DFT and beyond: Hands-on Tutorial Workshop 2011

Tutorial 1: Basics of Electronic Structure Theory

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The ultimate goal!

$$H\Psi = E\Psi$$

Second order differential equation for a $3N_e$ -variable function $\Psi \Rightarrow$ Complex problem

Unsolved issues at the simplest level of approximations (mulitiple solutions, generalized Hartree-Fock method, \ldots)

Goals of this tutorial

- Familiarize with practical aspects of electronic structure theory in general and density functional theory (DFT) in particular
- Hartree-Fock (HF) method and Kohn-Sham DFT (non-periodic)
- Numerical solution of the approximate equations (tool: FHI-aims)
- Exploring potential energy surfaces (total energies at fixed nuclei, local minima, transition states, vibrational spectra)
- Electronic structure analysis (visualization tools, electron density, Kohn-Sham orbitals and spectrum)

Solving the Kohn-Sham equations

Hohenberg-Kohn Theorem $\Psi(\mathbf{r}_1...\mathbf{r}_{N_e}) \Leftrightarrow n(\mathbf{r})$

Kohn-Sham scheme

$$\left(-\frac{1}{2}\nabla^2 + \int d^3r' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + v_{xc} + v\right)\phi_i = \varepsilon_i\phi_i \quad \Rightarrow n = \sum_i f_i |\phi_i|^2$$

KS Orbitals $\{\phi_i\}$ $\langle \phi_i, \phi_j \rangle = \delta_{ij}$

XC Potential v_{xc} unknown, but \exists many approximations

LDA, PBE, ...

External potential *v* contains ionic contributions

Hartree-Fock method

$$\Psi = \det |(\phi_1(r_1)...\phi_{N_e}(r_{Ne})|$$

$$\left(-\frac{1}{2}\nabla^2 + \sum_j \int d^3r' \frac{|\phi_j(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} \underbrace{-\sum_j \int d^3r' \frac{\phi_j^*(\mathbf{r}')\phi_i(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}}_{\mathbf{exact (HF) exchange}} + v\right) \phi_i = \varepsilon_i \phi_i$$

- (1) Single Slater determinant
- (2) Mean-field approximation

No self-interaction error But also: no correlation

Hybrid functionals: DFT + fraction of exact exchange B3LYP, PBE0, HSE06, ...

Basis sets

Expand in a finite basis $\{\varphi_i\}$: $\phi_j = \sum_{i=1}^N c_{ij} \varphi_i$

Finite Basis Numeric atom centered

Gaussians

Plane waves + Pseudopotenials

Slater type Grid based

Projector augmented waves (PAW)

... many more

Basis sets

Expand in a finite basis $\{\varphi_i\}$: $\phi_j = \sum_{i=1}^N c_{ij}\varphi_i$

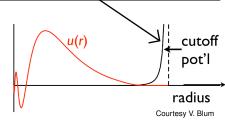
Numeric atom centered (FHI-aims)

$$arphi_i(r) = rac{u_i(r)}{r} Y_{lm}(\Omega)$$

$$\left[-rac{1}{2} rac{d^2}{dr^2} + rac{l(l+1)}{r^2} + v_i(r) + v_{ ext{cut}}
ight] u_i(r) = arepsilon_i u_i(r)$$

Flexible:

- » Free-atom like
- » Hydrogen like
- » Free lons, harmonic osc...



Basis sets

Expand in a finite basis $\{\phi_i\}$: $\phi_j = \sum_{i=1}^N c_{ij}\phi_i$



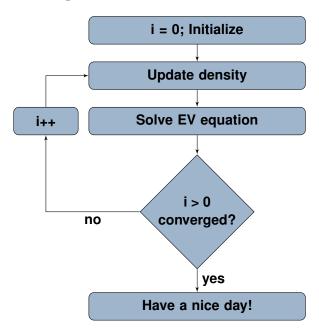
Generalized matrix eigenvalue equation in c_{ij}

$$\hat{h}^{KS}\phi = E\phi \quad \Rightarrow \quad \sum_{j}h_{ij}(c)c_{jl} = arepsilon_{l}\sum_{j}s_{ij}c_{jl}$$
Overlap matrix $s_{ij} = \langle arphi_{i}, arphi_{j}
angle$
Hamilton matrix $h_{ij} = \langle arphi_{i}, \hat{h}^{KS}arphi_{j}
angle$

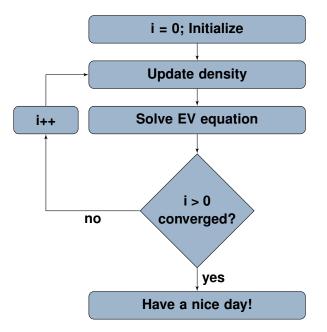


Self-consistent solution

Finding the self-consistent solution



Finding the self-consistent solution



Mixing (Pulay)

(Sca)Lapack ELPA

Criteria

Energy

Charge density
Sum of eigenvalues

Force

FHI-aims: 2 input files geometry.in control.in

geometry.in

control.in

Atomic structure

```
# x y z
atom 0.0 0.0 0.0 N
atom 1.1 0.0 0.0 N
# That's a comment
```

Units: Positions in Å

Energies in eV

geometry.in

Atomic structure

```
# x y z
atom 0.0 0.0 0.0 N
atom 1.1 0.0 0.0 N
# That's a comment
```

control.in

Physical model settings

```
xc pw-lda
charge 0.
spin none
relativistic none
```

Units:

Positions in Å

Energies in eV

geometry.in

Atomic structure

```
atom 0.0 0.0 0.0 N
atom 1.1 0.0 0.0 N
# That's a comment
```

Units: Positions in A

Energies in eV

control.in

Physical model settings

```
xc pw-lda
charge 0.
spin none
relativistic none
```

SCF convergence settings

```
occupation_type gaussian 0.01
mixer pulay
n_max_pulay 10
charge_mix_param 0.2
sc_accuracy_rho 1E-4
sc_accuracy_eev 1E-2
sc_accuracy_etot 1E-5
sc iter limit 100
```

geometry.in

Atomic structure

```
# x y z
atom 0.0 0.0 0.0 N
atom 1.1 0.0 0.0 N
# That's a comment
```

Units: Positions in Å Energies in eV

Manual, chap. 2.1

control.in

Physical model settings

```
xc pw-lda
charge 0.
spin none
relativistic none
```

SCF convergence settings

```
occupation_type gaussian 0.01
mixer pulay
n_max_pulay 10
charge_mix_param 0.2
sc_accuracy_rho 1E-4
sc_accuracy_eev 1E-2
sc_accuracy_etot 1E-5
sc_iter_limit 100
```

Species specifics

...

/usr/local/aimsfiles/species_default

Predefined species Copy-paste into control.in

- light
- tight
- really tight

/usr/local/aimsfiles/species_default

Predefined species Copy-paste into control.in Manual, chap. 2.2

light
 Fast, many production tasks

 Fast pre-relaxation

 tight
 Used to verify important results

 Converged settings

 really tight
 Heavily converged numerical settings

 Explicit convergence tests

/usr/local/aimsfiles/species_default

Predefined species Copy-paste into control.in Manual, chap. 2.2

light

tight

really tight

Increased accuracy:

Basis

Hartree potential

Basis cutoff potential

Integration grids

/usr/local/aimsfiles/species_default

Predefined species Copy-paste into control.in Manual, chap. 2.2

Increased accuracy:

Basis

• tight Hartree potential

Basis cutoff potential

Additionally converge basis ("tiers")!

Invoking FHI-aims ...

Introduction

Summary of control.in file

```
Invoking FHI-aims ...
Reading file control.in.
Reading geometry description geometry.in.
Preparing all fixed parts of the calculation.
```

Geometry independent preparations Basis set generation

Begin self-consistency loop: Initialization.

Date: 20110615, Time: 003756.746

Geometry dependent preparations Integration grid Initialization of charge density

5

```
Begin self-consistency loop: Initialization.

Date: 20110615, Time: 003756.746

Begin self-consistency iteration # 1
Date: 20110615, Time: 003756.810
```

First SCF cycle

```
Begin self-consistency loop: Initialization.
            Date: 20110615, Time: 003756.746
            Begin self-consistency iteration # 1
            Date: 20110615, Time: 003756.810
       First SCF cycle
               » Energy
THIS
TUTORIAL ——
               | Total energy
                                          -2920.4718774 eV
                 Total energy,
               | Electronic free energy : -2920.4718774 eV
Periodic metals only
```

```
Begin self-consistency loop: Initialization.

Date: 20110615, Time: 003756.746

Begin self-consistency iteration # 1
Date: 20110615, Time: 003756.810
```

First SCF cycle

» Self-consistency convergence accuracy

```
| Change of charge density : 0.4491E-04
| Change of sum of eigenvalues : -.7874E-02 eV
| Change of total energy : 0.2591E-07 eV
```

```
Begin self-consistency loop: Initialization.

Date: 20110615, Time: 003756.746

Begin self-consistency iteration # 2
Date: 20110615, Time: 003756.810
```

Second SCF cycle

» Self-consistency convergence accuracy

```
| Change of charge density : 0.4491E-04
| Change of sum of eigenvalues : -.7874E-02 eV
| Change of total energy : 0.2591E-07 eV
```

Self-consistency cycle converged.

7 Self-consistency cycle converged.

» Energy and forces

Total energy uncorrected : -0.290332172209443E+04 eV
| Total energy corrected : -0.290332172209443E+04 eV
| Electronic free energy : -0.290332172209443E+04 eV

Electronic free energy : -0.290332172209443E+04 eV

» SCF info

| Number of self-consistency cycles : 7

» Timings

7 Self-consistency cycle converged.

» Energy and forces

| Total energy uncorrected : -0.290332172209443E+04 eV | Total energy corrected : -0.290332172209443E+04 eV | Electronic free energy : -0.290332172209443E+04 eV

» SCF info

```
| Number of self-consistency cycles : 7
```

» Timings

8 Have a nice day.

7

Self-consistency cycle converged.

Postprocessing

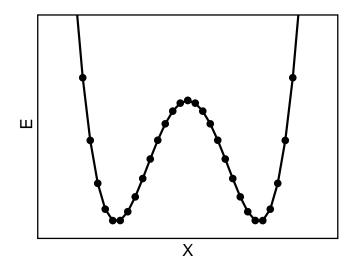
Structure optimization

- » Get next relaxation step
- » Redo SCF for new geometry

8

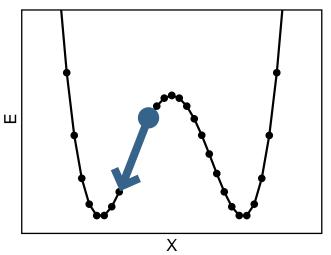
Have a nice day.

Forces



Forces

Energy gradient



Forces

Force component α on atom I

$$E^{tot} = E^{tot}(R_I, c_j)$$

$$F_{\alpha}^I = \frac{dE^{tot}}{dR_I^{\alpha}} = \frac{\partial E^{tot}}{\partial R_I^{\alpha}} + \sum_j \underbrace{\frac{\partial E^{tot}}{\partial c_j}}_{=0} \underbrace{\frac{dc_j}{dR_I^{\alpha}}}_{=0}$$

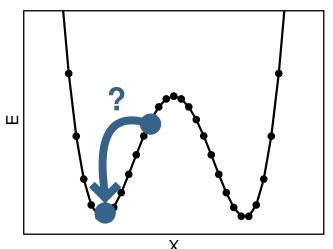
 E^{tot} is minimized with respect to c_j In FHI-aims basis functions depend on atomic postitions.

Keyword in control.in

sc_accuracy_forces 1E-4

Structure optimization

Aim: Find local minimum on potential energy surface (PES)

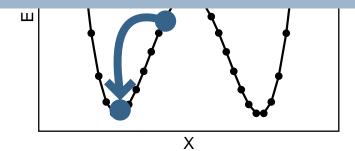


Structure optimization

Aim: Find local minimum on potential energy surface (PES)

Many methods!

Industry standard: quasi Newton methods



Structure optimization in FHI-aims

Basic idea: local harmonic model of PES

$$\tilde{E}(X_0 + X) = E(X_0) - F^T(X_0)X + \frac{1}{2}X^TB(X_0)X$$

Next relaxation step

Trust radius method (TRM) BFGS + simple line search

recommended also implemented

Keyword in control.in

relax_geometry trm 1E-3



Force convergence criterion eV/Å

Manual, chap. 3.10

How do atoms move in a potential V?

⇒ Solve equations of motion!

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{R}_i} + \frac{\partial V}{\partial R_i} = 0$$

Kinetic energy: TPotential energy: V

How do atoms move in a potential V?

⇒ Solve equations of motion!

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{R}_i} + \frac{\partial V}{\partial R_i} = 0$$

Kinetic energy: *T* Potential energy: *V*

 \Rightarrow Taylor expansion of V around equilibrium position R_0 + harmonic approximation

$$V = V_0 \underbrace{-F(R_0)R}_{\text{=0 equilibrium}} + \frac{1}{2}R^TB(R_0)R \underbrace{+...higher\ terms}_{\text{=0 harmonic approximation}}$$

F: Forces
B: Hessian

How do atoms move in a potential V?

⇒ Solve equations of motion!

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{R}_i} + \frac{\partial V}{\partial R_i} = 0$$

Kinetic energy: TPotential energy: V

⇒ Solution

$$R \sim ue^{i\omega t}$$
, with $Bu - \omega^2 Mu = 0$

Eigenmodes *u*Mass-weighted diagonal matrix *M*

(1) Harmonc approximation valid(2) Equilibrium geometry

Solve $\mathbb{R}^{3N} \times \mathbb{R}^{3N}$ eigenvalue equation

$$\det(B - \omega^2 M) = 0$$

Hessian B

$$B^{ij} := \frac{\partial^2 E}{\partial R_i \partial R_j}$$

In practice: finite central numerical differences (of forces)

Wrapper

> aims_vibrations.mpi.pl

Manual, chap 4.6

Solve $\mathbb{R}^{3N} \times \mathbb{R}^{3N}$ eigenvalue equation

$$\det(B - \omega^2 M) = 0$$

- » Eigenmodes $\{Q_i, i \in 1...3N\}$
- » Eigenfrequencies

Solve $\mathbb{R}^{3N} \times \mathbb{R}^{3N}$ eigenvalue equation

$$\det(B - \omega^2 M) = 0$$

- » Eigenmodes $\{Q_i, i \in 1...3N\}$
- » Eigenfrequencies
- » 6 (almost) zero frequency modes (if molecule non-linear) translations + rotations
- » Imaginary frequency ⇒ Saddle point

Solve $\mathbb{R}^{3N} \times \mathbb{R}^{3N}$ eigenvalue equation

$$\det(B - \omega^2 M) = 0$$

- » Eigenmodes $\{Q_i, i \in 1...3N\}$
- » Eigenfrequencies
- » 6 (almost) zero frequency modes (if molecule non-linear) translations + rotations
- » Imaginary frequency ⇒ Saddle point
- » Infrared intensities (derivative of dipole moment μ)

$$I_i \sim \left| rac{d\mu}{dQ_i}
ight|^2$$

Based on harmonic approximation !

Limitation: Exercise 2.6 Beyond: Tutorial 5 (MD)

- » Eigenmodes $\{Q_i, i \in 1...3N\}$
- » Eigenfrequencies
- » 6 (almost) zero frequency modes (if molecule non-linear) translations + rotations
- » Imaginary frequency ⇒ Saddle point
- » Infrared intensities (derivative of dipole moment μ)

$$I_i \sim \left| rac{d\mu}{dQ_i}
ight|^2$$

Visualization

Orbitals and densities

Keyword in control.in

output cube eigenstate 52
output cube total_density

Get: *.cube file - values on a regular 3D grid.

Software: molden (jmol, gdis, xcrysden)

⇒ Appendix of handout



Practical issues

Each calculation one directory

```
> mkdir tutorial1
> cd tutorial1
> mkdir N2
```

2 input files

```
geometry.in control.in
```

Launching FHI-aims calculation

```
mpirun -np 4 aims.hands-on-2011.scalapack.mpi.x
| tee calculation.out
```

... scripting helps!

(Sample scripts in appendix of handout)

Timeline

PART 1: Basic electronic structure PART 2: Born-Oppenheimer surface PART 3: Visualization	90 mins 90 mins 30 mins
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Next: exercises

PART 1 Basic electronic structure

90 (CPU 20) mins

PART 2 Born Oppenheimer Surface

Born Oppenheimer Surface

90 (CPU < 20) mins

Exercise 4: Planar NH₃ (!)

geometry.in

atom 0.0 0.0 0.0 N atom 0.8 -0.5 0.0 H atom -0.8 -0.5 0.0 H atom 0.0 1.0 0.0 H

10 min

control.in

```
xc pw-lda
charge 0
spin none
relax_geometry trm 1E-3
sc_accuracy_eev 1E-2
sc_accuracy_rho 1E-4
sc_accuracy_etot 1E-5
sc_iter_limit 300
```

+ copy/paste light species

How can I do a geometry optimization?

Relax the NH₃ molecule starting from a planar initial guess.

Exercise 4: Planar NH₃ (!)

10 min

geometry.in

control.in

atom 0.0 0.0 0.0 N atom 0.8 -0.5 0.0 H atom -0.8 -0.5 0.0 H atom 0.0 1.0 0.0 H

```
xc pw-lda
charge 0
spin none
relax_geometry trm 1E-3
sc_accuracy_eev 1E-2
sc_accuracy_rho 1E-4
sc_accuracy_etot 1E-5
sc_iter_limit 300
```

Visualization: Molden ⇒ Appendix of handout

How does the fully relaxed structure look like?

Exercise 4: Planar NH₃ (!)

10 min

geometry.in

control.in

 atom
 0.0
 0.0
 0.0
 N

 atom
 0.8
 -0.5
 0.0
 H

 atom
 -0.8
 -0.5
 0.0
 H

 atom
 0.0
 1.0
 0.0
 H

```
xc pw-lda
charge 0
spin none
relax_geometry trm 1E-3
sc_accuracy_eev 1E-2
sc_accuracy_rho 1E-4
sc_accuracy_etot 1E-5
sc_iter_limit 300
```

Visualization: (/pub/tutorial1/utilities)

- > create_relax_movie.pl aims.NH3.out > NH3.molden
- > molden NH3.molden

How does the fully relaxed structure look like?

Exercise 4: Planar NH₃ (!) 10 min

Solution

Stays planar



Exercise 5: Planar NH₃ (!) 15 min

Is this geometry stable?

 \Rightarrow perform a vibrational analysis

```
> aims.vibrations.hands-on-2011.mpi.pl NH3_planar
```

control.in + geometry.in in same folder!

From Exercise 4:

- control.in
- geometry.in from output file grep Final atomic structure

Exercise 5: Planar NH₃ (!) 15 min

Do you find negative frequencies? How do they look like?

```
> aims.vibrations.hands-on-2011.mpi.pl NH3_planar
```

control.in + geometry.in in same folder!

From Exercise 4:

- control.in
- geometry.in from output file grep Final atomic structure

Visualization: /pub/tutorial1/utilities

- > troublemaker -xyz2molden NH3_planar.xyz > NH3.molden
- > molden NH3.molden

Exercise 5: Planar NH₃ (!) 15 min

Solution

Unstable Imaginary frequency: out of plane mode



Exercise 6: Getting NH₃ right 10 min

Can I find a stable geometry?

(1) Distort the geometry of the planar NH₃ molecule along the imaginary mode ← *xyz from Exercise 5 (/pub/tutorial1/utilities).

```
> ./troublemaker.pl NH3_planar.xyz | tail -4
```

(2) Relax the geometry again.

Same control.in as before

How does the structure look like? What is the difference in energy with respect to the planar geometry? Exercise 6: Getting NH₃ right 10 min

Solution

3D geometry 0.2 eV lower in energy



Exercise 7: 3D NH₃ 15 min

Now stable?

For 3D geometry, perform a vibrational analysis for the 3D optimized NH_3 molecule.

Is the structure stable?
What does the mode lowest in energy that is not a rotation or translation look like?

Exercise 7: 3D NH₃ 15 min

Now stable?

For 3D geometry, perform a vibrational analysis for the 3D optimized NH₃ molecule.

Is the structure stable?
What does the mode lowest in energy that is not a rotation or translation look like?

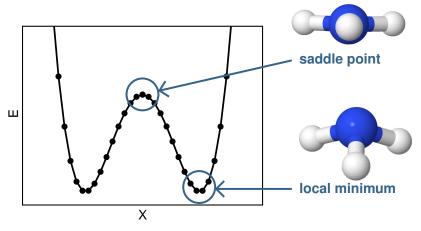
Solution

Stable
Umbrella mode (lowest non-trivial mode)



Summary

A: Saddle point or local minimum?



B: Do a vibrational analysis!

Exercise 8: Infrared (IR) intensities 20 min

Are (numerical) settings sensitive?

- ⇒ Compare IR spectra!
- (1) Optimize geometry
- (2) Perform vibrational analysis

Exercise 8: Infrared (IR) intensities 20 min

Are (numerical) settings sensitive?

- ⇒ Compare IR spectra!
- (1) Optimize geometry
- (2) Perform vibrational analysis

Vibration script does that for you!

relax_geometry trm 1E-3

control.in

Exercise 8: Infrared (IR) intensities 20 min

Important! relax_geometry trm 1E-3

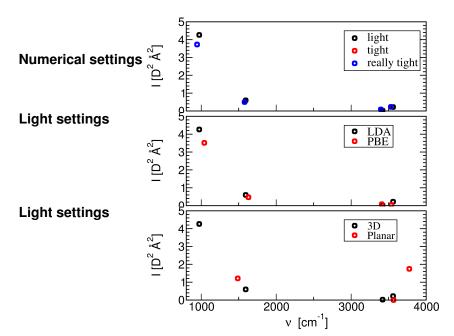
- Perform a convergence tests with respect to light, tight and really tight settings of the IR spectrum (xc pw-lda).
- For light setting compute the PBE IR spectrum and compare it with the LDA spectrum.
- Compare to experiment.
- (Bonus) For light settings compare the IR spectrum of the planar and 3D NH₃ molecule (xc pw-lda).

```
> aims.vibrations.hands-on-2011.mpi.pl NH3_foo
```

Visualization: 2 column *dat file (gnuplot/xmgrace) (/pub/tutorial1/utilities)

```
extract_harmivb.sh NH3_foo.vib.out > NH3_foo.dat
```

Solution

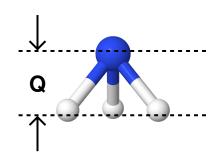


Exercise 9: Limits of the harmonic approximation (!)

Where does it break down?

Consider a cut through the Born Oppenheimer surface where the N atom is distorted perpendicular to the plane spaned by the H atoms.

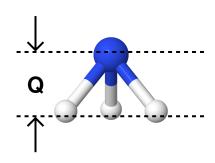
Define coordinate Q



Exercise 9: Limits of the harmonic approximation (!)

Compute total energies as a function of Q:

- Define 1D PES
 Constrained relaxation
 H atoms in one plane
- Out-of-plane mode relative to planar geometry
- Umbrella mode relative to 3D geometry



In which range is the process described by one mode only?

Exercise 9 - Part I 15 min

Constrained relaxation relax_geometry trm 1E-3 Q within [0,1.2] Å, stepwidth 0.04 Å.

geometry.in

```
atom 0.000 0.000 <Q> N

constrain_relaxation .true.

atom 0.873 -0.504 0.000 H

constrain_relaxation z

atom -0.873 -0.504 0.000 H

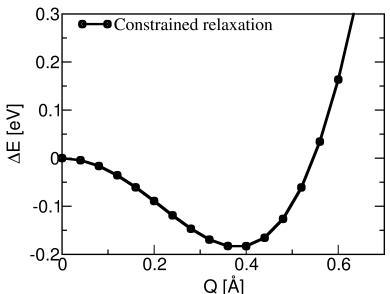
constrain_relaxation z

atom 0.000 1.008 0.000 H

constrain_relaxation z
```

Reference energy with respect to saddle point (Q = 0) Sample script? \rightarrow Appendix A of handout

Solution



Exercise 9 - Part II 30 min

Approximate PES along planar out-of-plane mode

- (1) get *xyz file from planar vibrational analysis
- (2) For Q within [0,1.2] Å, stepwidth 0.04 Å generate the corresponding geometry.in files

```
./troublemaker -norm N -mode 1 NH3_planar.xyz
```

Important: $Q \neq N$

N describes the extension of the whole mode. Can you find the relation $Q \leftrightarrow N$ by inspecting the *xyz file ?

(3) Calculate the energy for this geometry.

Reference it to the saddle point energy (Q=0)

Do NOT perform a structure relaxation!
... similar for umbrella mode of 3D geometry

Exercise 9 - Part II 30 min

Approximate PES along planar out-of-plane mode

- (1) get *xyz file from planar vibrational analysis
- (2) For Q within [0,1.2] Å, stepwidth 0.04 Å generate the corresponding geometry.in files

./troublemaker -norm
$$N$$
 -mode 1 NH3_planar.xyz

Important: $Q \neq N$

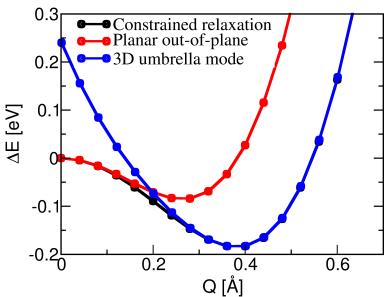
$$N = Q/0.6966$$

(3) Calculate the energy for this geometry.

Reference it to the saddle point energy (Q=0)

Do NOT perform a structure relaxation! ... similar for umbrella mode of 3D geometry

Solution



Exercise 10: Mode mixing (!) (Bonus)

Consider the 'reaction coordinate' Q of the previous exercise. For every geometry step project the 'real PES' (i.e. Part I of exercise 9) on the modes of the planar NH₃ molecule.

$$\mathbf{X} = \mathbf{X_0} + \sum_{i=1}^{3N} \lambda_i \mathbf{M_i}, \qquad \mathbf{M_i}, \mathbf{X}, \mathbf{X_0} \in \mathbb{R}^{3N} \qquad \lambda_i \in \mathbb{R}$$

Was our assumption that the system can by described by the out-of-pane mode only appropriate?

Exercise 10: Mode mixing (!) (Bonus)

- (1) Get the *xyz file from the planar vibrational analysis.
- (2) Get the constrained relaxed geometries from EX 9 (/pub/tutorial1/utilities).

```
./FinalAtomicStructure.sh aims.NH3_PES_Q.out > geo_Q.in
```

(3) For every Q: get projection coefficients

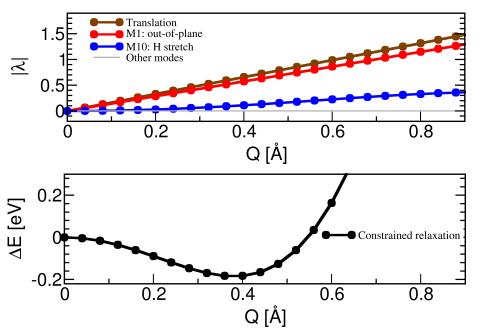
 \Rightarrow Plot λ as a function of Q. Which modes contribute ?

```
> troublemaker.pl -project geo_Q.in NH3_planar.xyz
```

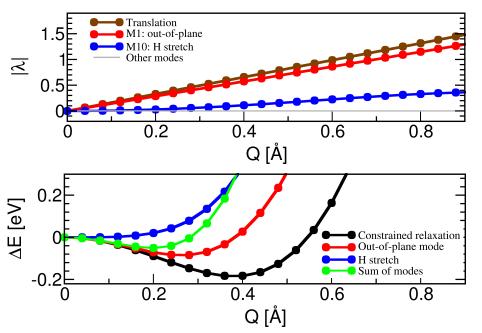
(4) For contributing modes compute the energetic contribution as a function of Q

> ./troublemaker.pl -mode N -norm λ NH3_planar.xyz

Solution



Solution



PART 3

30 (CPU < 5) mins

Visualization

That's it!