nicola Marzari, Nicolas Mounet, Adriano Mosca Conte, Alfredo Pasquarello, Lorenzo Paulatto, Pasquale Pavone, Mickael Profeta, Dario Rocca, Guido Roma, Riccardo Sabatini, Carlo Sbraccia, Sandro Scandolo, Gabriele Sclauzero, Manu Sharma, Alexander Smogunov, Kurt Stokbro, Pascal Thibaudeau, Antonio Tilocca, Andrea Trave, Paolo Umari, Brent Walker, Renata Wentzcovitch, Yudong Wu, Xiaofei Wang ...

... and let me apologize to anybody I may have forgotten.



*That's all Folks!*

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