

# Scaling Relations: A Computational Approach to Catalyst Search

Aleksandra Vojvodic





*We now stand at the brink of a “Control Age” that could spark revolutionary changes in how we inhabit our planet, paving the way to a bright and sustainable future for us all.\**



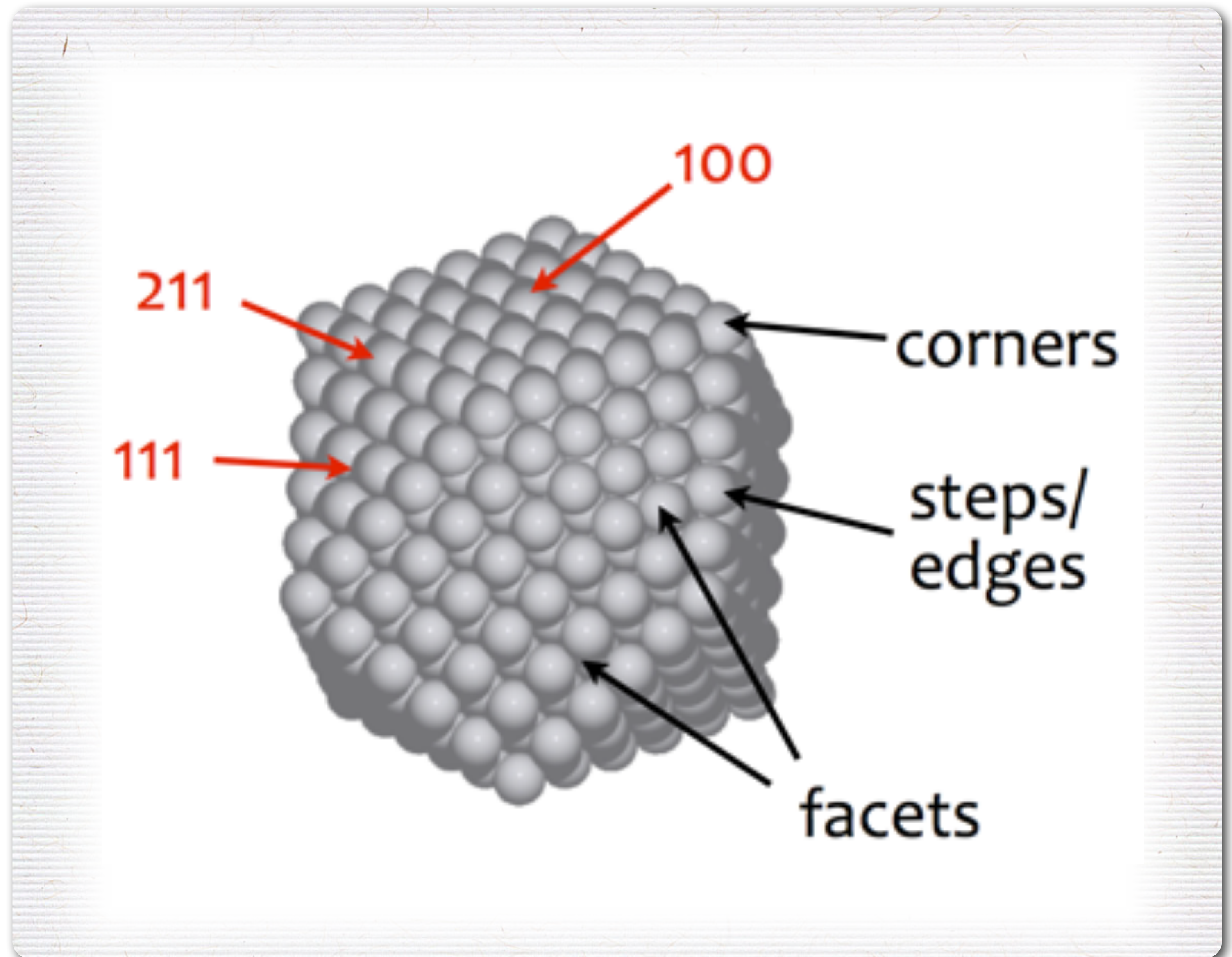
- How to design energy- & atom-efficient synthesis of new materials with tailored properties to reduce environmental impact?
  - Computer guided catalyst and materials design rooted in fundamental understanding
  - Search for the “materials genome”

\* Directing Matter and Energy: Five Challenges for Science and the Imagination, Basic Energy Sciences Report, U.S. DOE (2009).

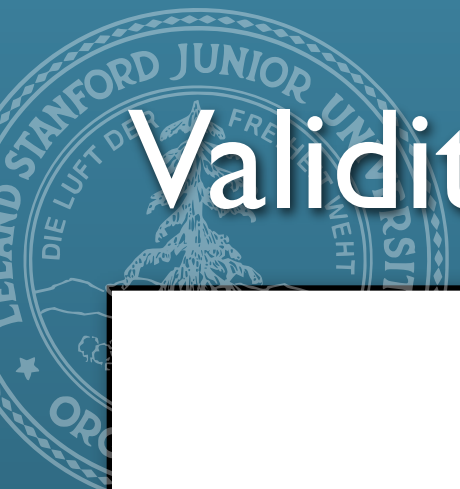


## Nano-particles:

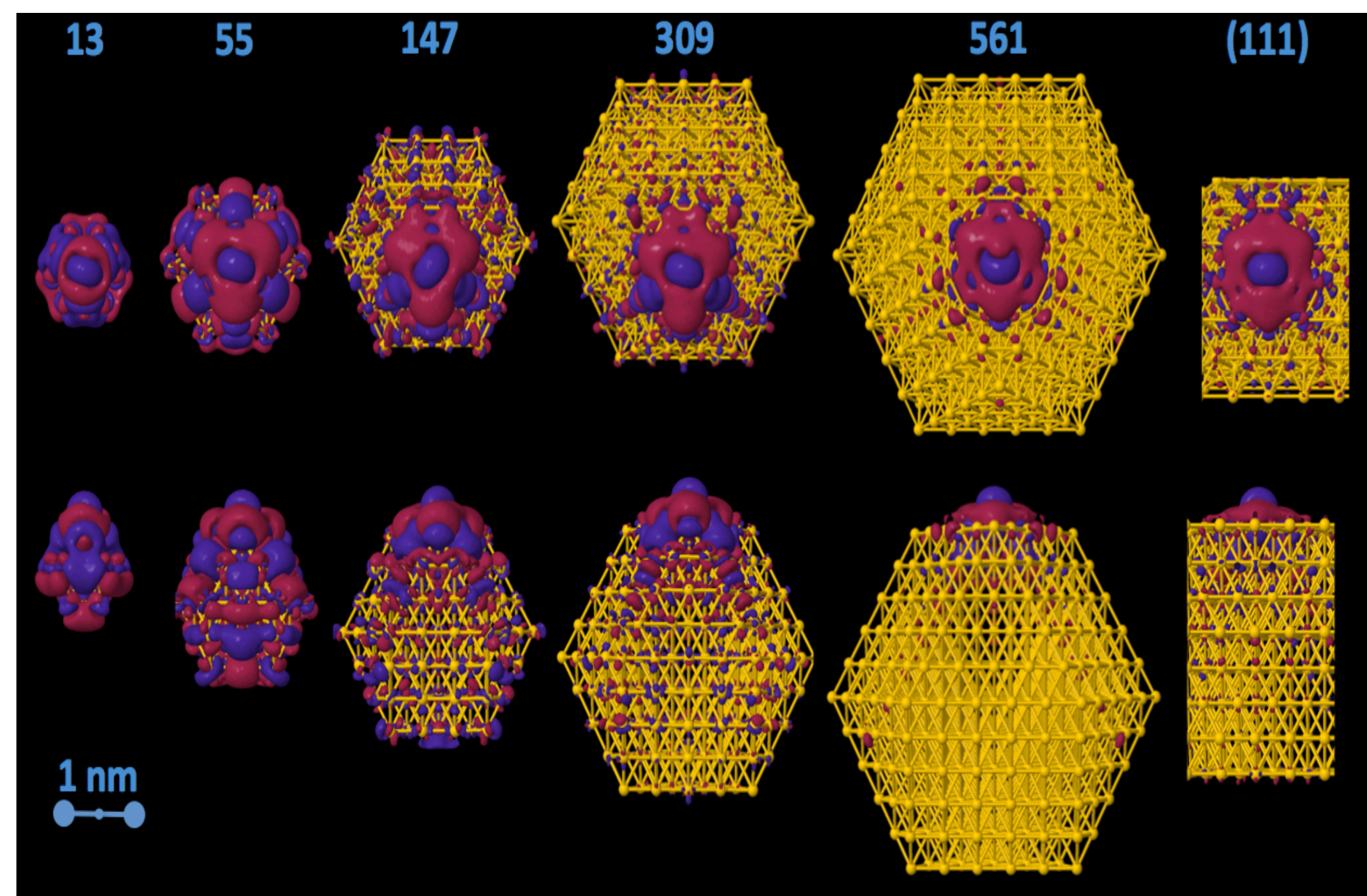
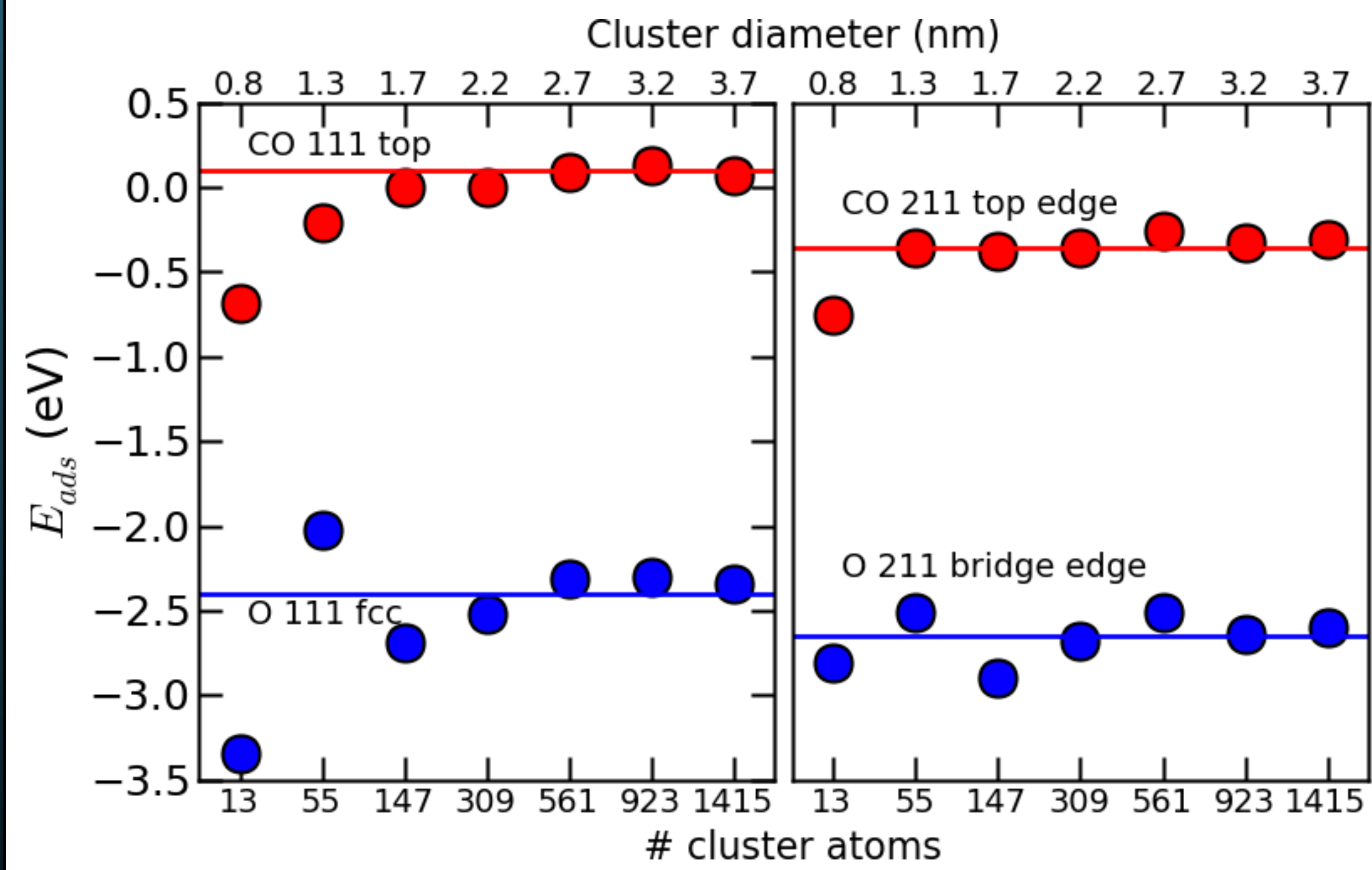
- Finite size
- Many facets
- Steps/Edges
- Corners
- Defects
- Support





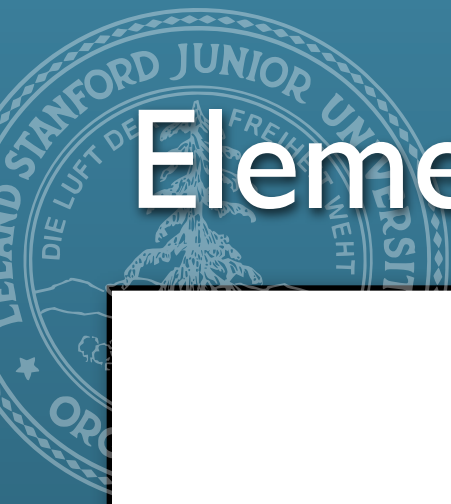


## adsorption on Au nano-particles

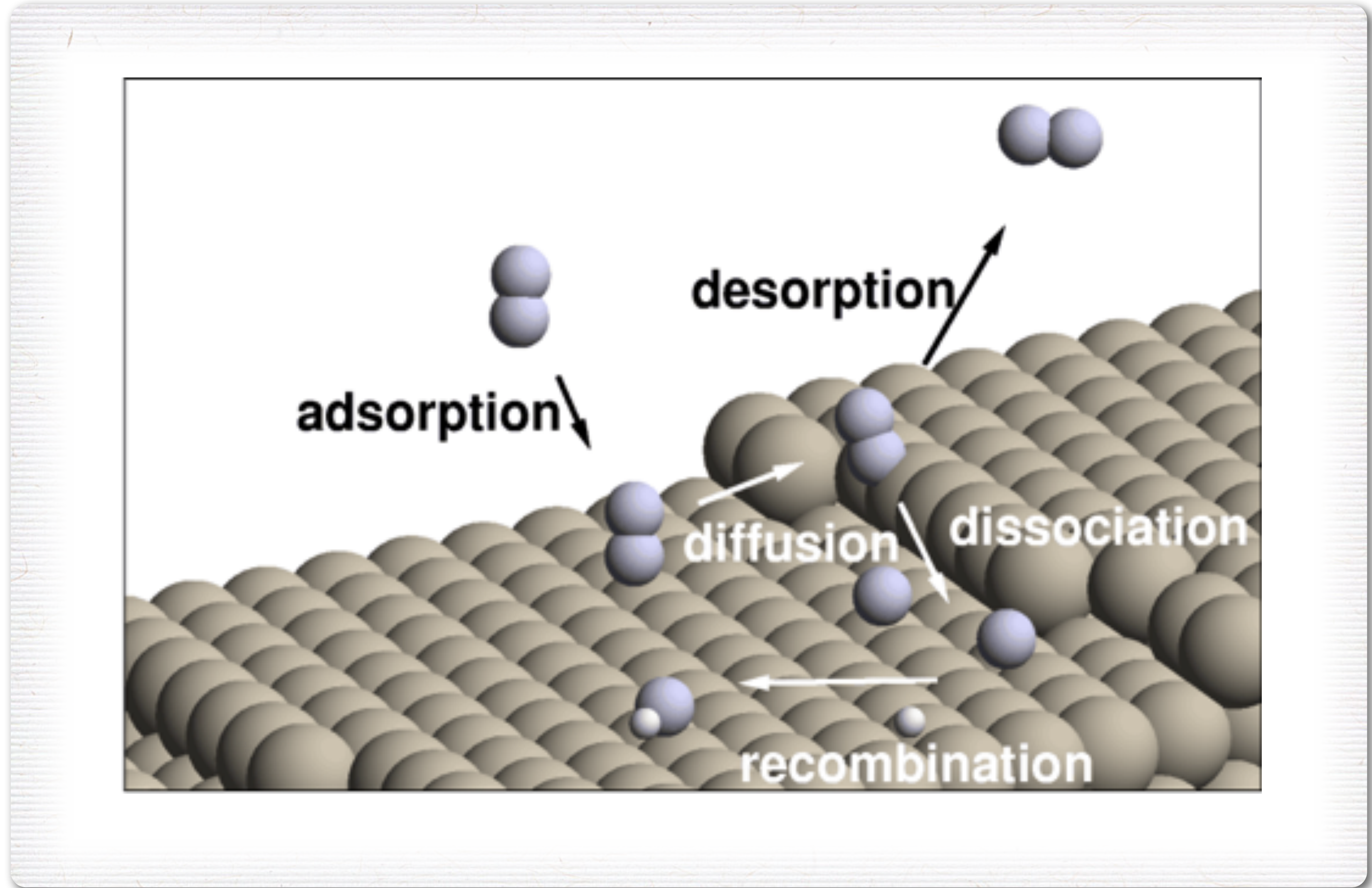


A. Larsen, J. Kleis, K. Thygesen, J. K. Nørskov, K. Jacobsen, Phys. Rev. B **84** (2011)





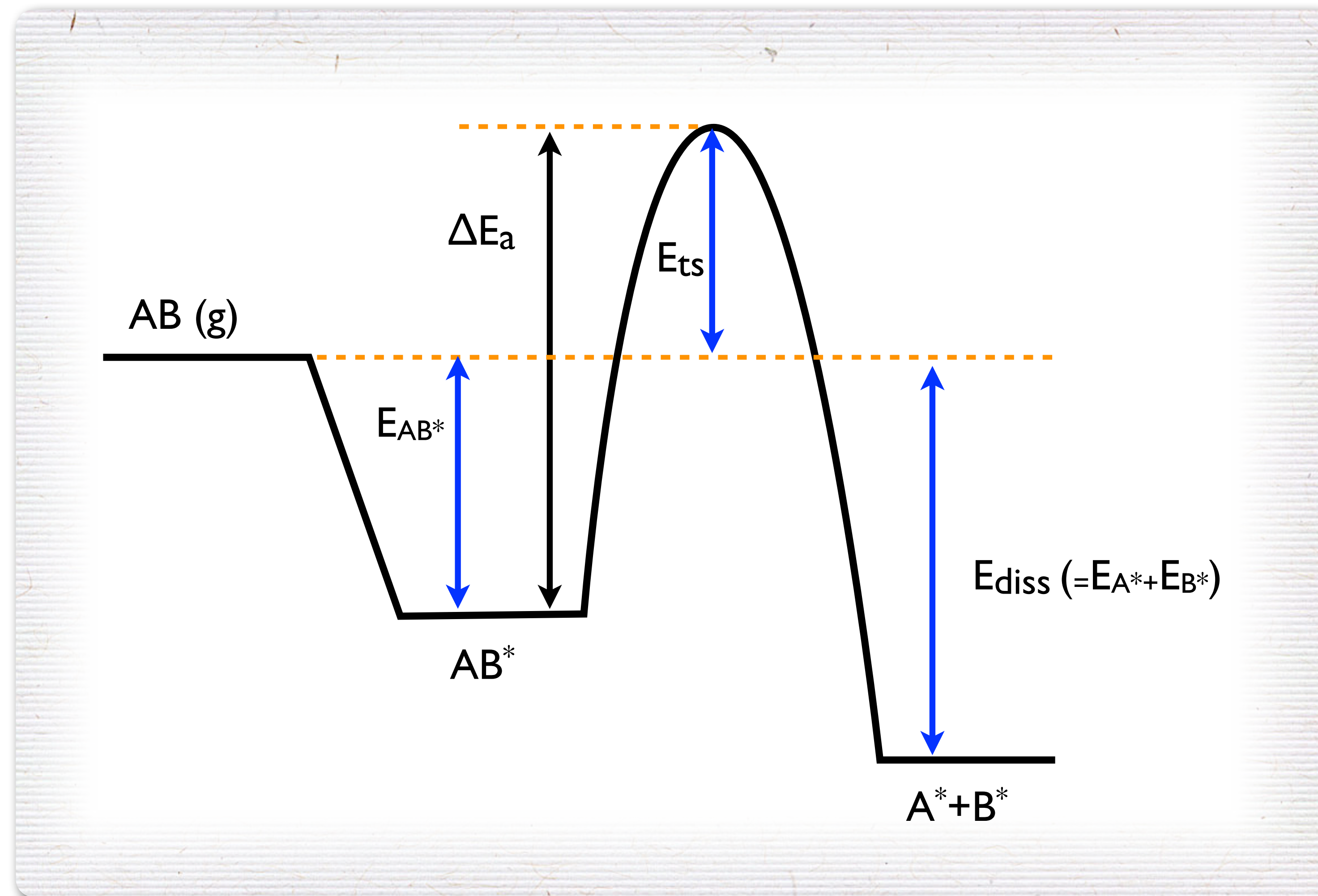
# Elementary Surface Reaction Steps





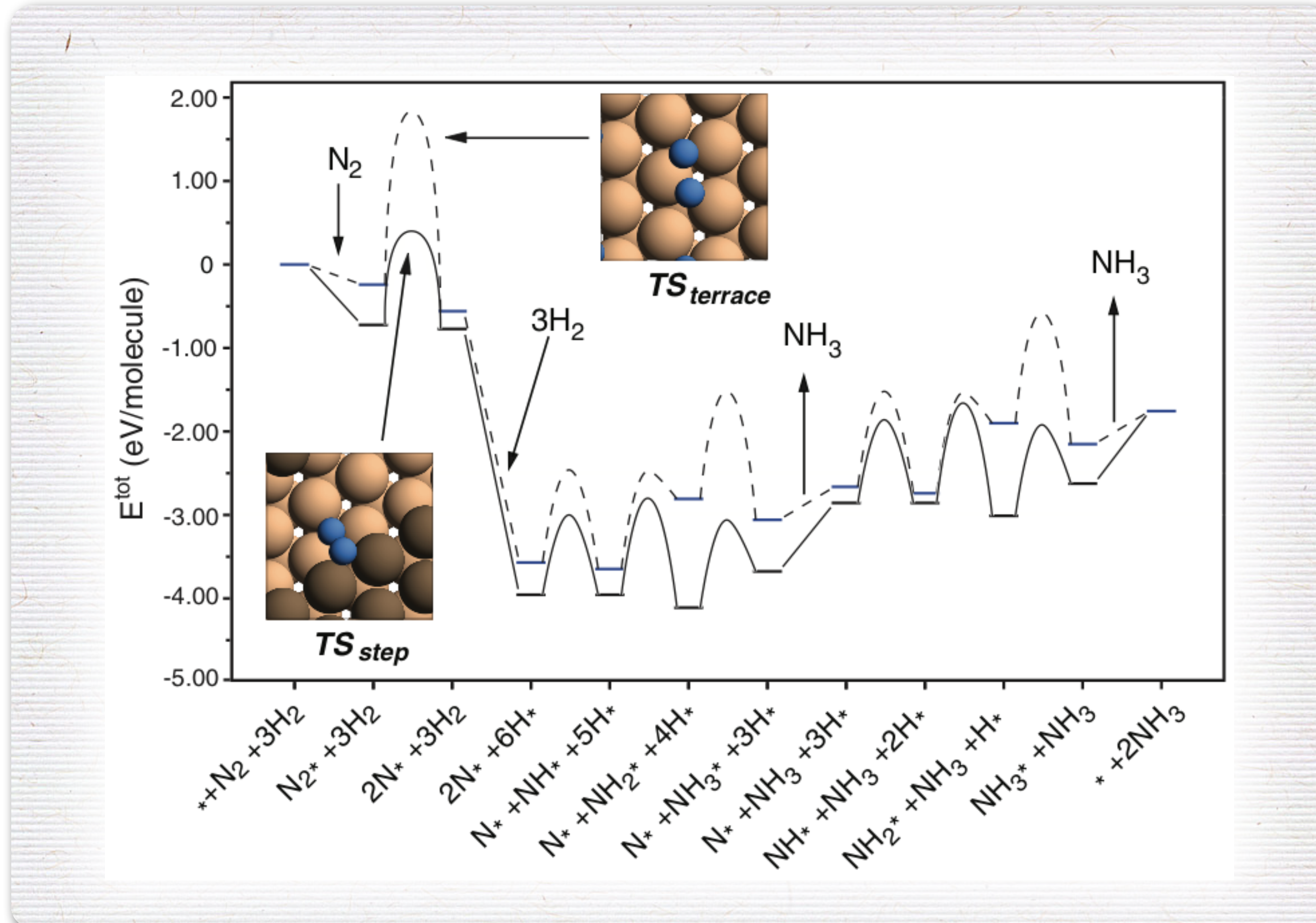
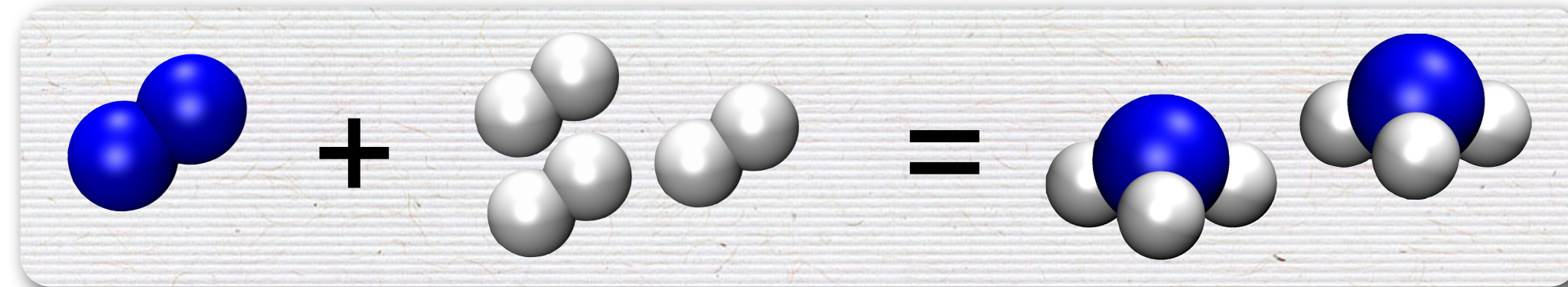


# Key Ingredients to Surface Reactions: Adsorption and Barriers

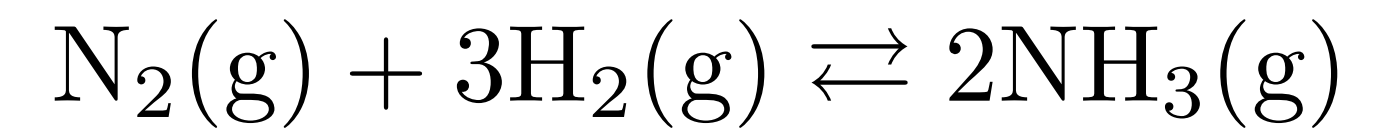




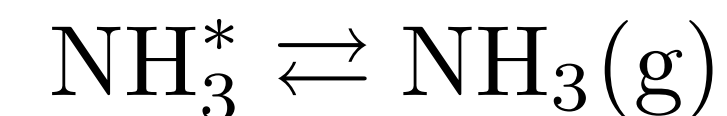
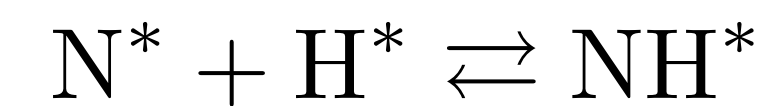
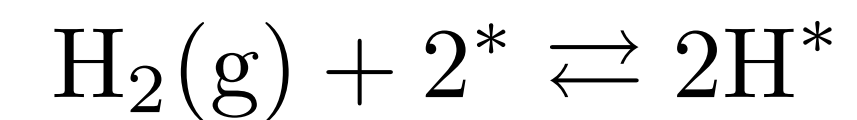
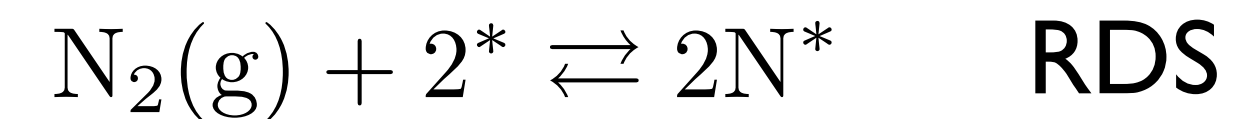
# Ammonia Synthesis, Stepped Ru(0001) Surface



- Overall reaction



- Elementary steps



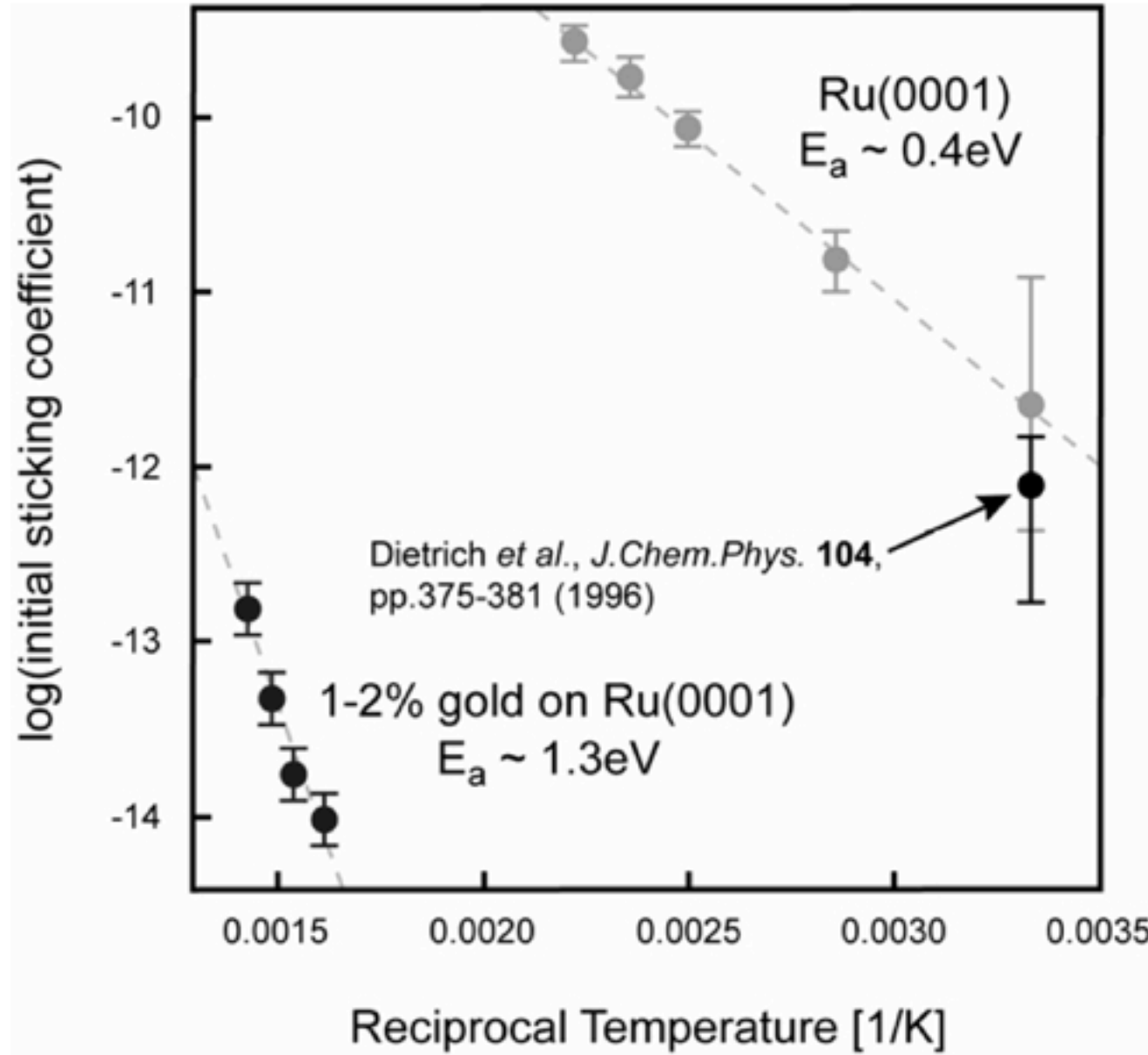
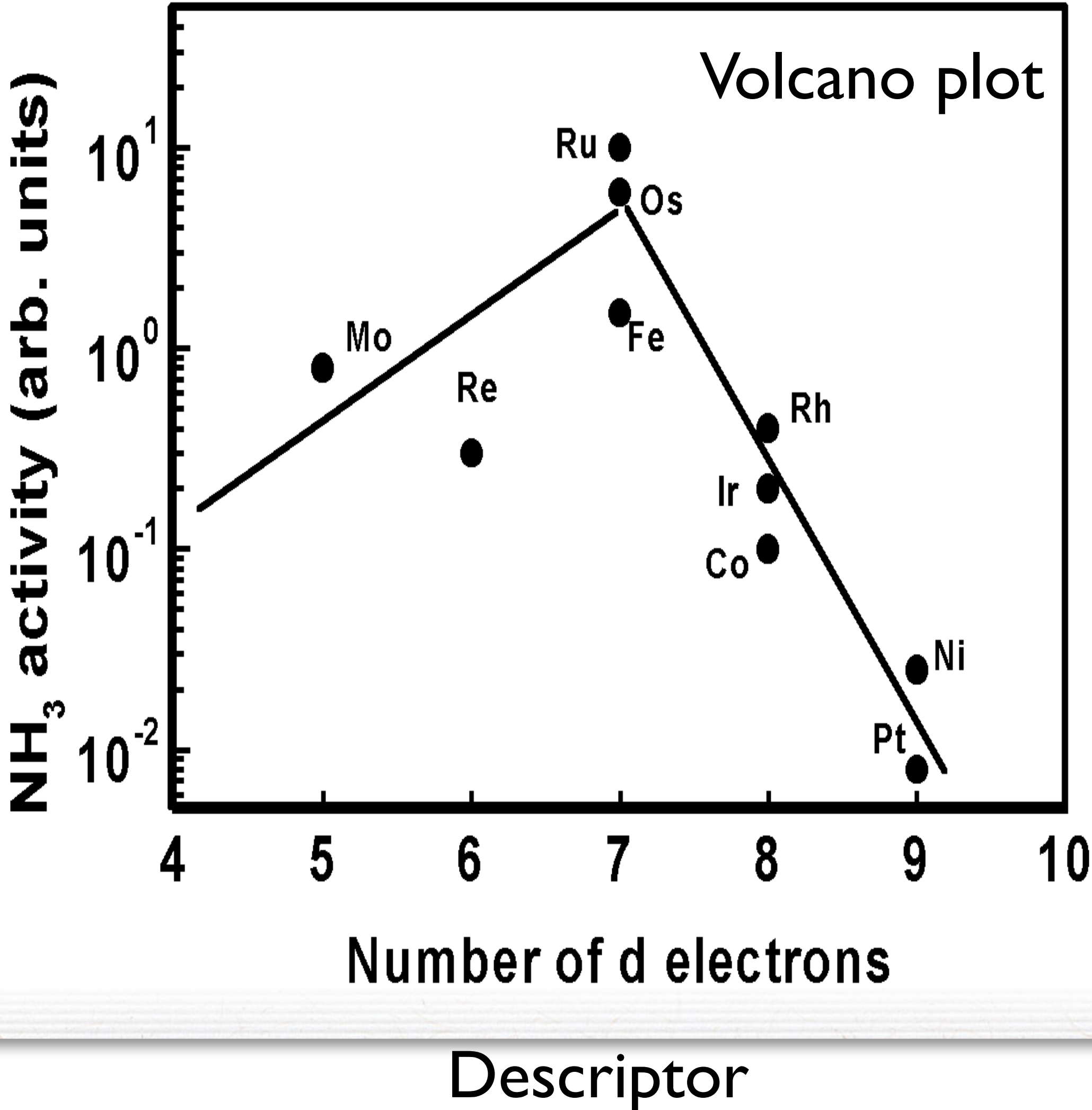
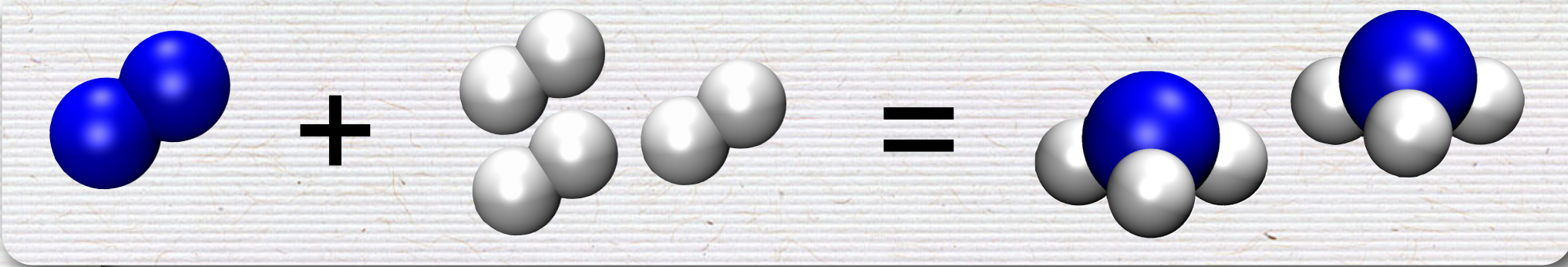
- Rate of reaction

$$R = R_1 = k_1 p_{\text{N}_2} \theta_*^2 (1 - \gamma)$$

K. Honkala et al., Science **307**, 555 (2005)



# What About Other Catalysts?



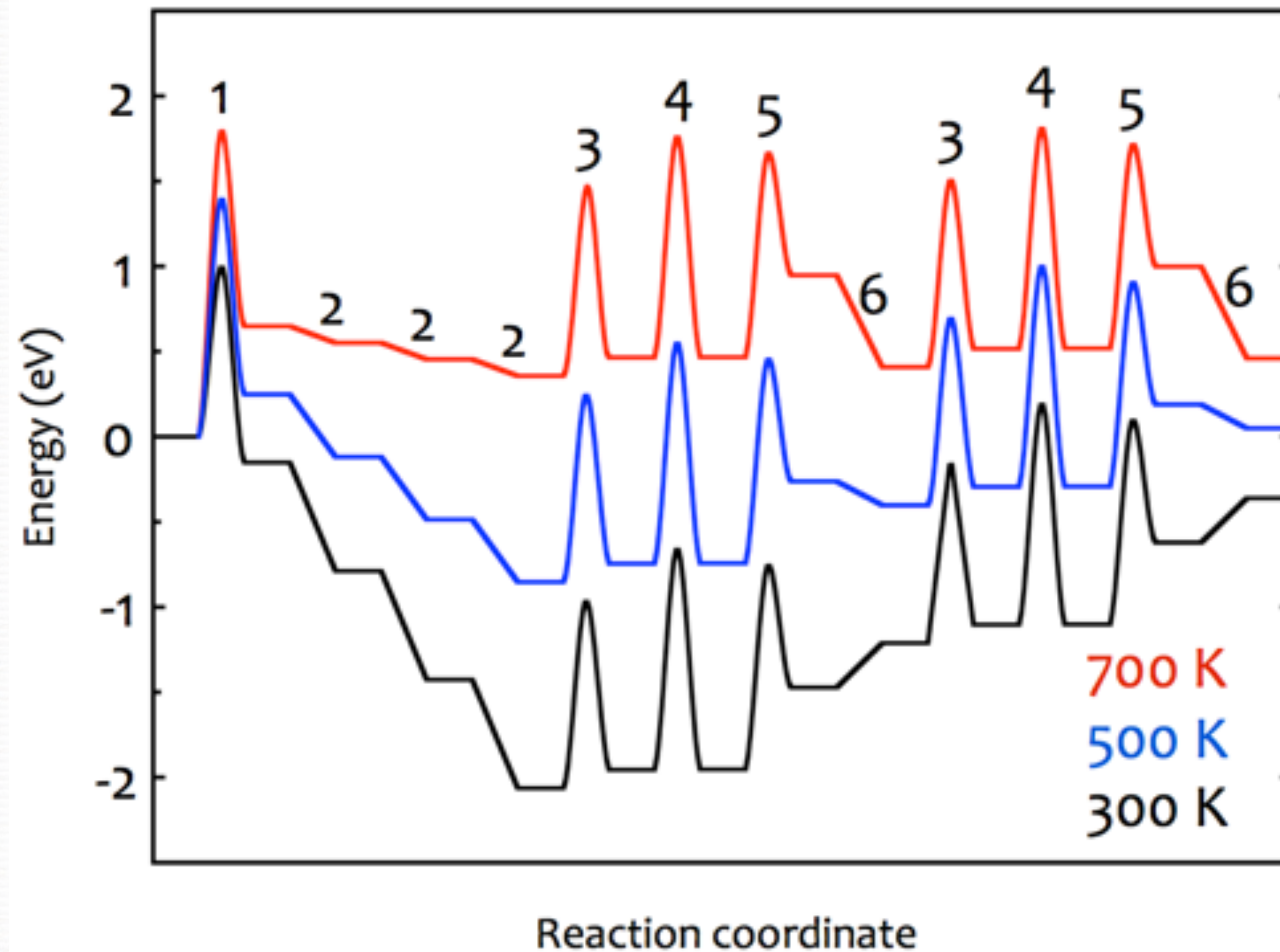
S. Dahl et al., *Phys. Rev. Lett.* **83**, 1814 (1999)

- Why is Ru good and Ni poor?
- What determines reactivity?

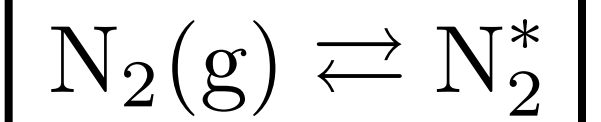
Ozaki and Aika, *Catalysis I* (Anderson and Boudart, Ed.)



## Ammonia Synthesis on Ru(0001)



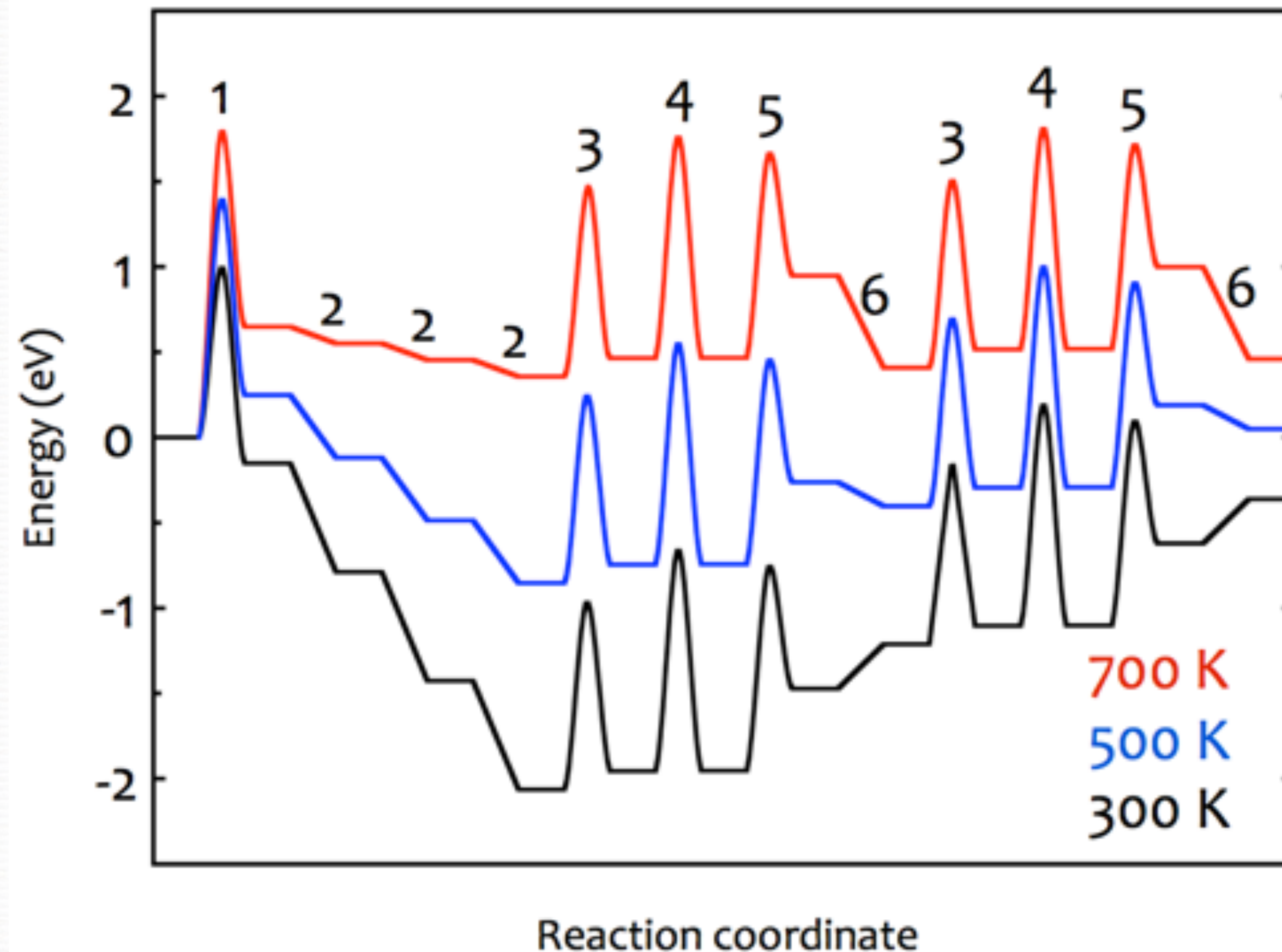
1.  $\text{N}_2(\text{g}) + 2^* \rightleftharpoons 2\text{N}^*$
2.  $\text{H}_2(\text{g}) + 2^* \rightleftharpoons 2\text{H}^*$
3.  $\text{N}^* + \text{H}^* \rightleftharpoons \text{NH}^*$
4.  $\text{NH}^* + \text{H}^* \rightleftharpoons \text{NH}_2^*$
5.  $\text{NH}_2^* + \text{H}^* \rightleftharpoons \text{NH}_3^*$
6.  $\text{NH}_3^* \rightleftharpoons \text{NH}_3(\text{g})$



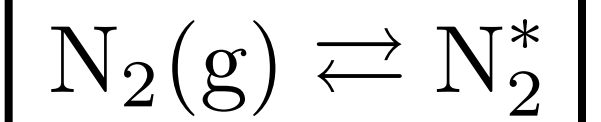
• How many energetic variables are there?



## Ammonia Synthesis on Ru(0001)



1.  $\text{N}_2(\text{g}) + 2^* \rightleftharpoons 2\text{N}^*$
2.  $\text{H}_2(\text{g}) + 2^* \rightleftharpoons 2\text{H}^*$
3.  $\text{N}^* + \text{H}^* \rightleftharpoons \text{NH}^*$
4.  $\text{NH}^* + \text{H}^* \rightleftharpoons \text{NH}_2^*$
5.  $\text{NH}_2^* + \text{H}^* \rightleftharpoons \text{NH}_3^*$
6.  $\text{NH}_3^* \rightleftharpoons \text{NH}_3(\text{g})$



• How many energetic variables are there?

6 intermediates

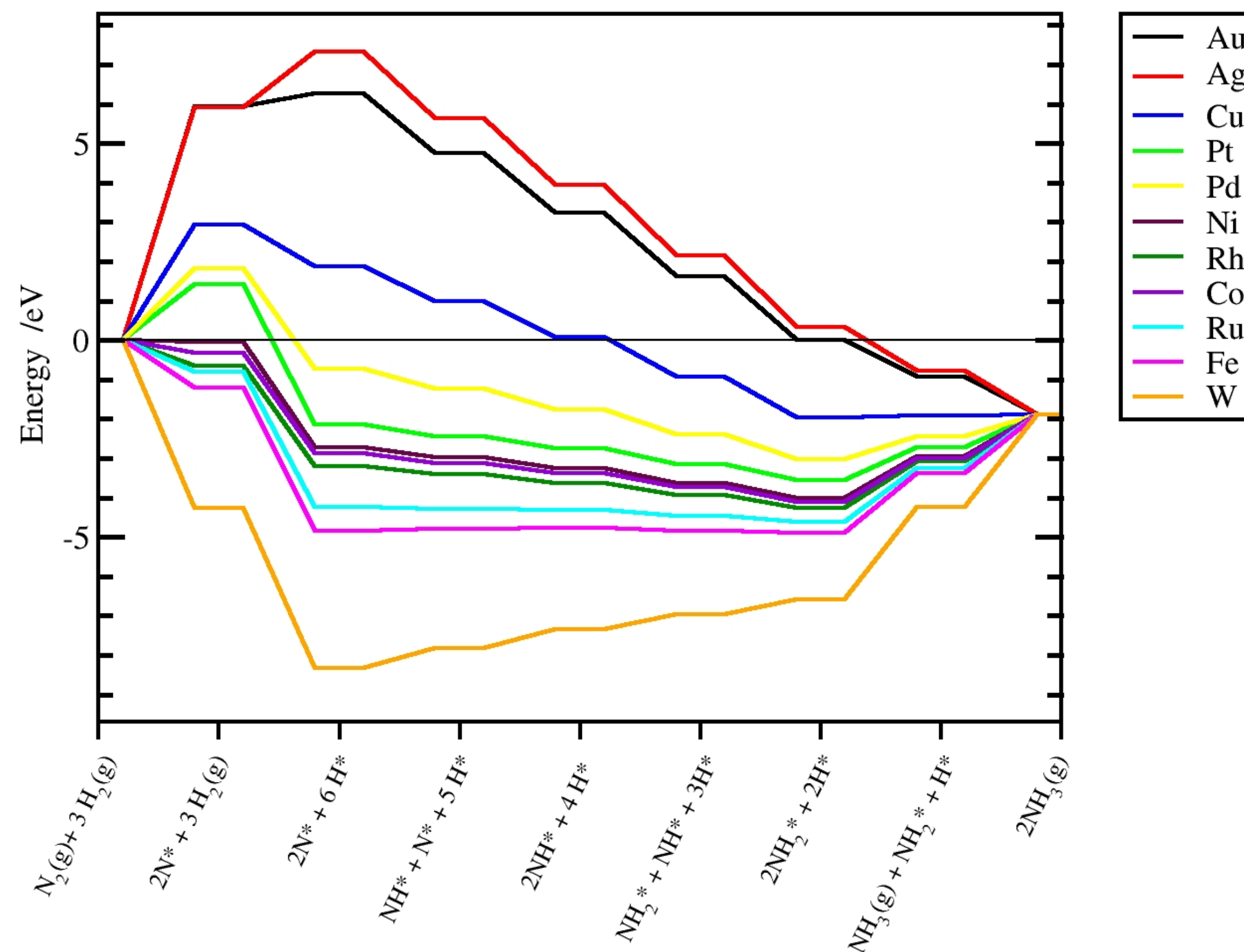
+

6 transition states

=

12 different energy parameters





## Adsorption scaling

- Relationships between bond energies of different intermediates

## Brønsted-Evans-Polanyi (BEP)/ Transition state scaling

- Relationships between activation energies and reaction energies

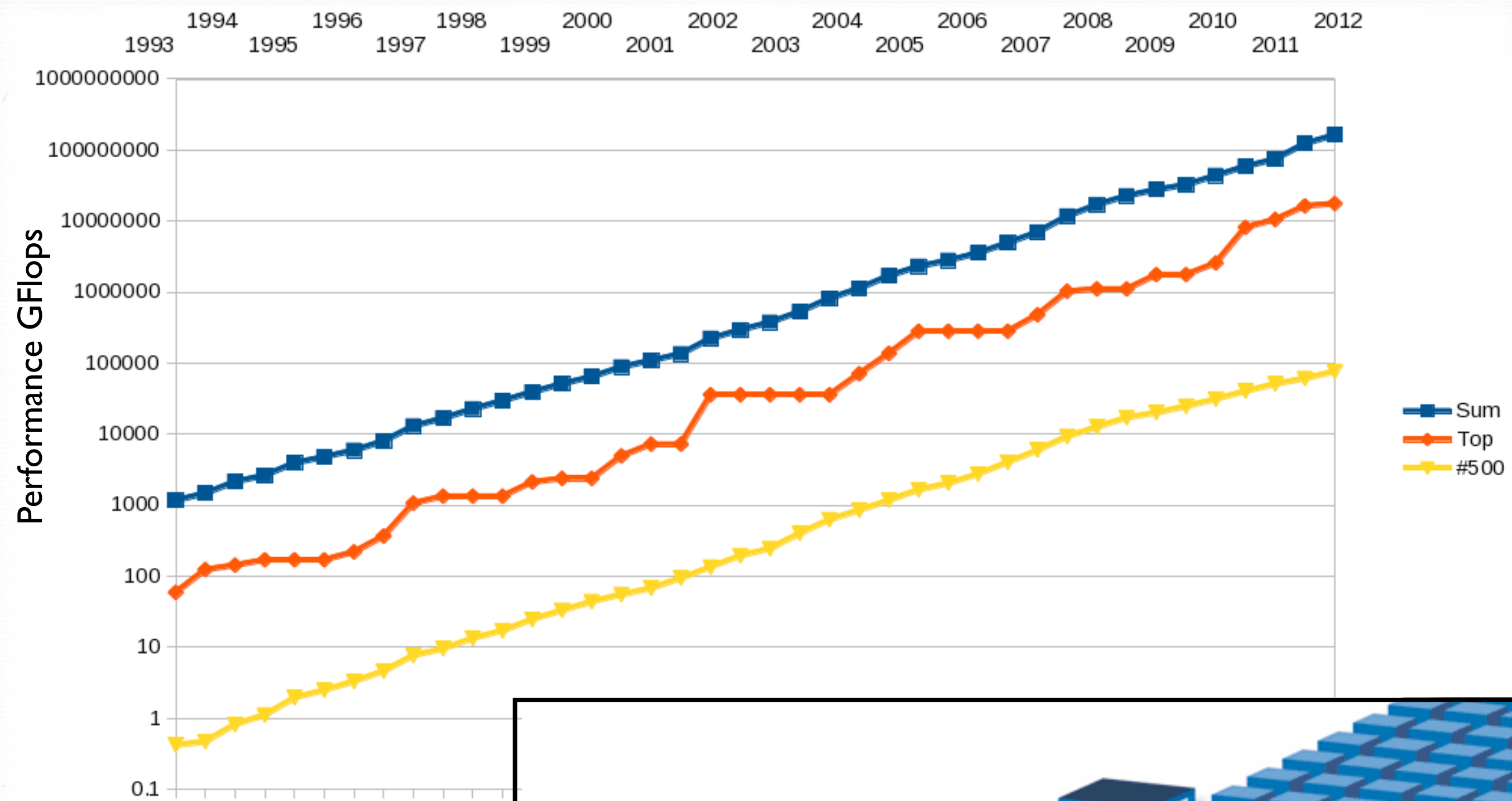




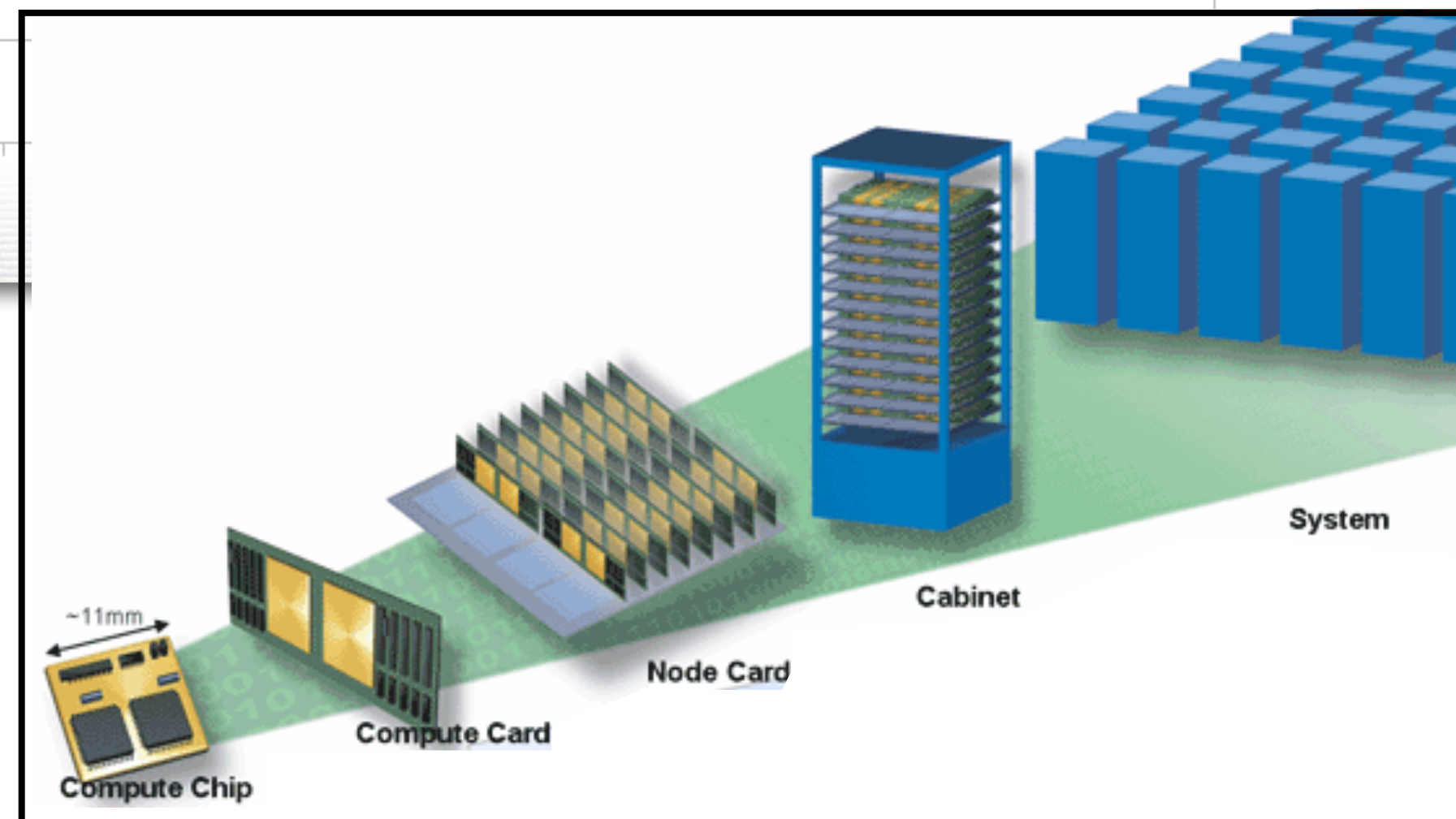
# Perform Computer Experiments



# Machines: Supercomputers



www.top500.org

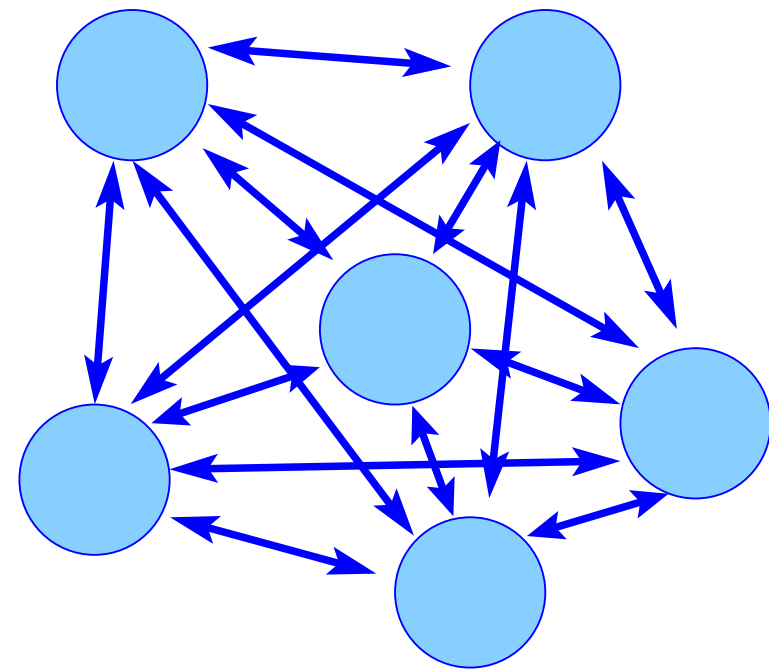


SUNCAT computer cluster



Blue Gene/Q Argonne National Laboratory #4

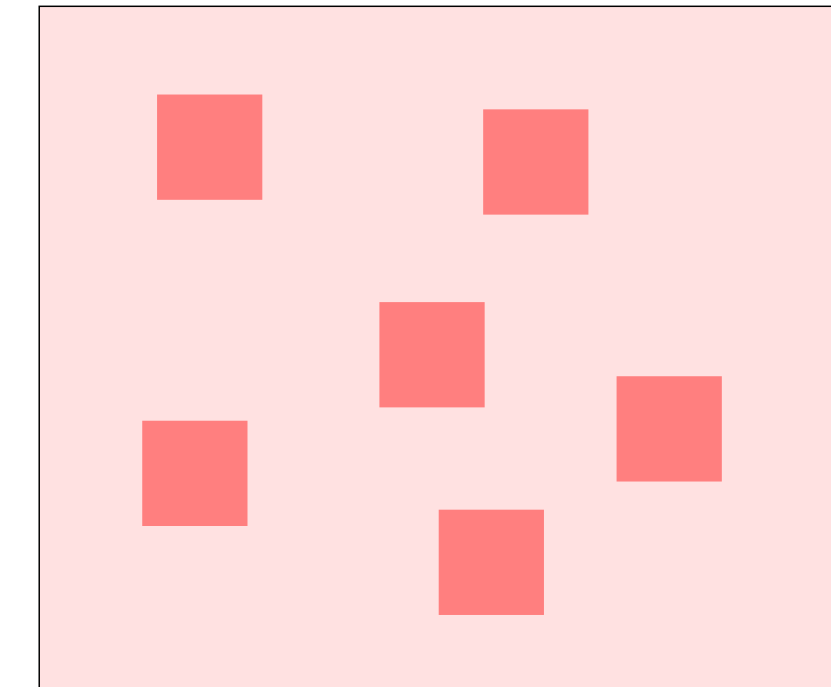




N interacting electrons  
+ real potential

$$H\Psi(\mathbf{r}) = \varepsilon\Psi(\mathbf{r})$$

$$H = -\frac{1}{2} \sum_i \nabla_i^2 + \frac{1}{2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} - \sum_{i,l} \frac{Z}{|\mathbf{r}_i - \mathbf{R}_l|}$$



N non-interacting, fictitious  
particles + effective potential

Kohn-Sham Equations:

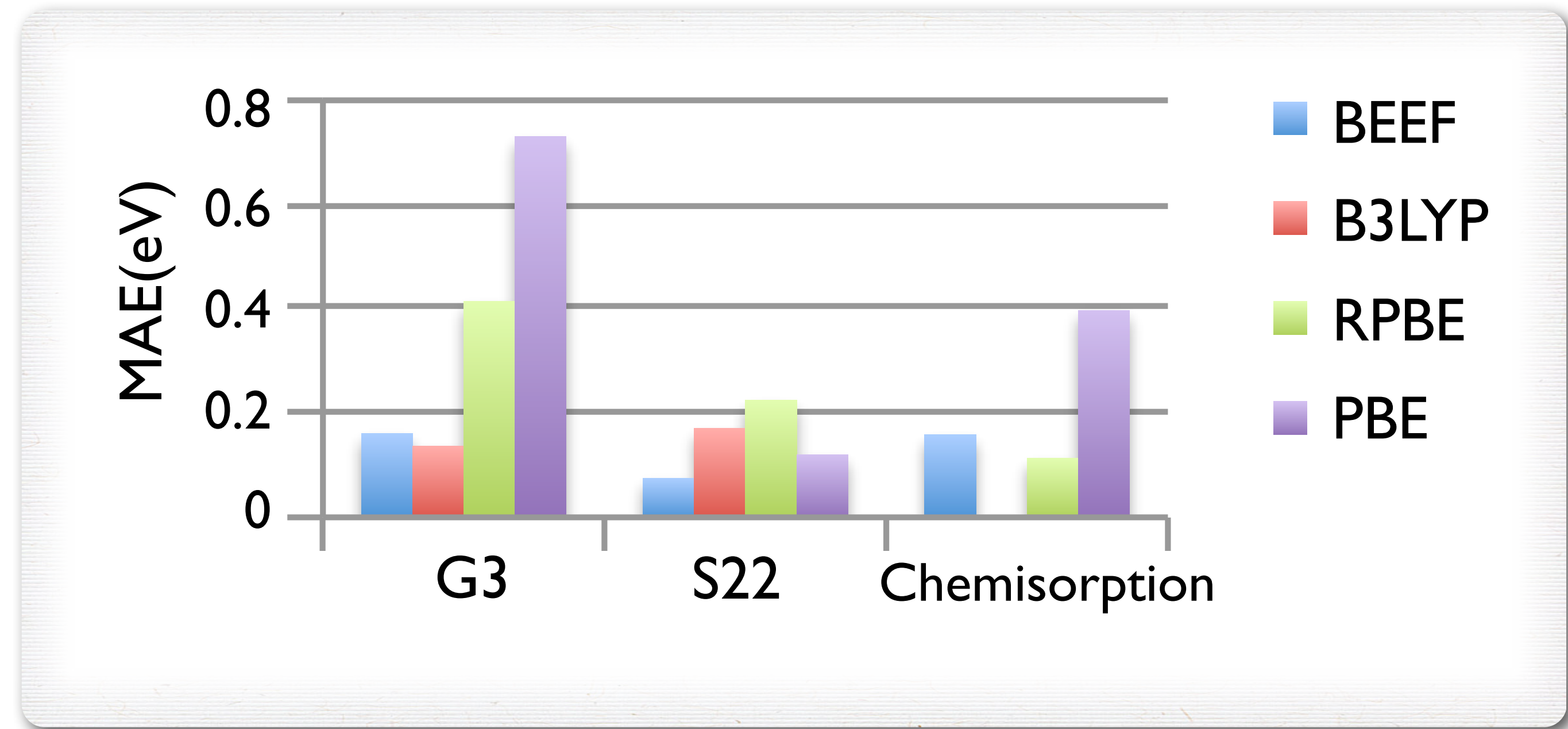
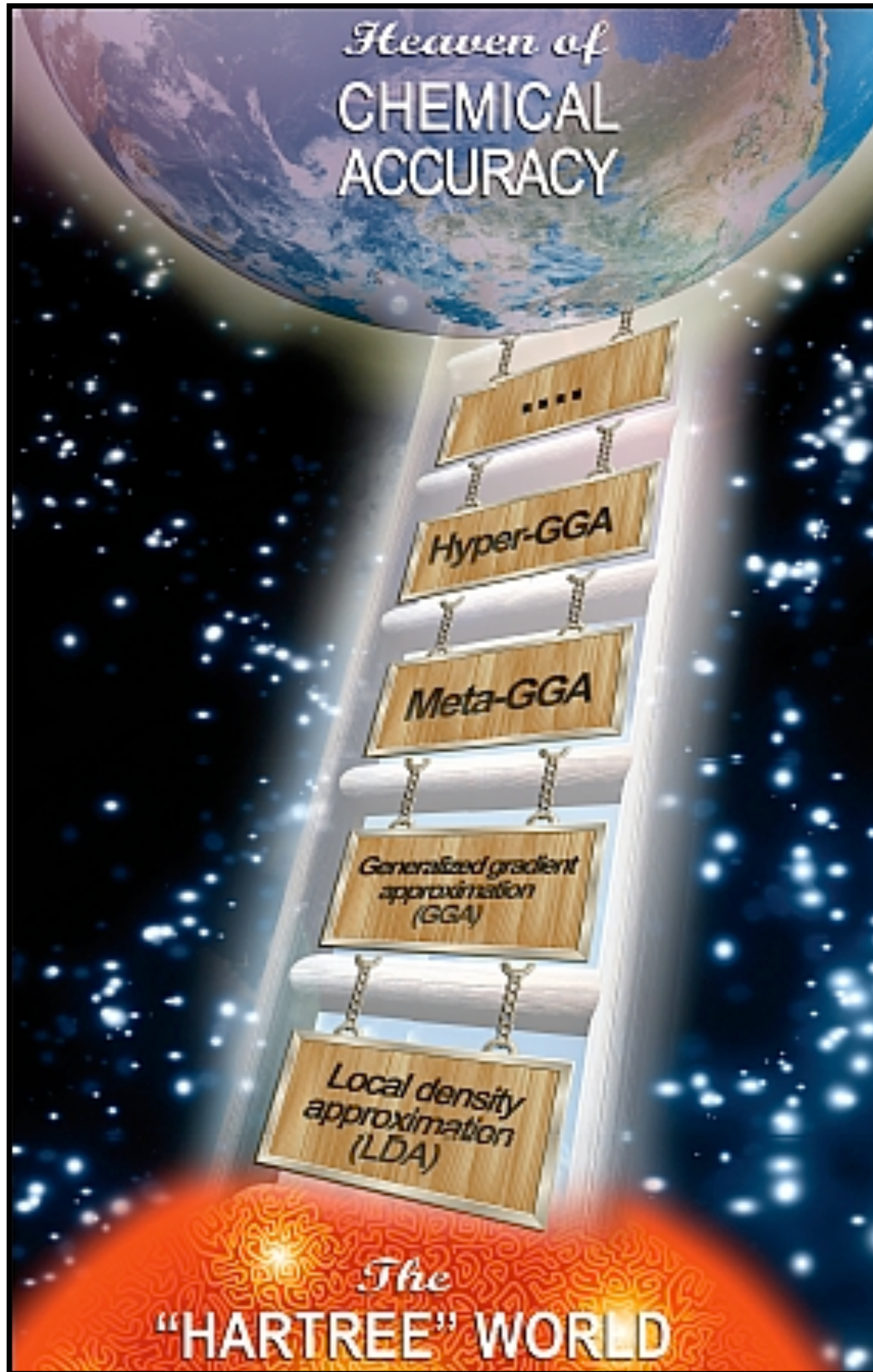
$$\left[ -\frac{1}{2} \nabla^2 + v_{\text{eff}} \right] \psi_i(\mathbf{r}) = \varepsilon_i \psi_i(\mathbf{r})$$

$$n(\mathbf{r}) = \sum_{i=1}^N |\psi_i(\mathbf{r})|^2$$

$$v_{\text{eff}} = v_{\text{ext}}(\mathbf{r}) + v_{\text{Hartree}}[n(\mathbf{r})] + v_{\text{xc}}[n(\mathbf{r})] \quad \text{with} \quad v_{\text{xc}}[n(\mathbf{r})] = \frac{\delta E_{\text{xc}}[n]}{\delta n(\mathbf{r})}$$



# DFT Performance: "It's All in the Exchange-Correlation Functional"

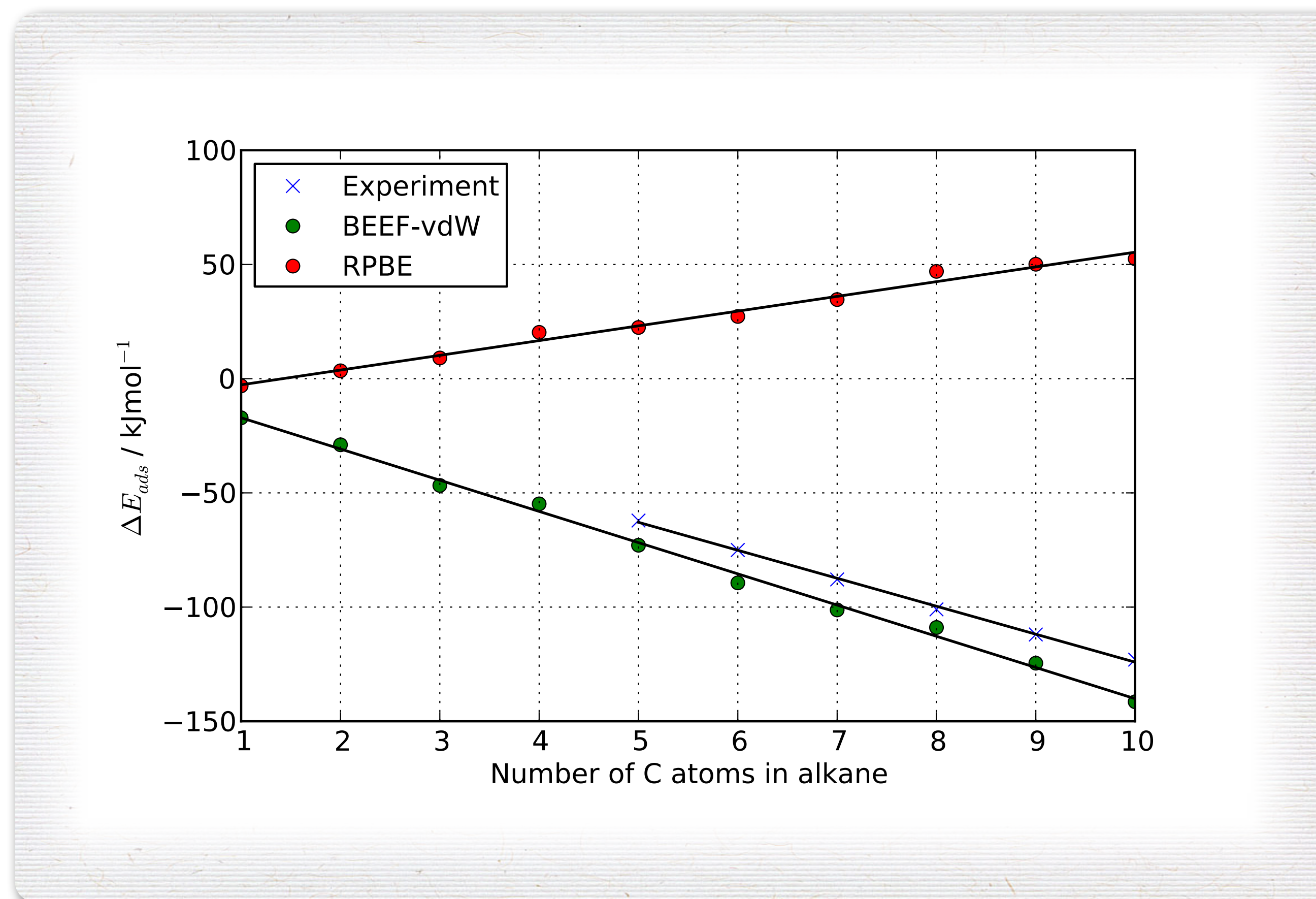
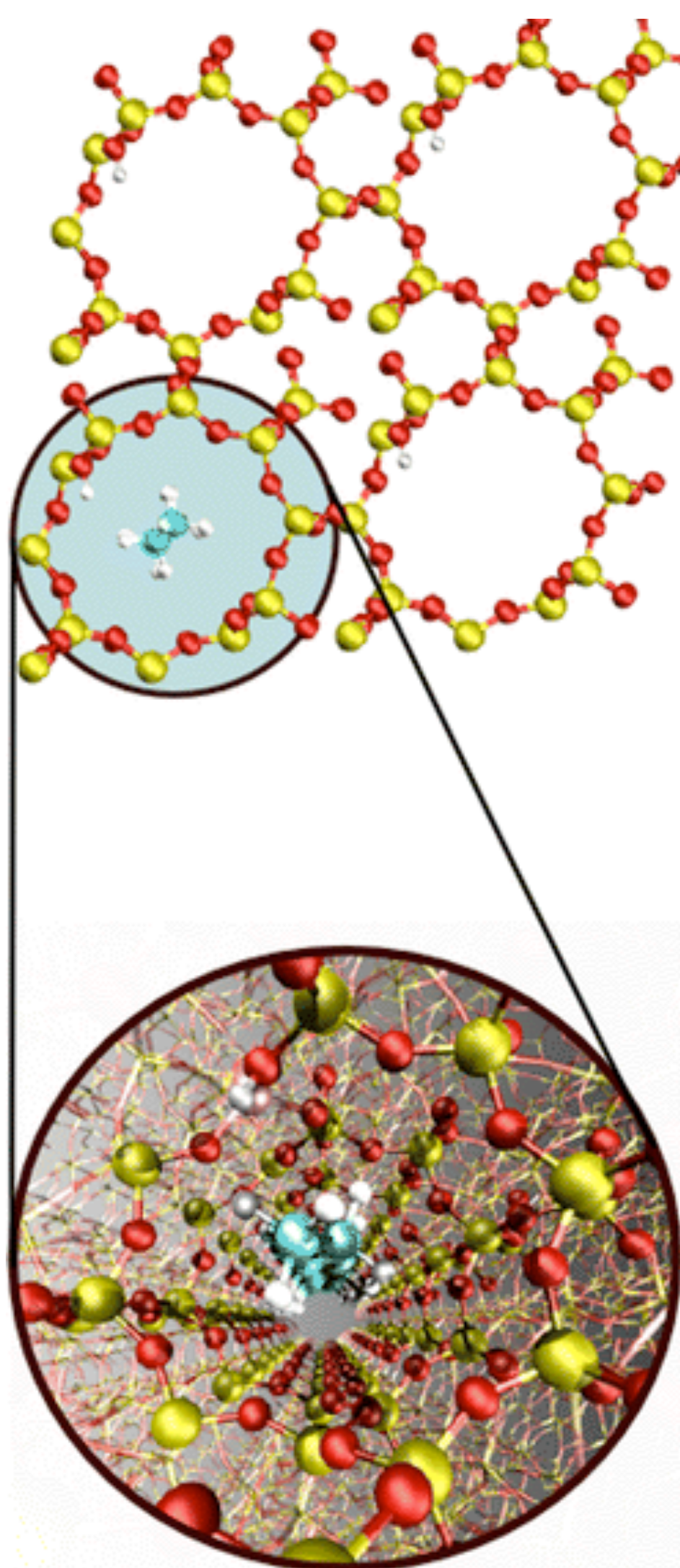


J. P. Perdew *et al.* in AIP Conf. Proc. **577** (2001),  
M.A. L. Marques *et al.*, Comp. Phys. Commun. **183**, 2272 (2012).

J. Wellendorff, *et al.*, Phys. Rev. B **85**, 23 (2012),

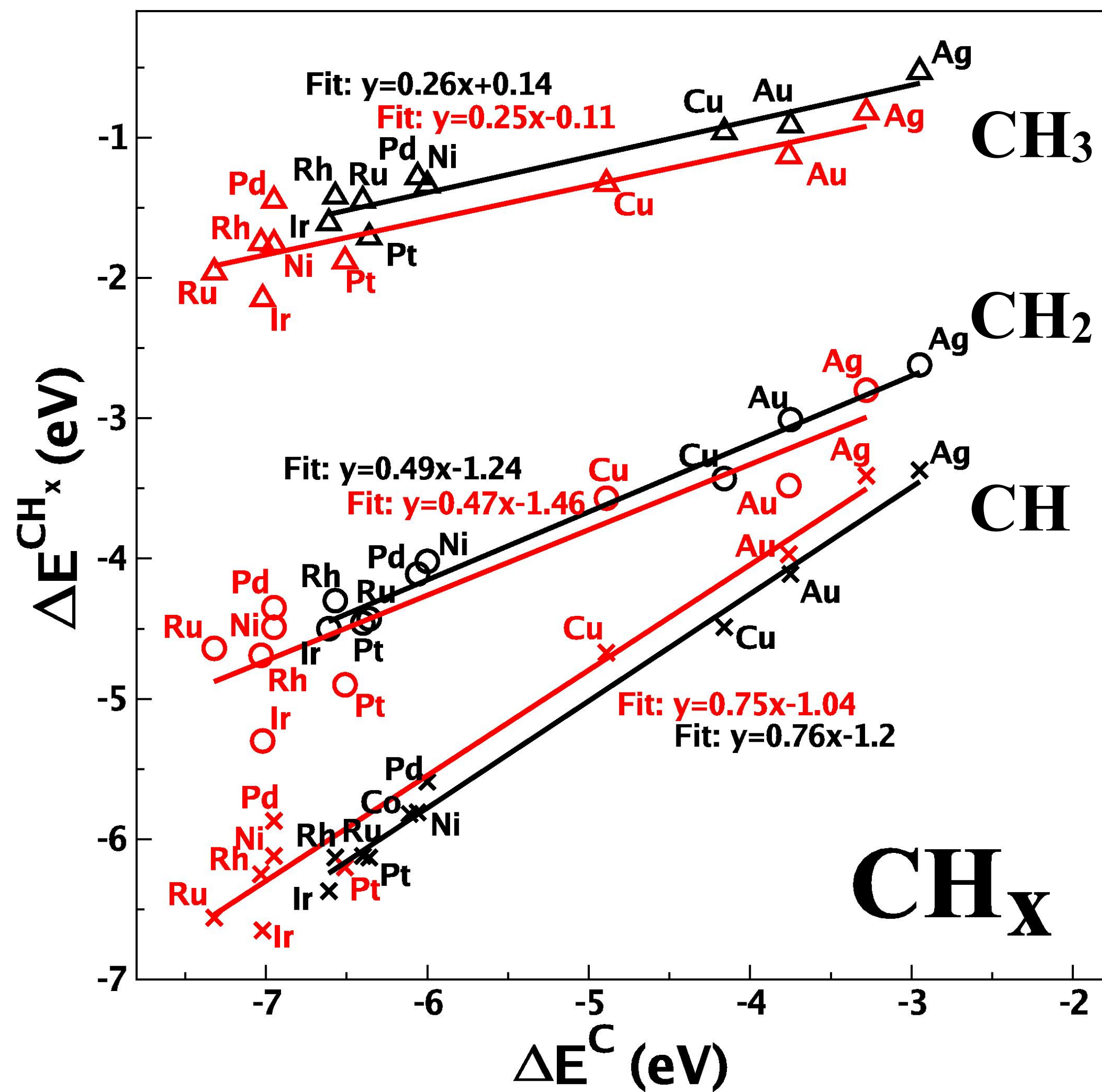


# Application of BEEF: Physorption of n-alkanes in Zeolites (ZSM22)

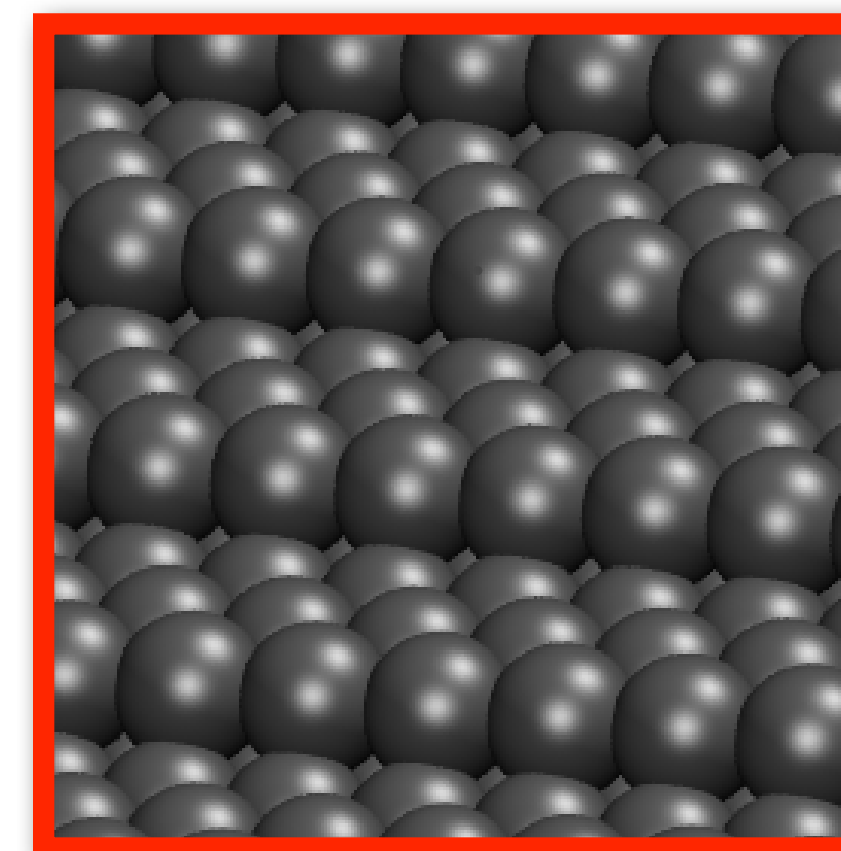


R. Brogaard, P. G. Moses, J. K. Nørskov, Catal. Lett. **142**, 1057 (2012).

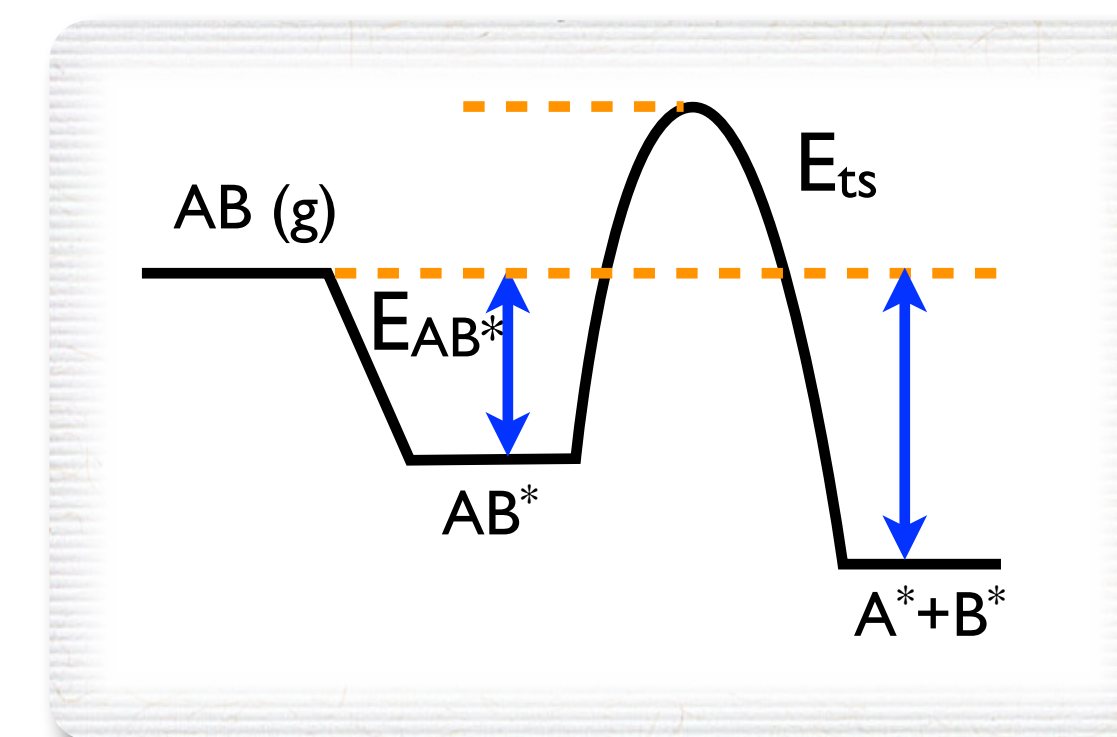
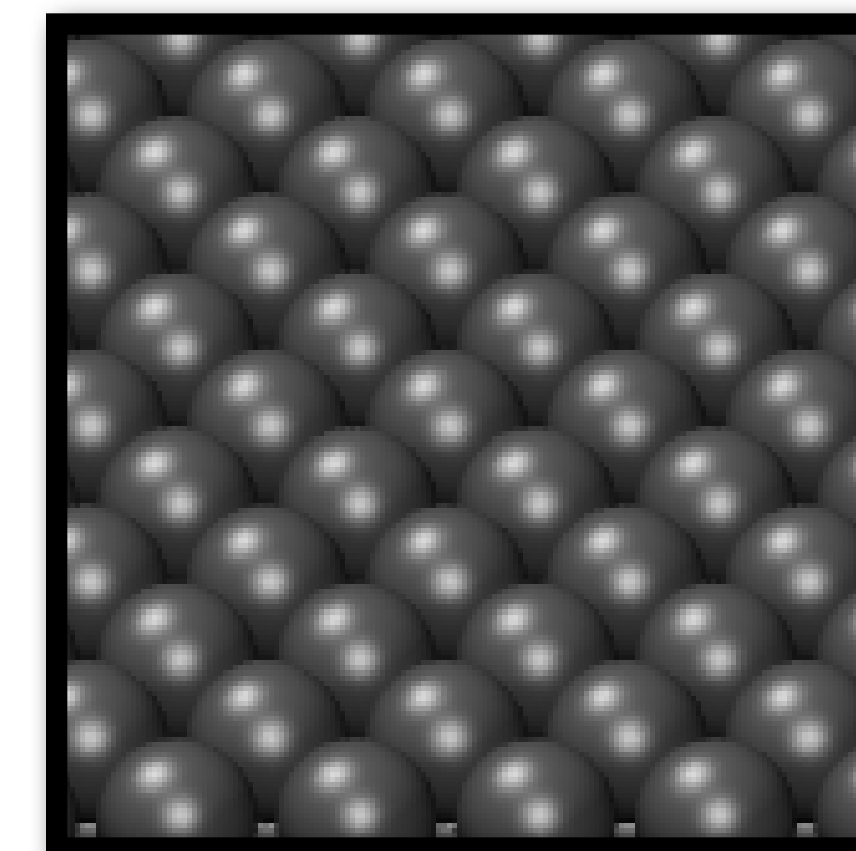




stepped surface

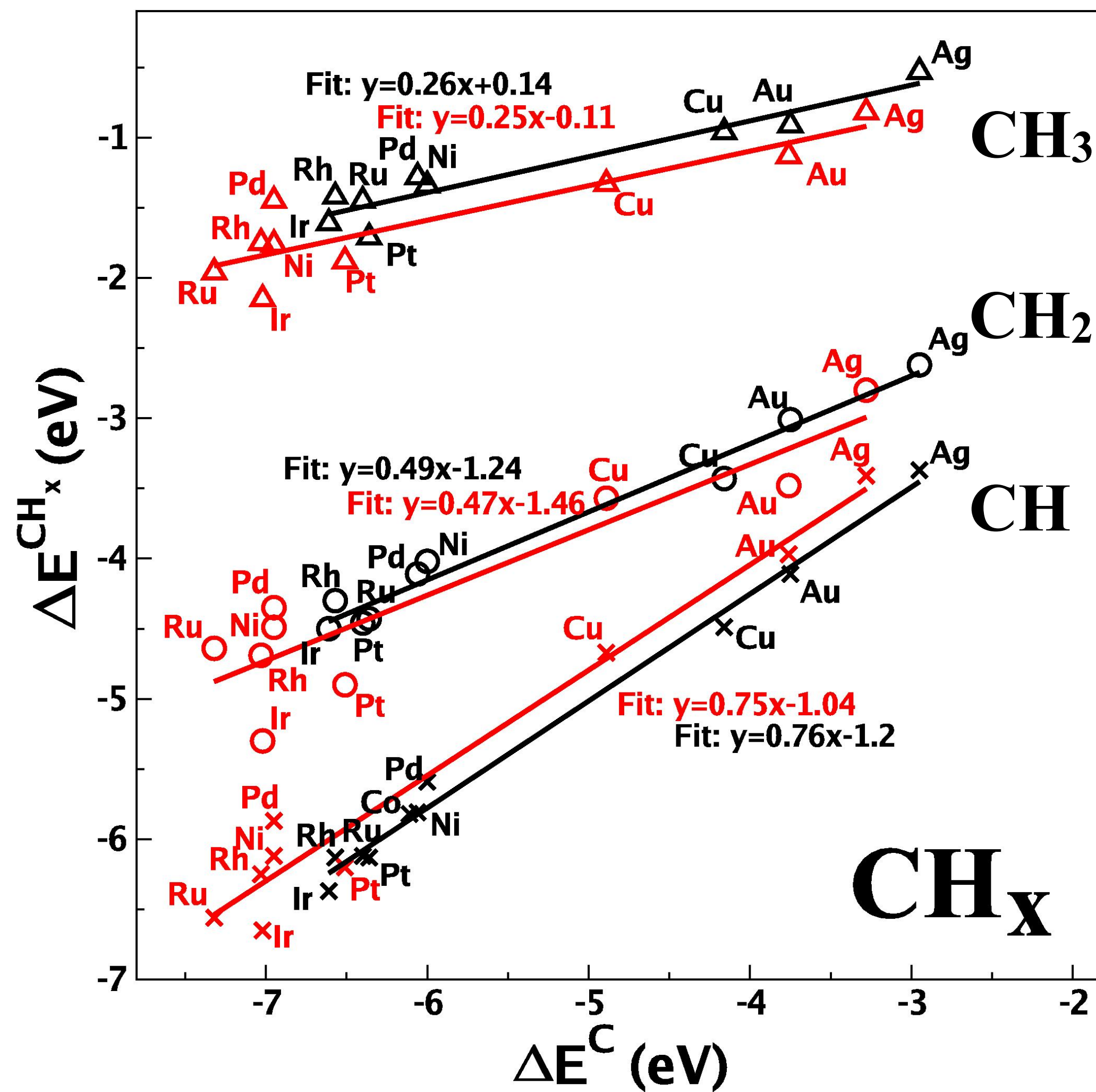


close-packed surface



F.Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).





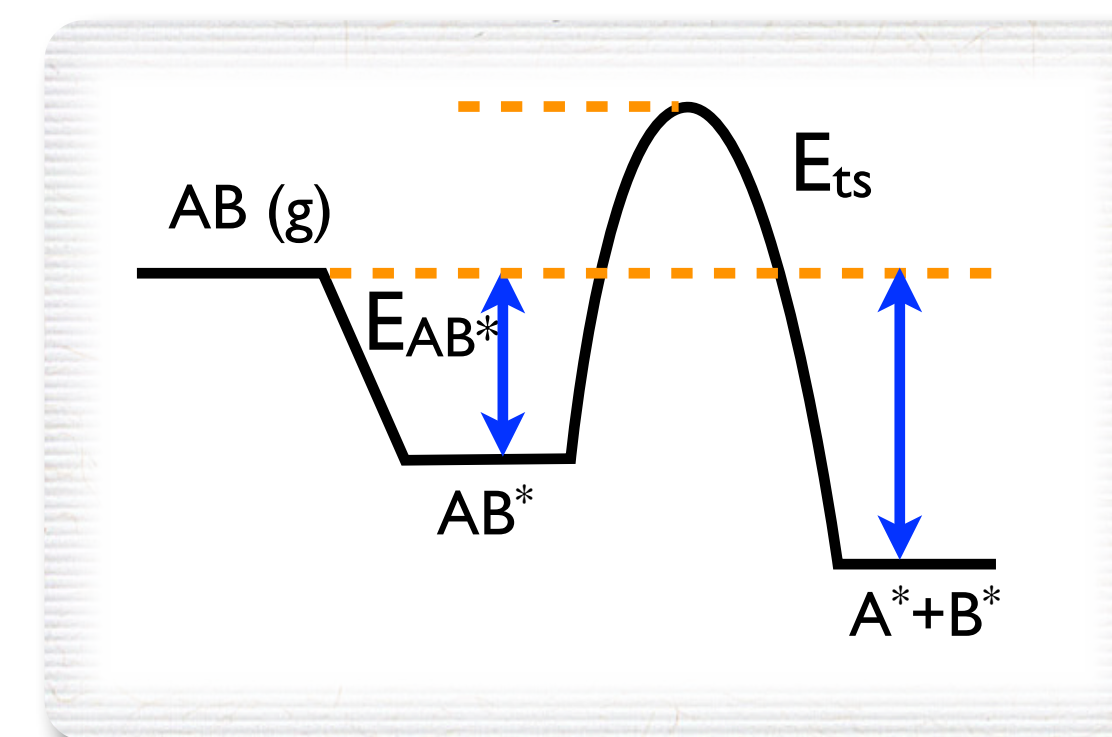
stepped surface

close-packed surface

$$y=0.26x+0.14$$

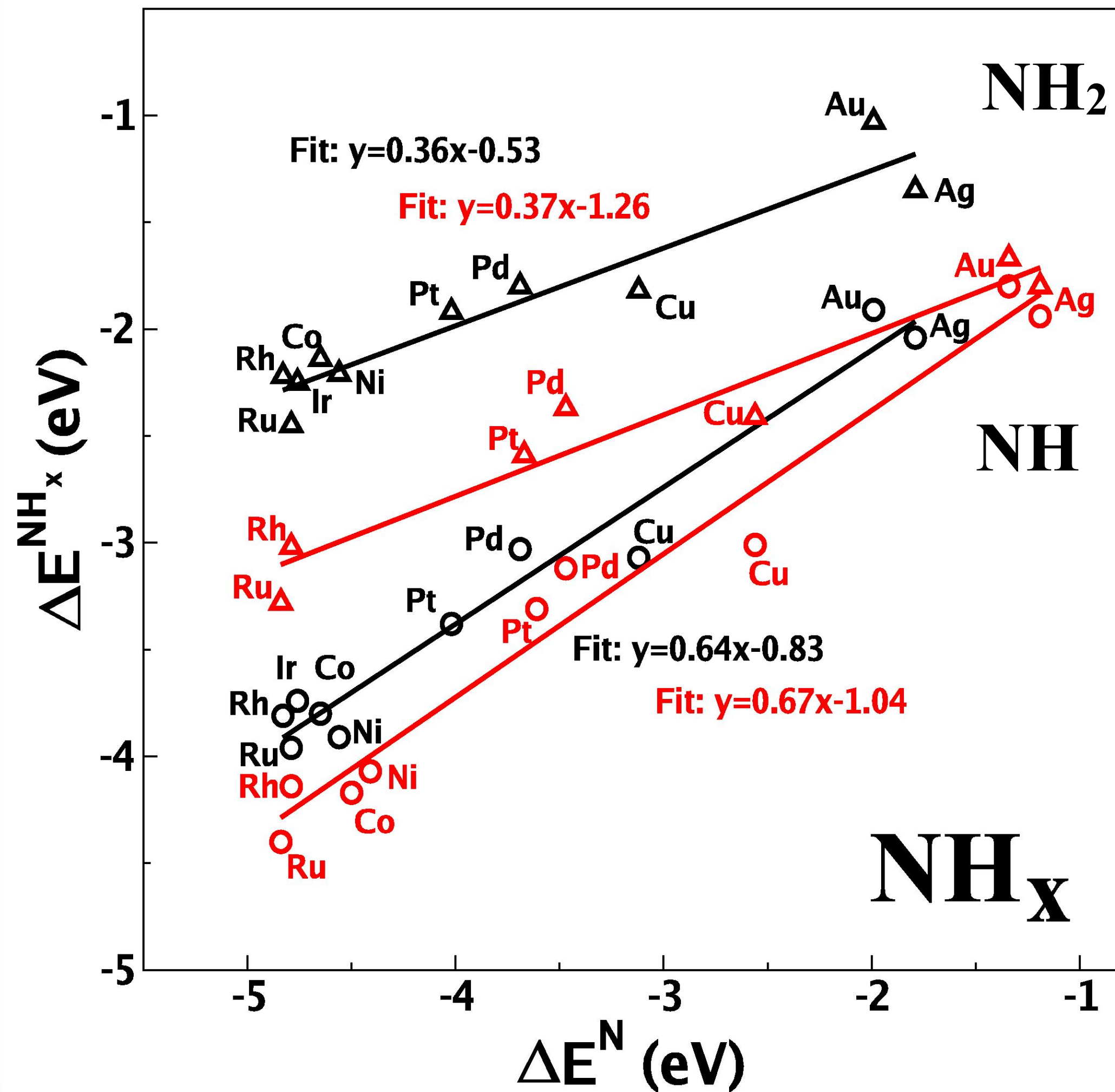
$$y=0.49x-1.24$$

$$y=0.76x-1.04$$

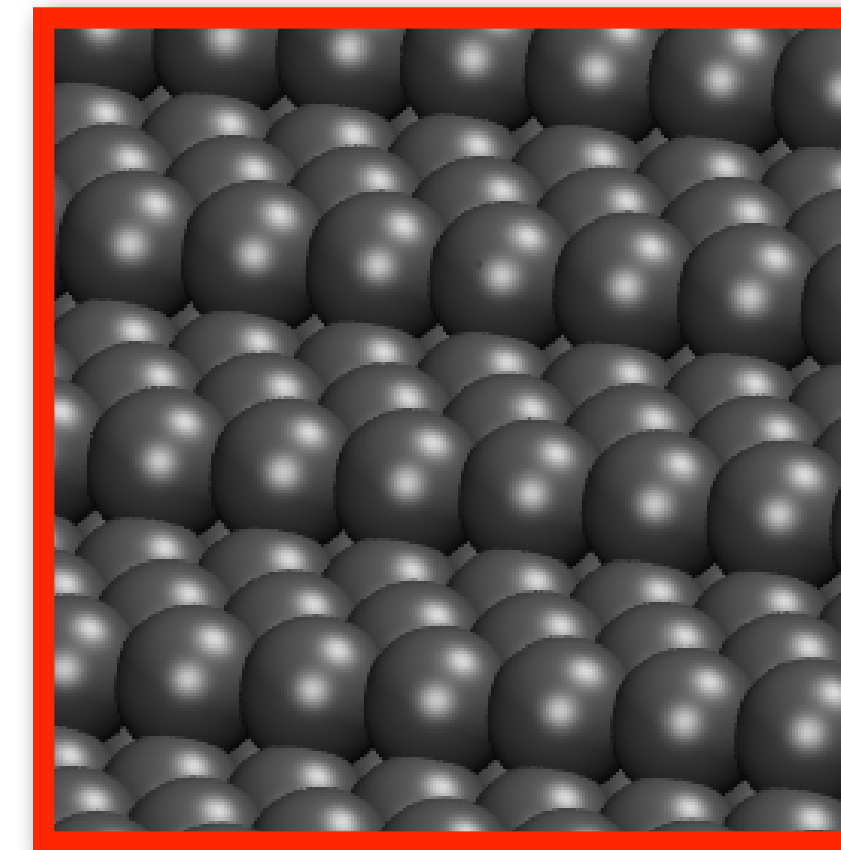


F.Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).

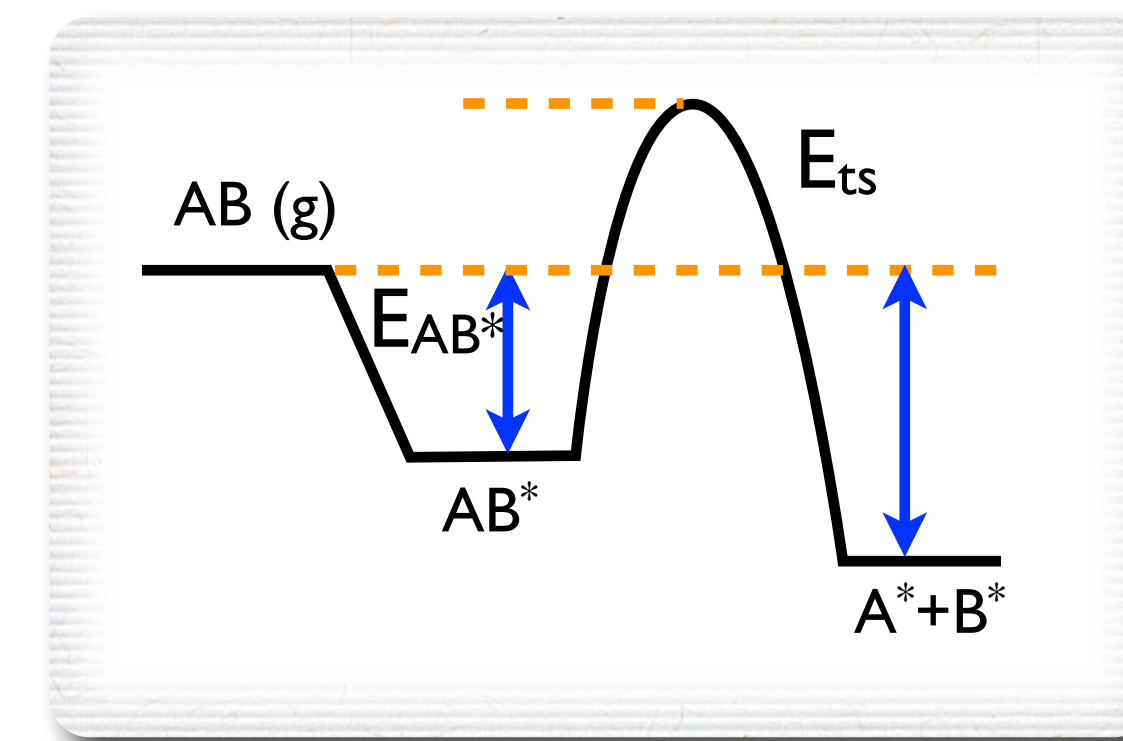
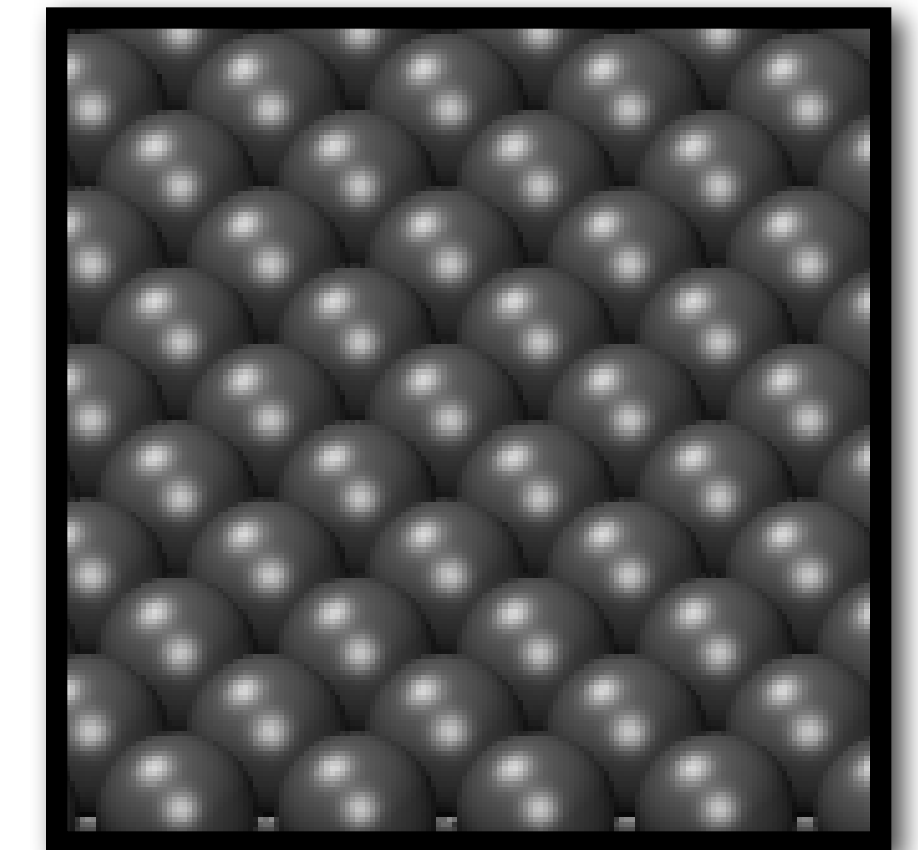




stepped surface

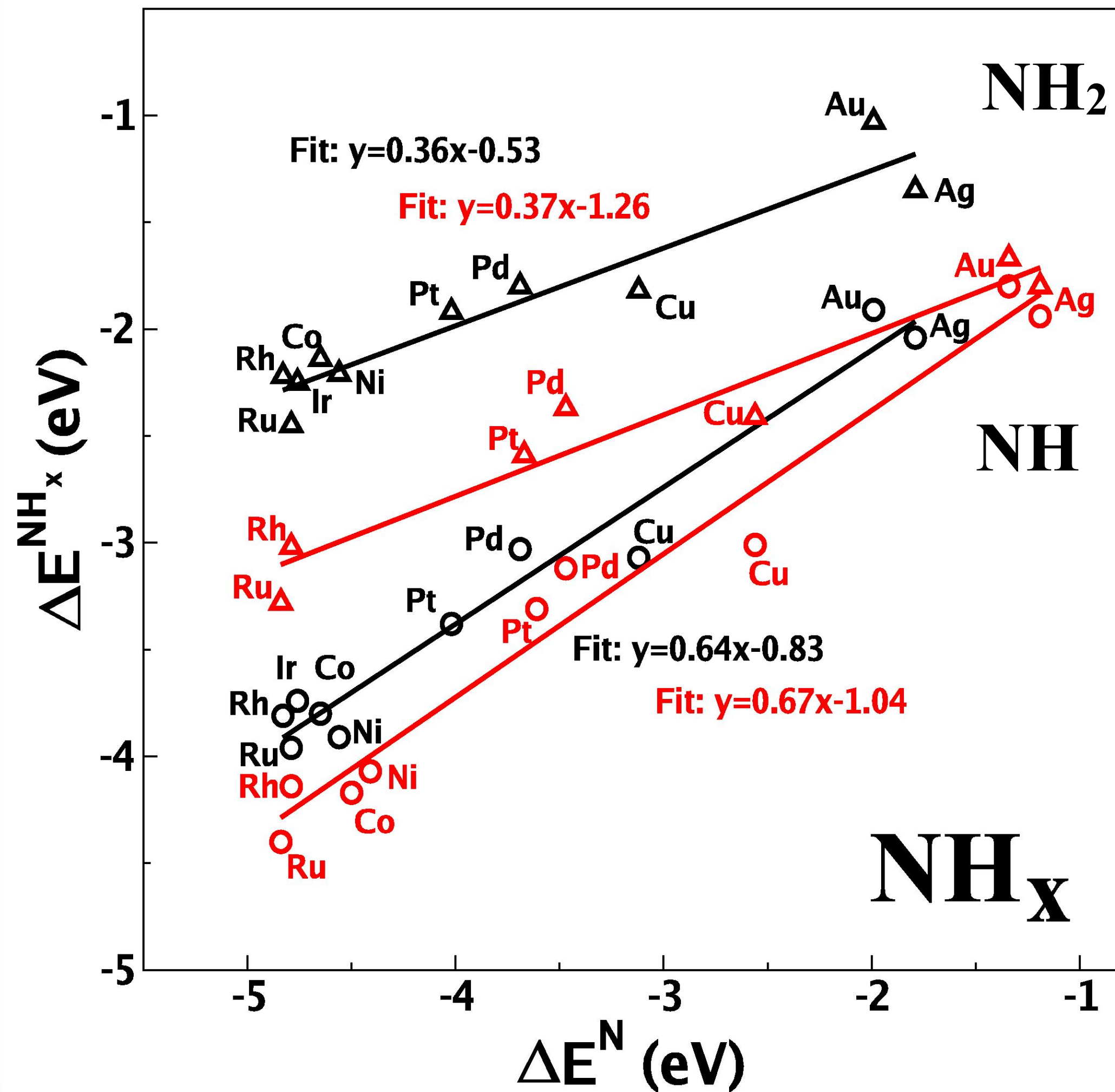


close-packed surface



F.Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).



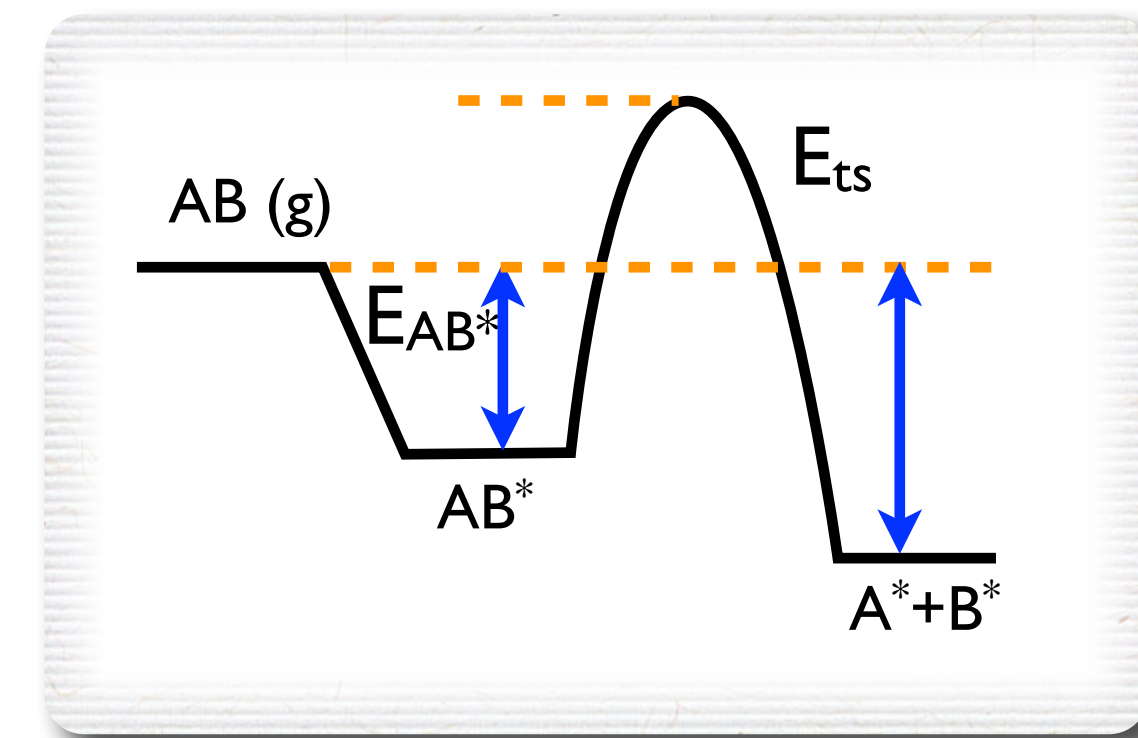


stepped surface

close-packed surface

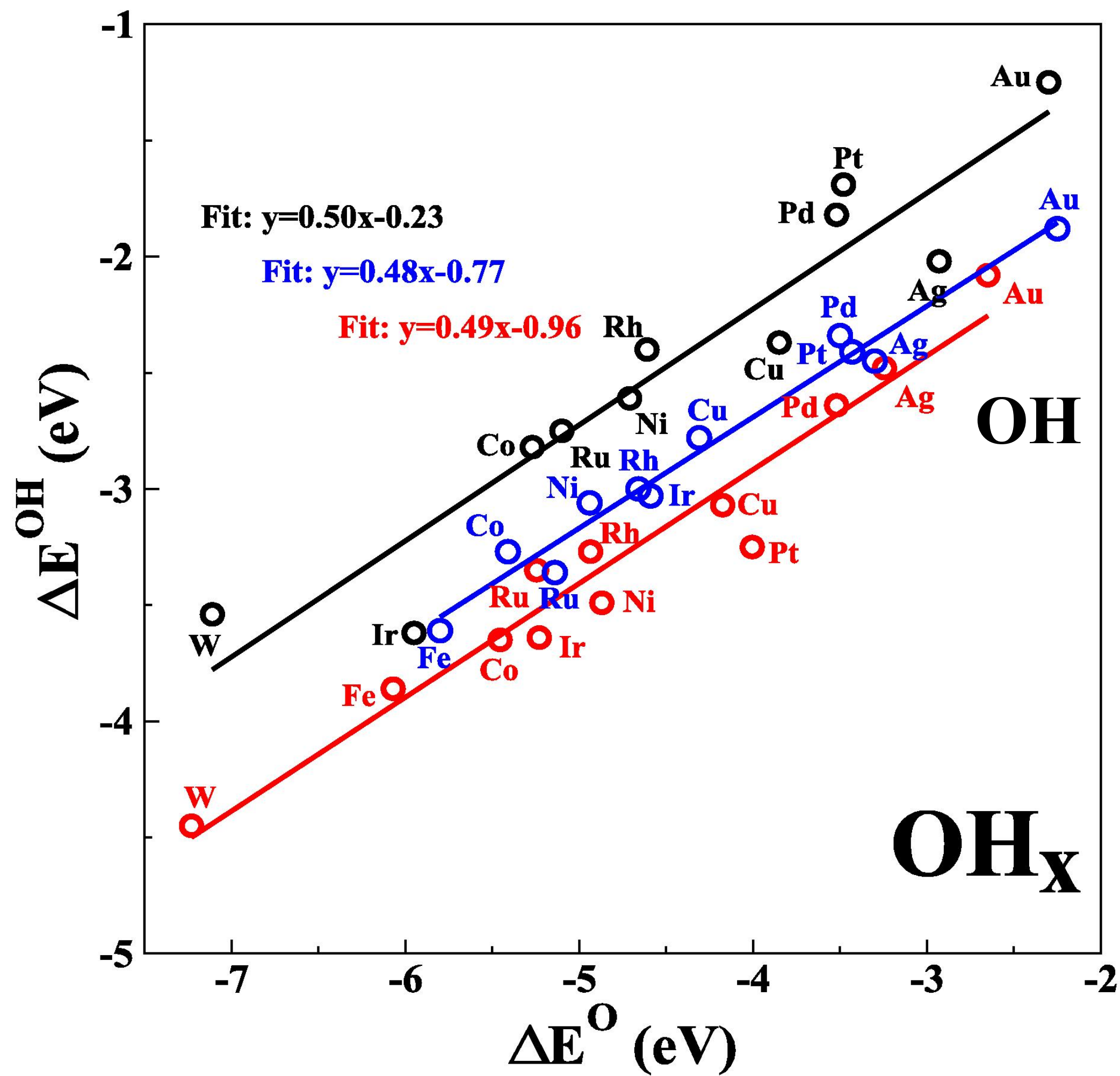
$$y=0.36x-0.53$$

$$y=0.64x-0.83$$

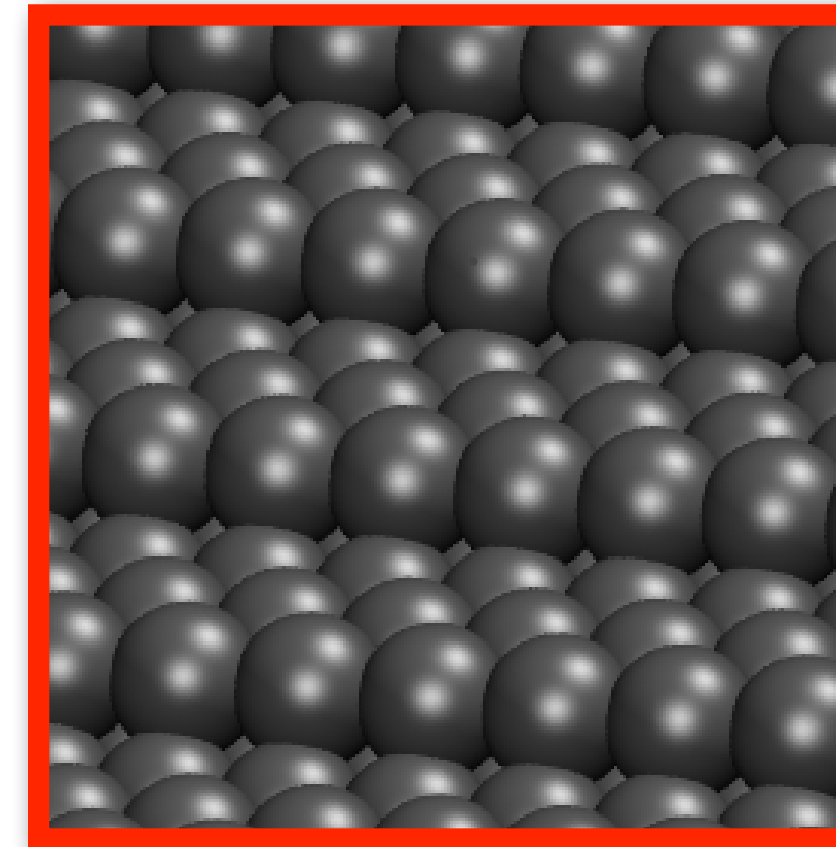


F.Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).

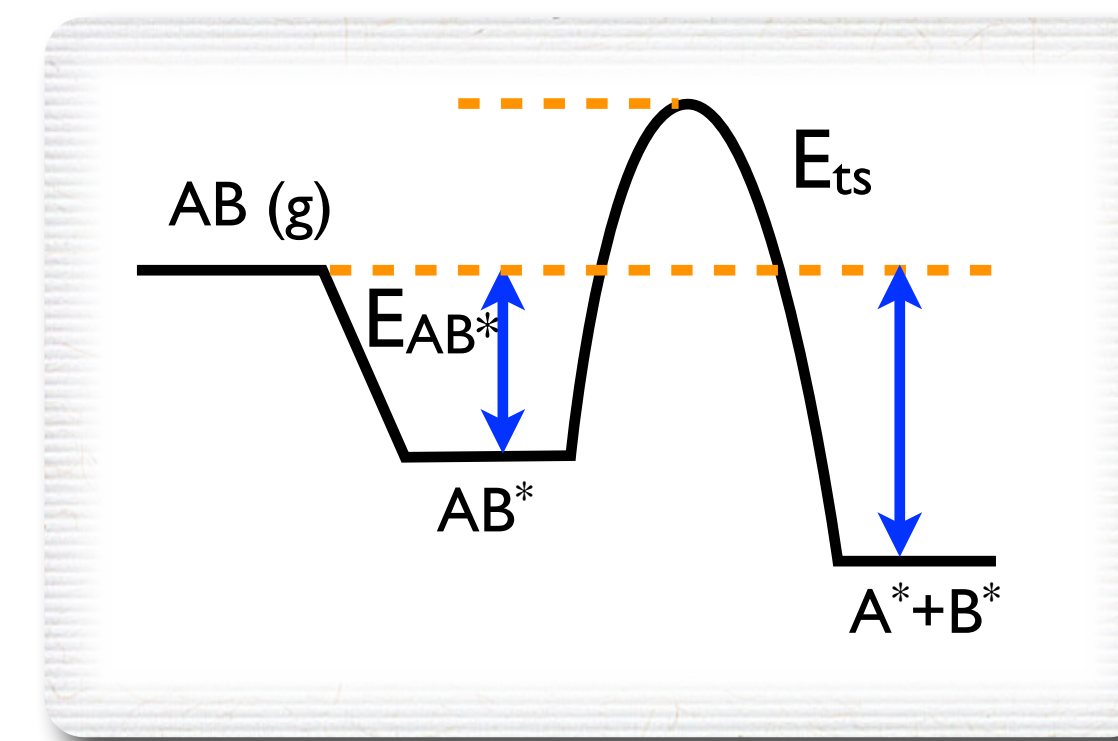
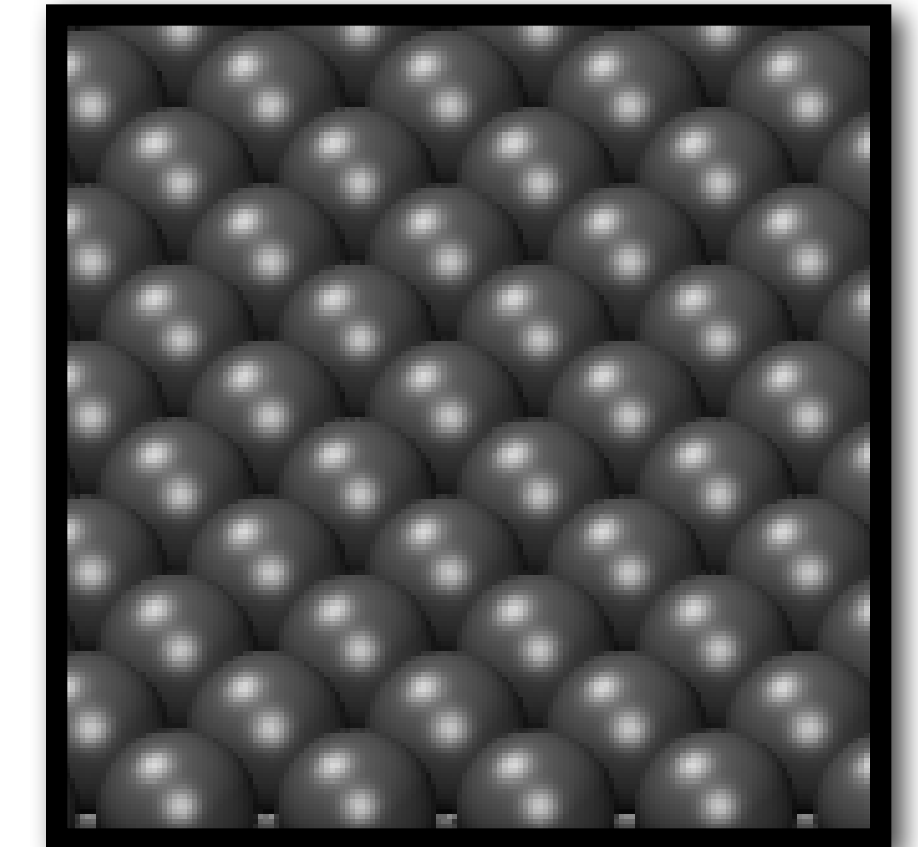




stepped surface

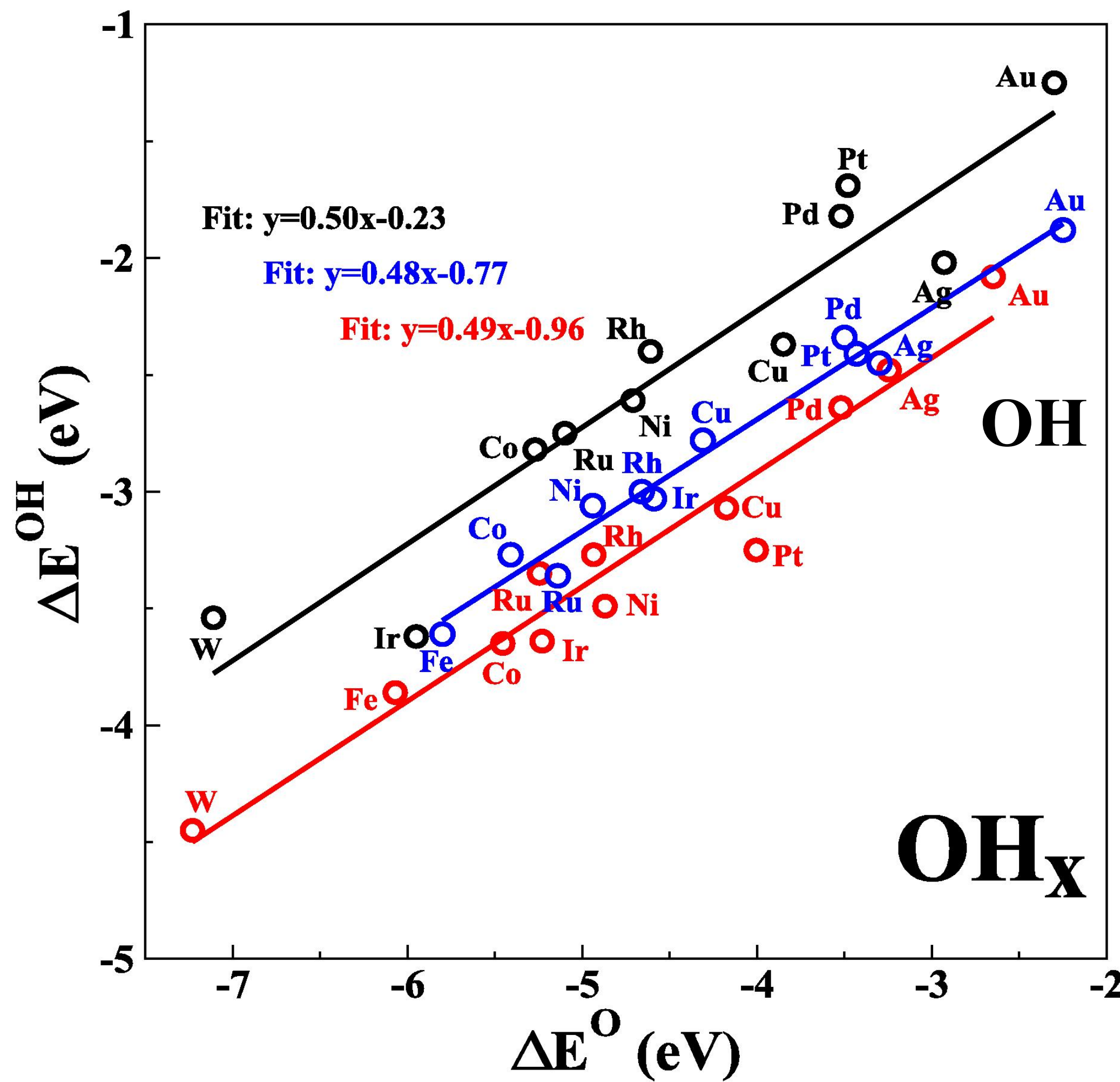


close-packed surface



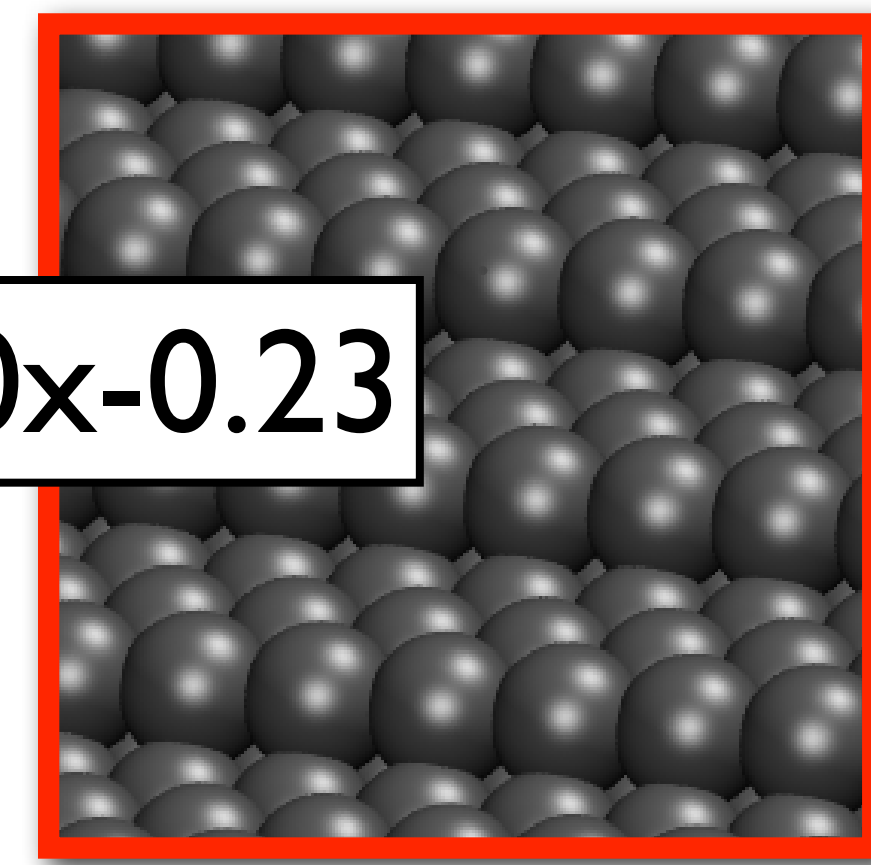
F.Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).



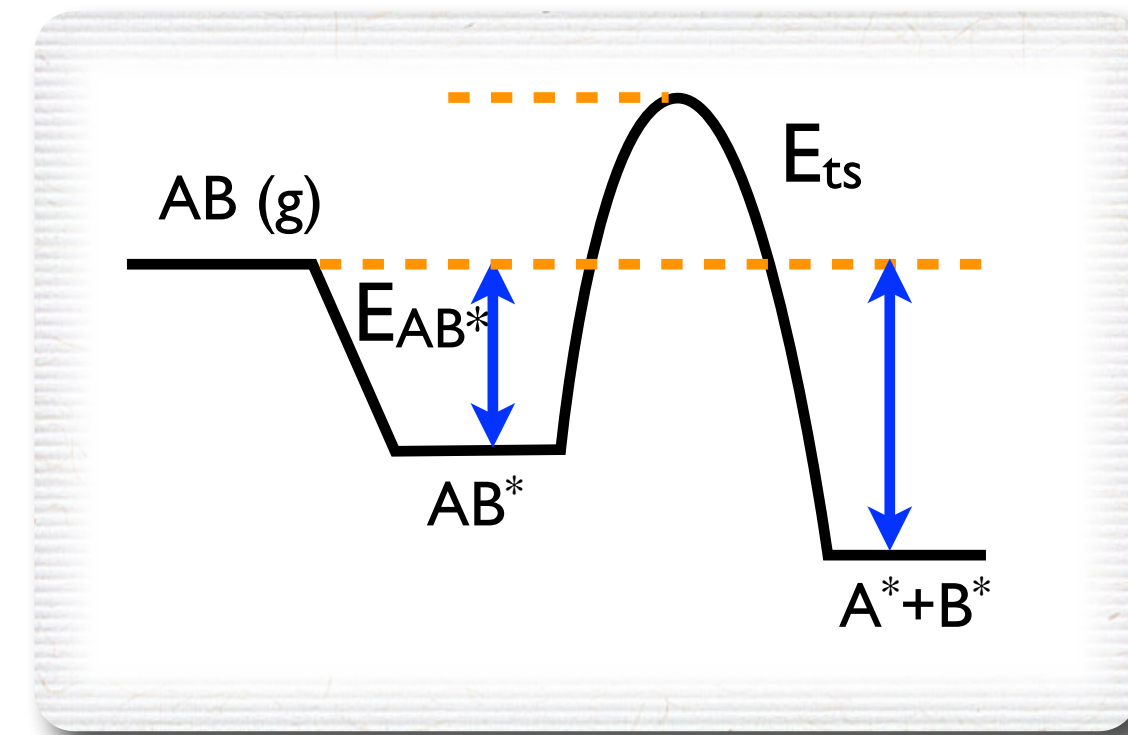
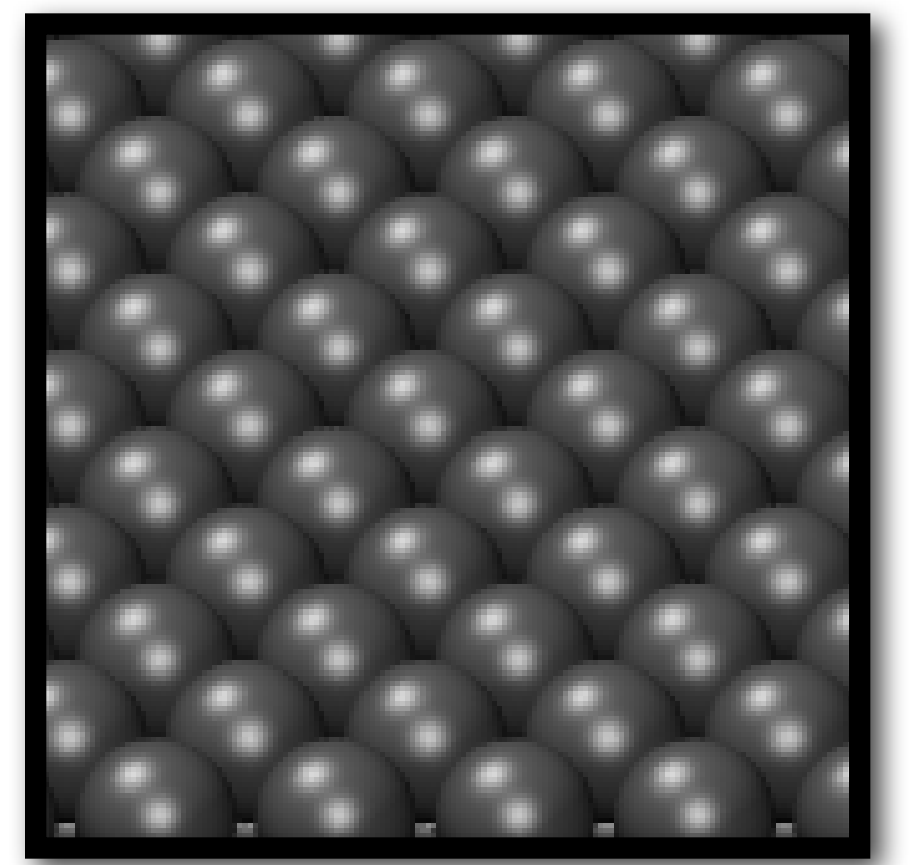


$y=0.50x-0.23$

stepped surface

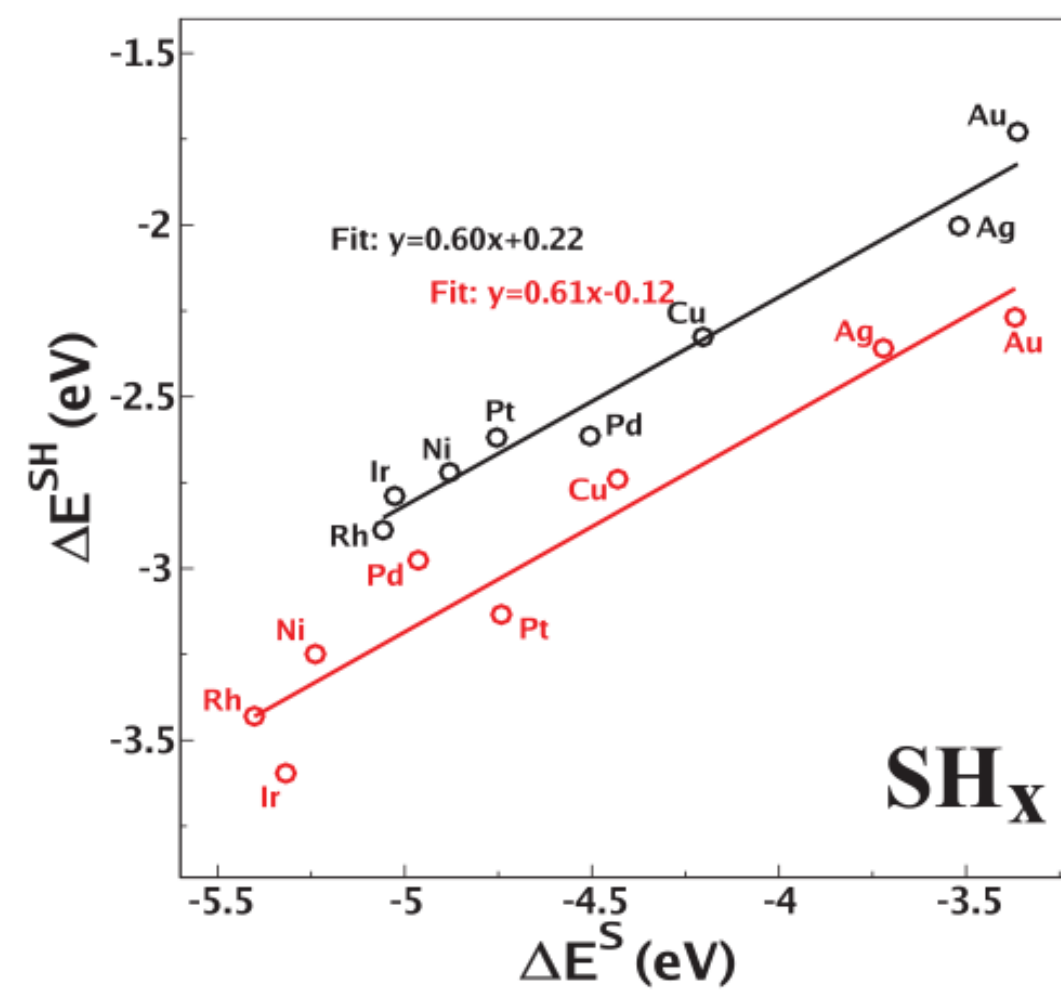
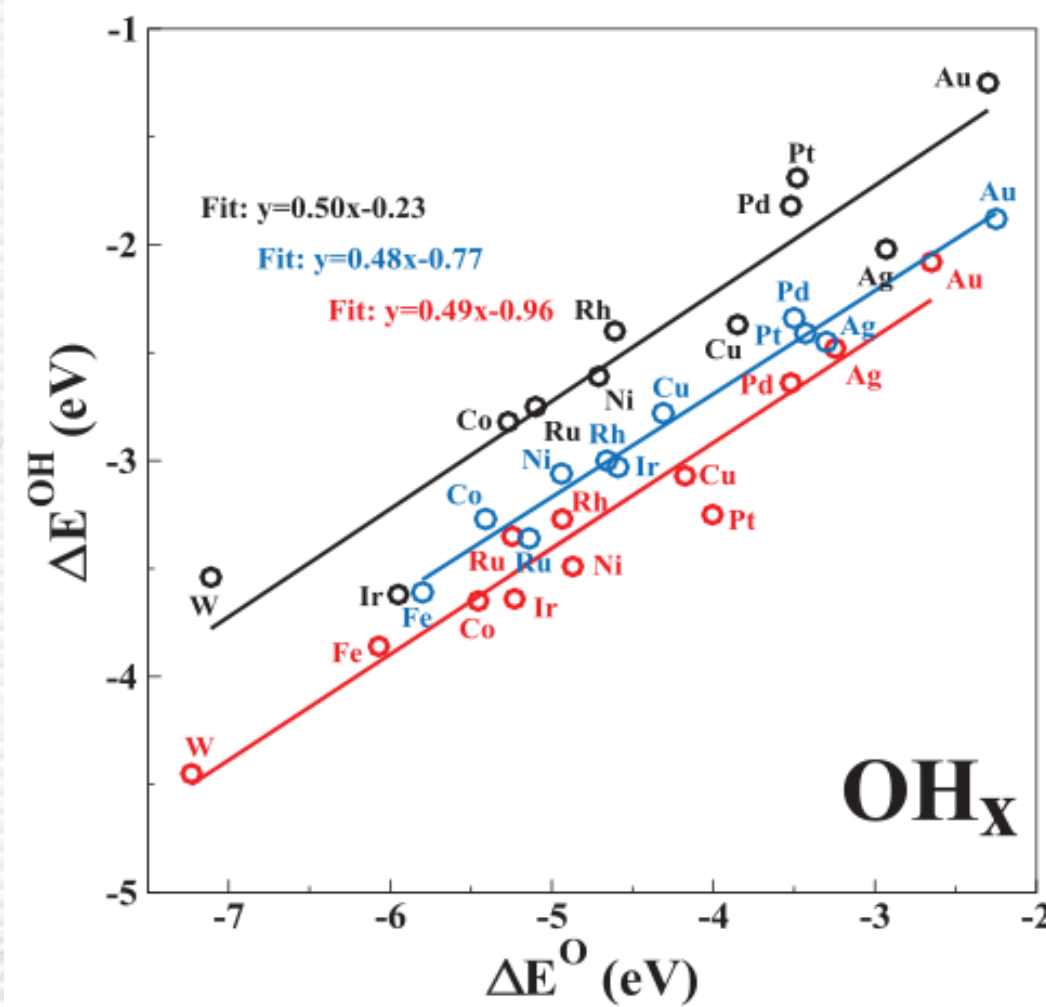
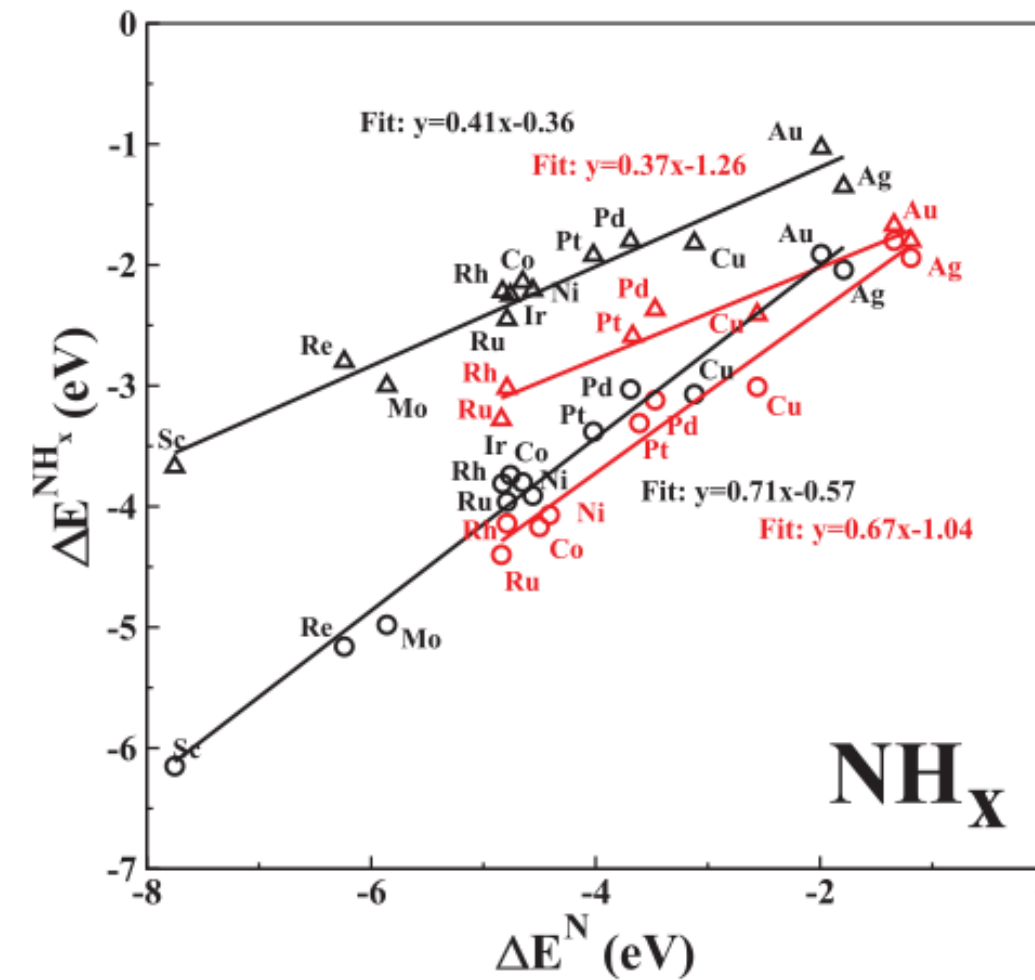
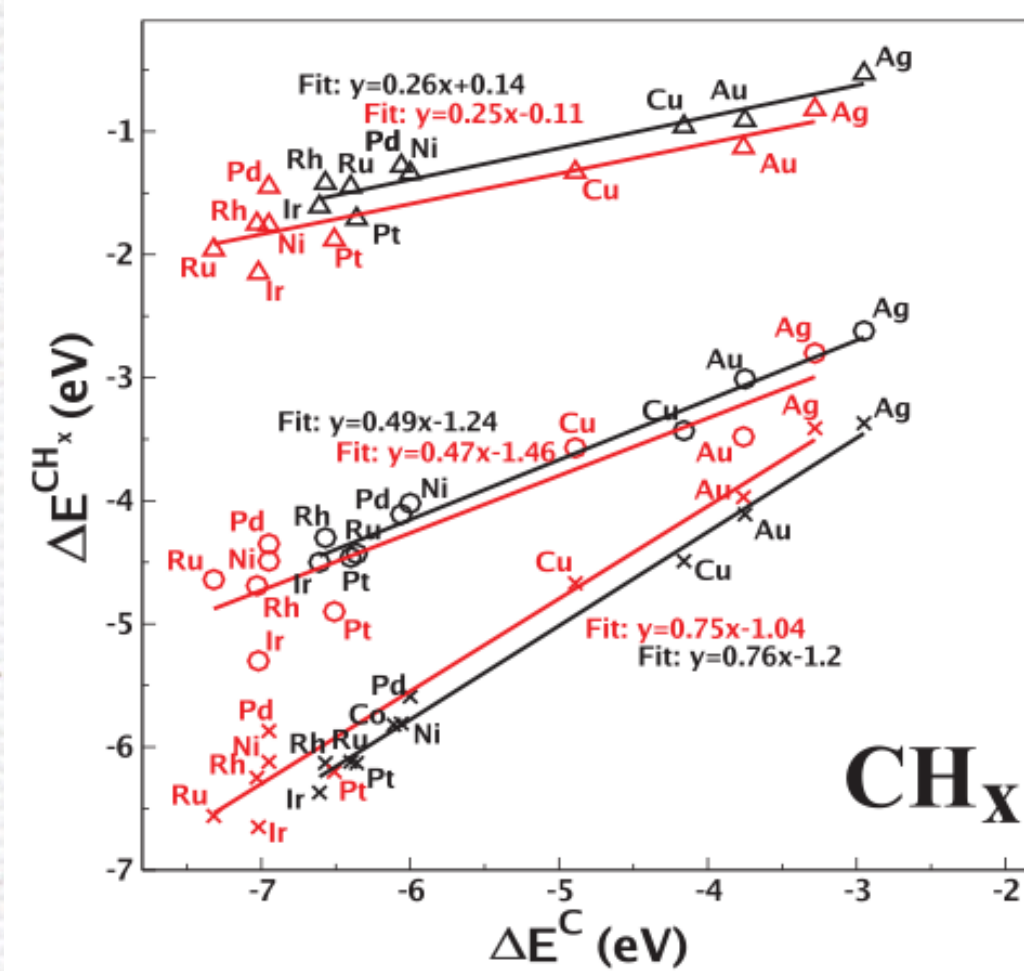


close-packed surface



F.Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).





$$\Delta E^{AH_x} = \gamma(x)\Delta E^A + \xi$$

$$\gamma(x) = (x_{\max} - x) / x_{\max}$$

Atom A:

- Valency:  $x_{\max}$
- # H atoms attached:  $x$
- # bonds to surface:  $x_{\text{surf}} = \gamma x_{\max}$

Bond order conservation

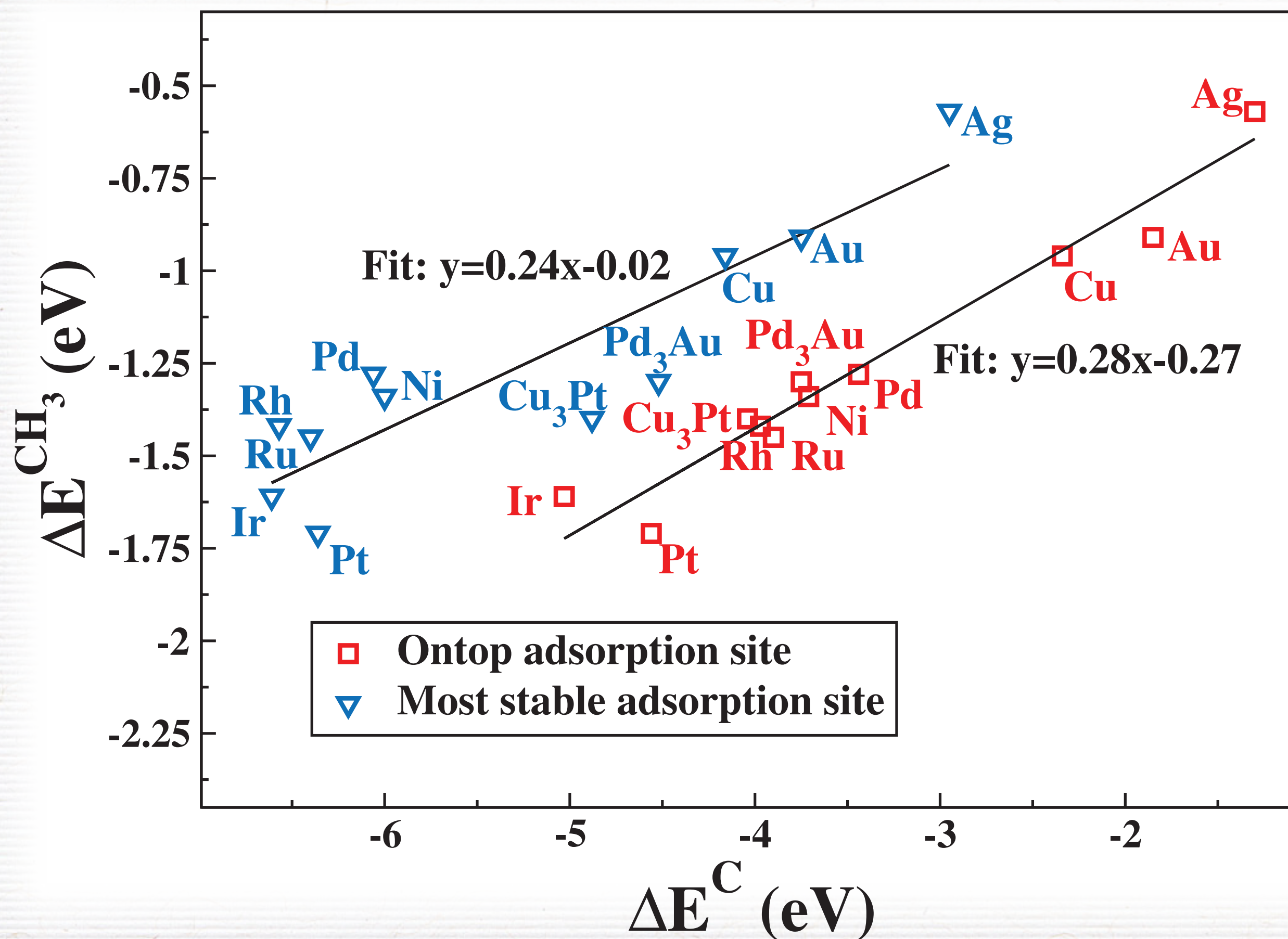
$$\gamma x_{\max} + x = x_{\max}$$

$$\Leftrightarrow$$

$$\gamma = \frac{x_{\max} - x}{x_{\max}}$$



# Adsorption Site and Scaling Relations



MAE=0.13eV

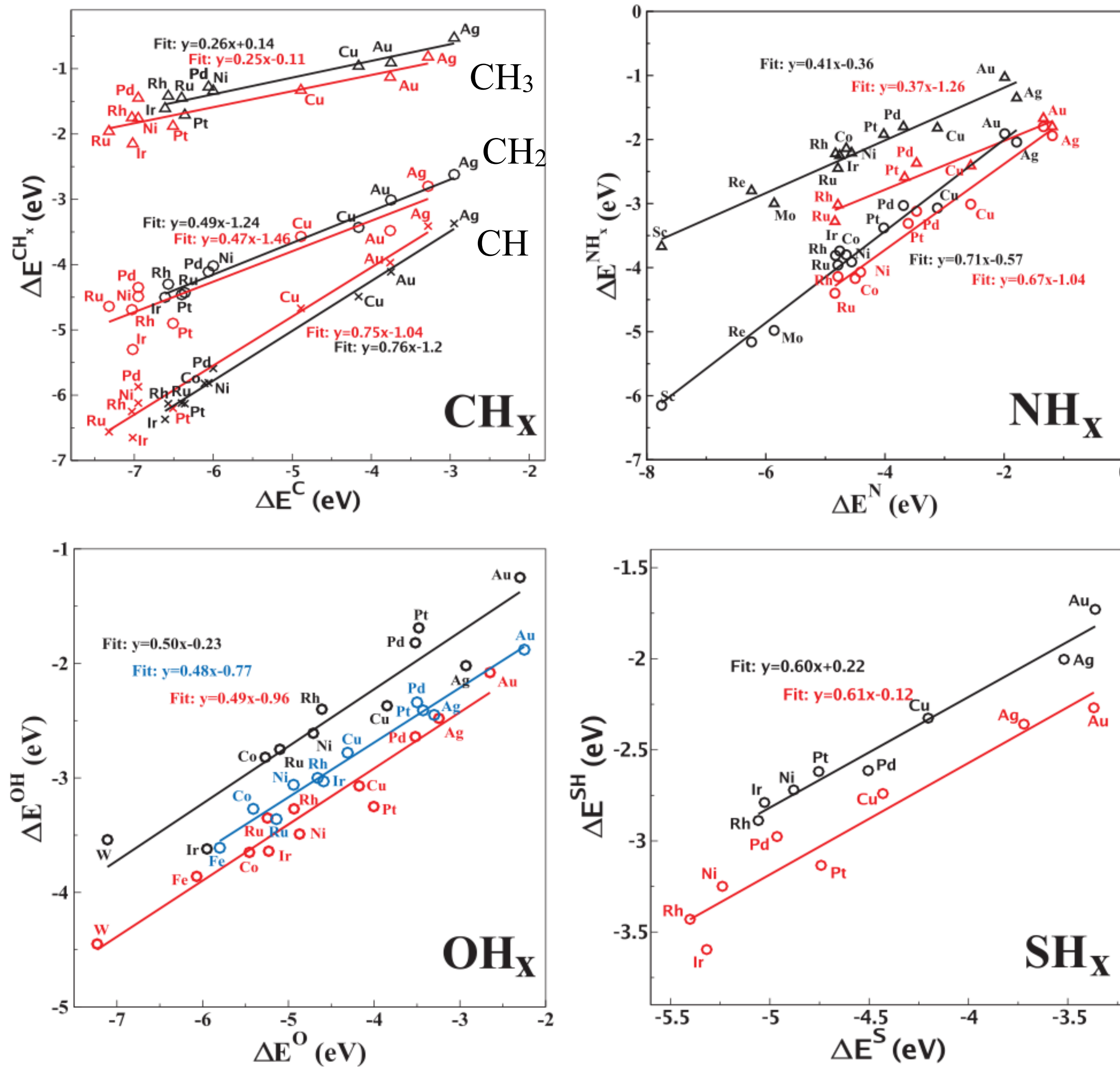
MAE=0.06eV

adsorbate	preferred adsorption site
CH <sub>3</sub>	1-fold
CH <sub>2</sub>	2-fold
CH	3-fold
C	3-fold
C	1-fold



Overall scaling independent of adsorption geometry and details of bonding  $\Rightarrow$  more general explanation





$$\Delta E^{\text{AH}_x} = \gamma(x)\Delta E^{\text{A}} + \xi$$

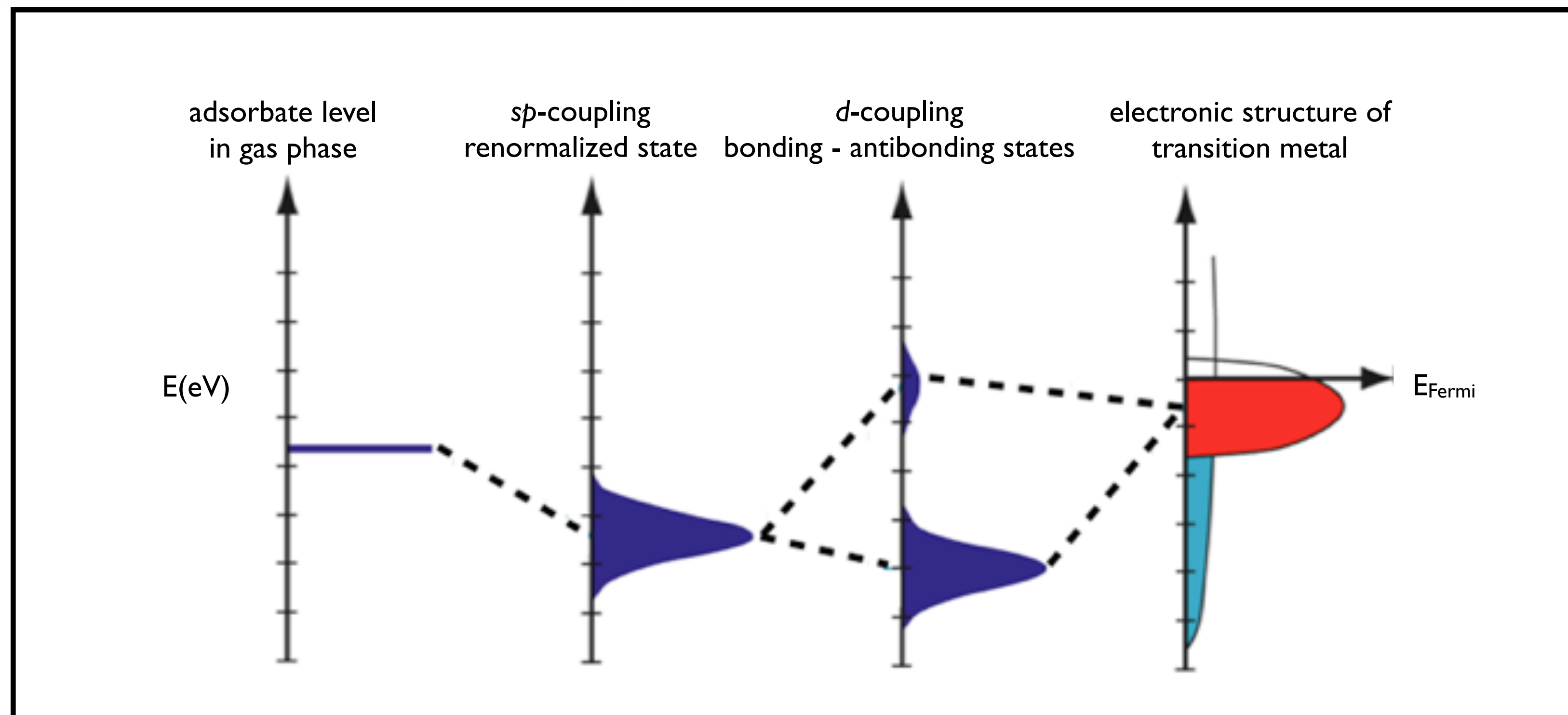
$$\gamma(x) = (x_{\text{max}} - x) / x_{\text{max}}$$

=  
d-band model  
+  
Effective medium theory

F. Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).



# The d-band Model of Chemical Bonding on Metal Surfaces



$$\Delta E_{ads} = \Delta E_{sp} + \Delta E_d \longrightarrow \Delta E_d = -2(1 - f) \frac{V_{ad}^2}{|\epsilon_d - \epsilon_a|} + 2(1 + f) \alpha V_{ad}^2$$

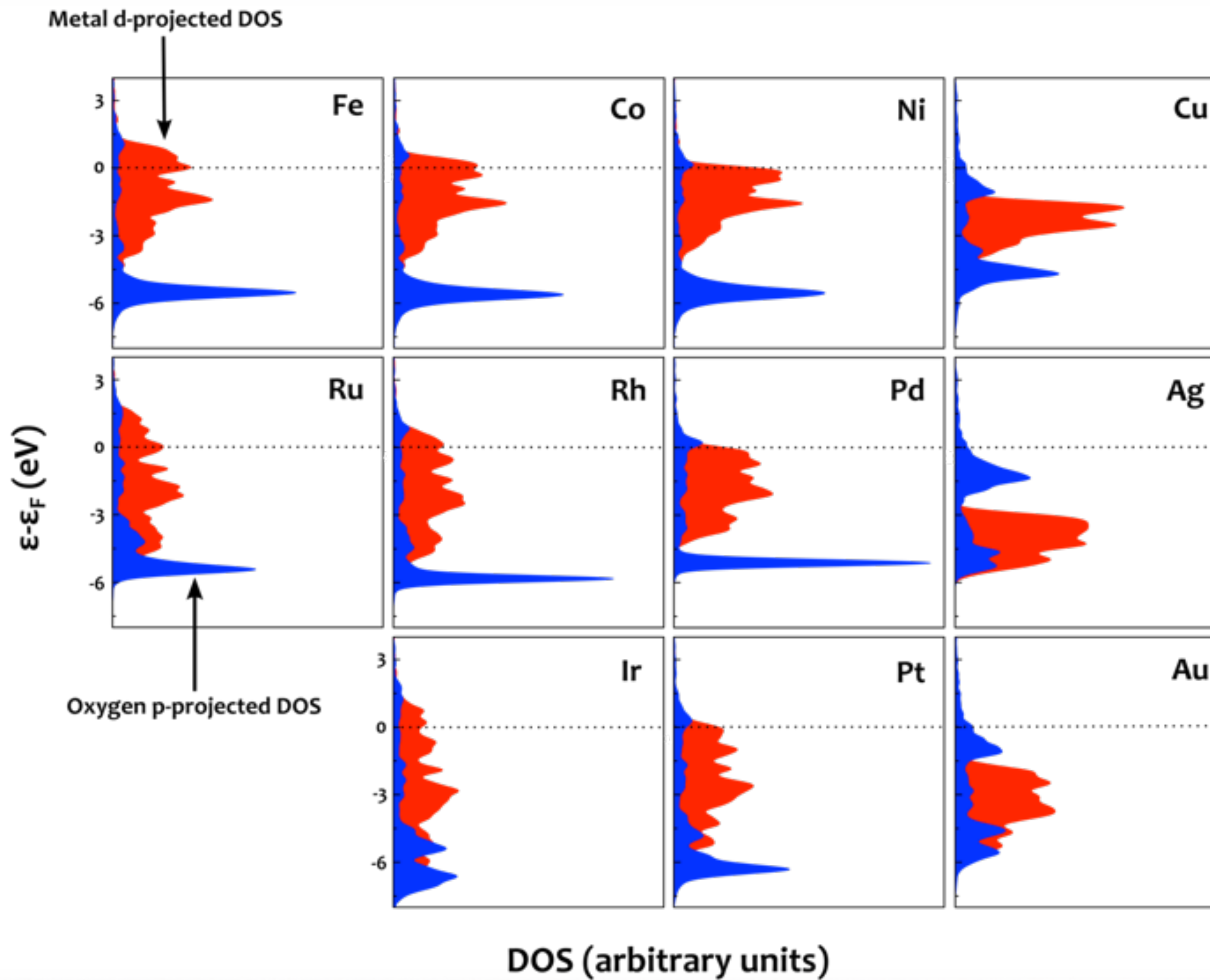
large and constant

small and varies

B. Hammer *et al.*, Surf. Sci. **343**, 211 (1995); Nature **376**, 238 (1995); Adv. Catal. **45**, 71 (2000).

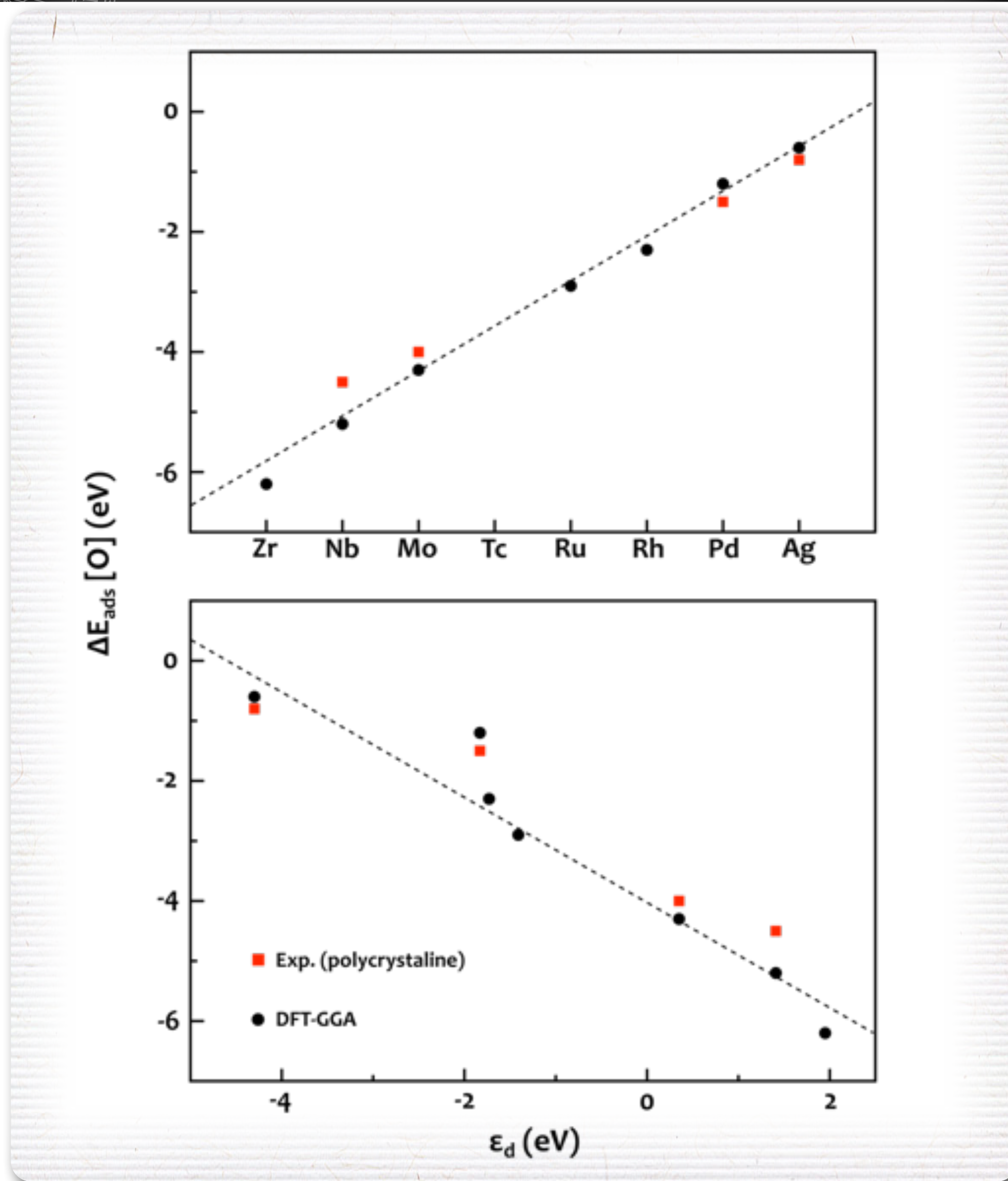


# Calculated Density of States

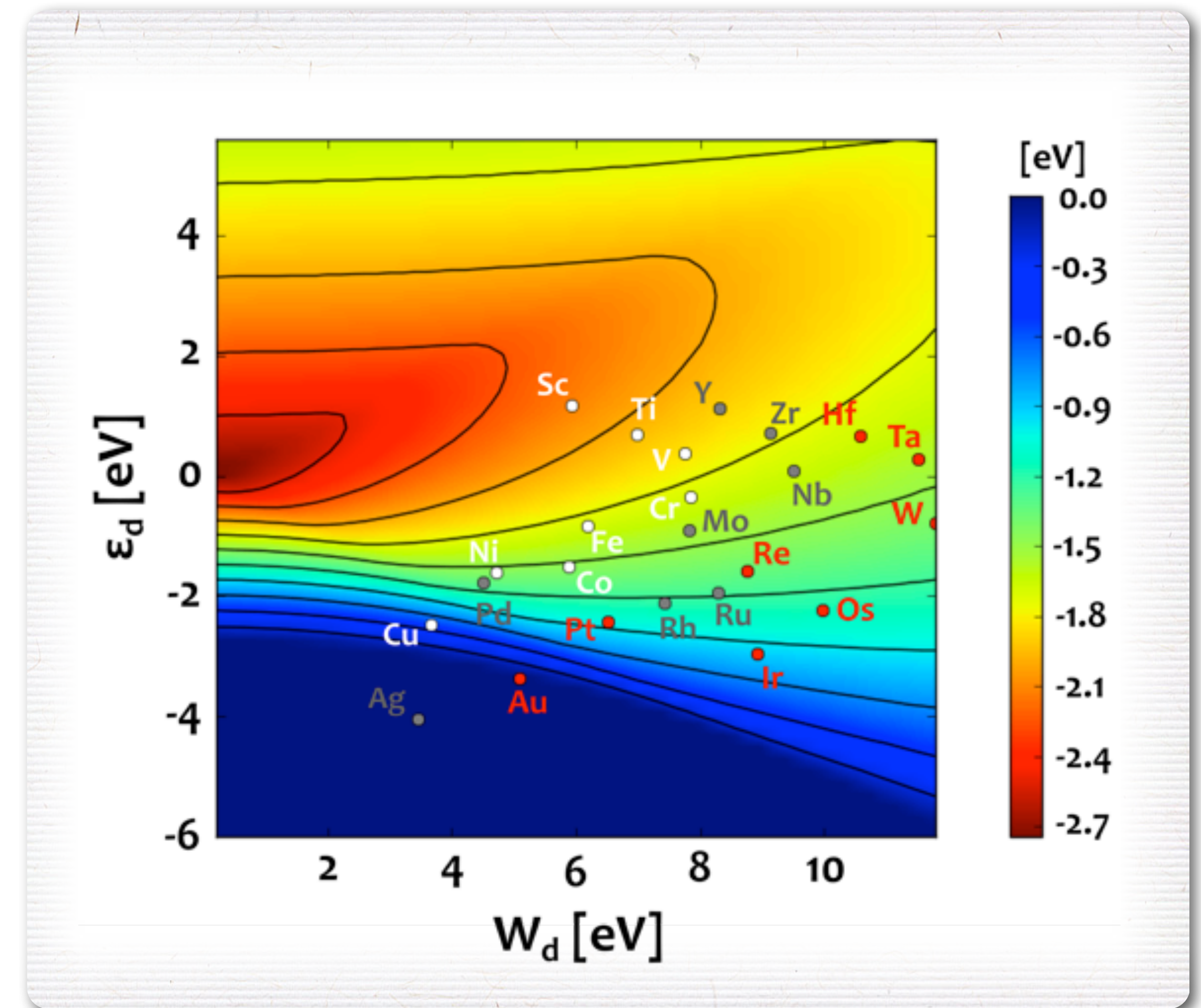




# Variations in O Adsorption Energies



B. Hammer, J. K. Nørskov, *Adv. Catal.* **45**, 71 (2000).



A. Vojvodic, J. K. Nørskov, F. Abild-Pedersen, *Top. Catal.*, accepted (2013).





From d-band model: variations between metals depend on  $\Delta E_d$

$$\Delta E^A = \Delta E_{sp}^A + \Delta E_d^A$$

$$\Delta E^{AH_x}(x) = \Delta E_{sp}^{AH_x} + \Delta E_d^{AH_x}$$

Assume

$$\Delta E_d^{AH_x}(x) = \gamma(x) \Delta E_d^A \Rightarrow$$

$$\begin{aligned} \Delta E^{AH_x}(x) &= \gamma(x) \Delta E_d^A + \Delta E_{sp}^{AH_x} \\ &= \gamma(x) \Delta E_d^A + \Delta E_{sp}^{AH_x} - \gamma(x) \Delta E_{sp}^A + \gamma(x) \Delta E_{sp}^A \\ &\equiv \gamma(x) \Delta E^A + \xi \end{aligned}$$

read off from figure for each  $AH_x/A$  combination

metal independent



$$\Delta E_d = \Delta E_d^{hyb} + \Delta E_d^{orth}$$

In the weak coupling limit  $\Delta E_d^{hyb} \propto \Delta E_d^{orth} \propto V_{ad}^2$ , where  $V_{ad}^2$  is the coupling strength between the adsorbate and the metal  $d$ -states.

$V_{ad}^2(\{r_{a_i}\})$  function of the number of metal neighbors and their distance to the adsorbate.

Add H atoms to the central C, N, O or S *i.e.*  $x \nearrow \Rightarrow$

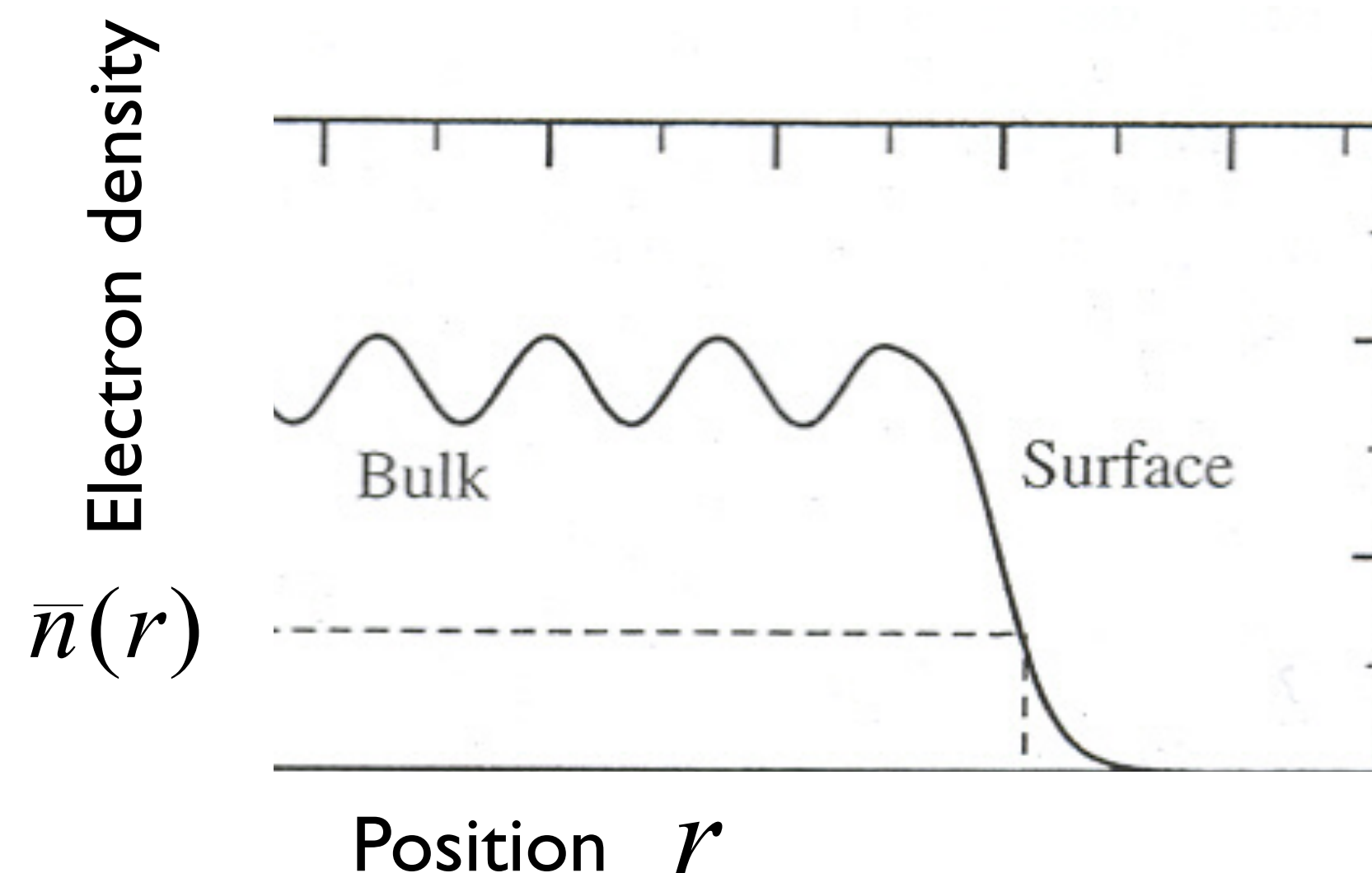
- $r_{a_i} \nearrow$
- $V_{ad}^2(\{r_{a_i}\}) \searrow$

$sp$ -coupling determines the adsorption bond lengths  $r_{a_i}$

Use effective medium theory (EMT)



- Interaction of atom A with atoms in the vicinity  $\approx$  interaction of A with a homogeneous electron gas (effective medium) of a density given by a spherical average  $n$  of the nearby atoms.  $\Delta E = \Delta E_{hom}(n)$ .
- $\Delta E_{hom}(n)$  has a minimum for a particular electron density  $n_0 \Rightarrow$  Equilibrium geometry is the position where A experiences  $n = n_0$

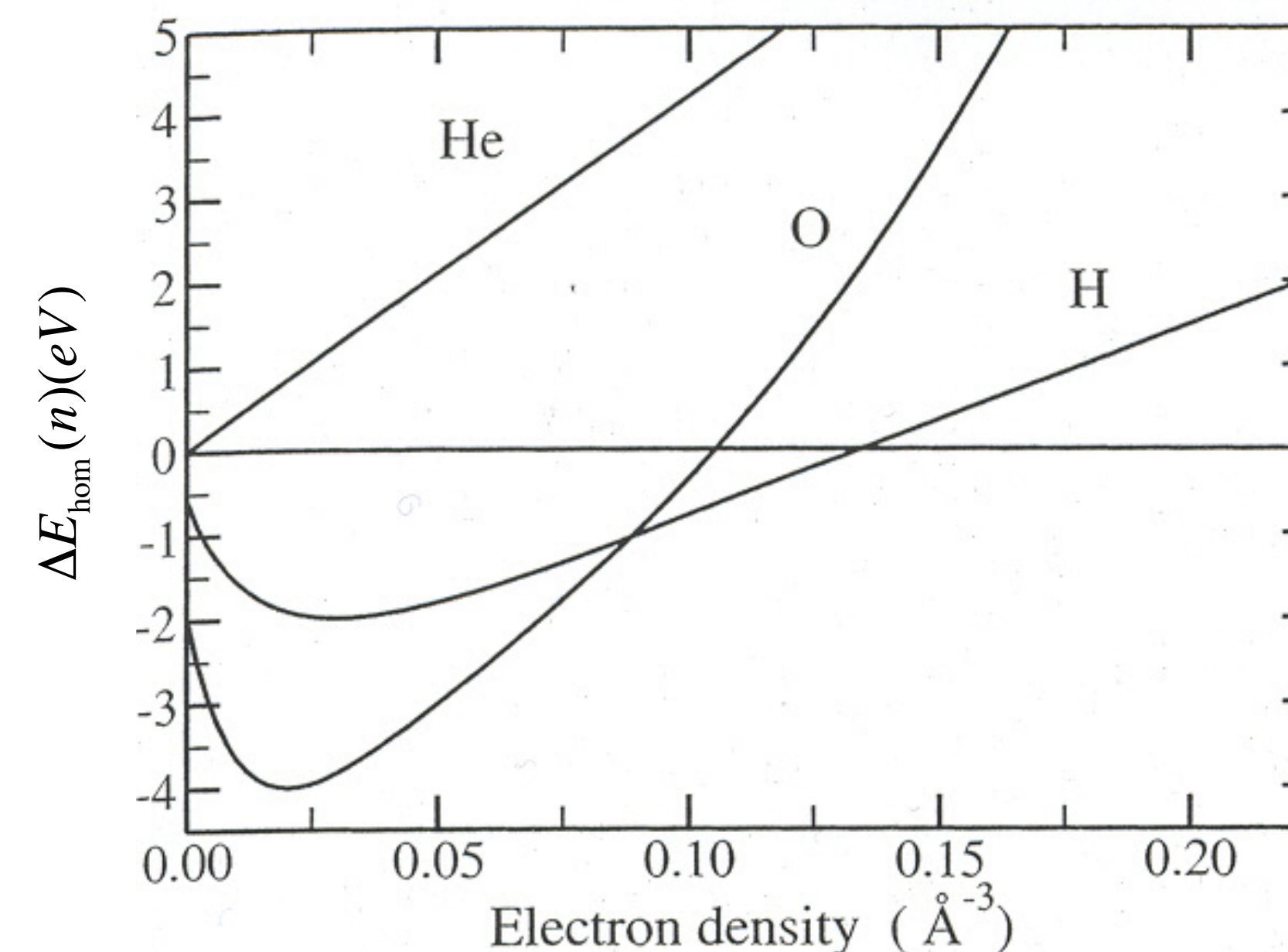


Surface electron density  $n_{surf}$  needed for  $n = n_0$  as  $x \nearrow$

$$\Rightarrow n_{surf} = \frac{x_{max} - x}{x_{max}} n_0 = \gamma(x) n_0$$

Assumption:  $d$ -states have approximately the same decay length as the  $sp$ -states  $\Rightarrow V_{ad}^2$  scales with  $n_{surf}$

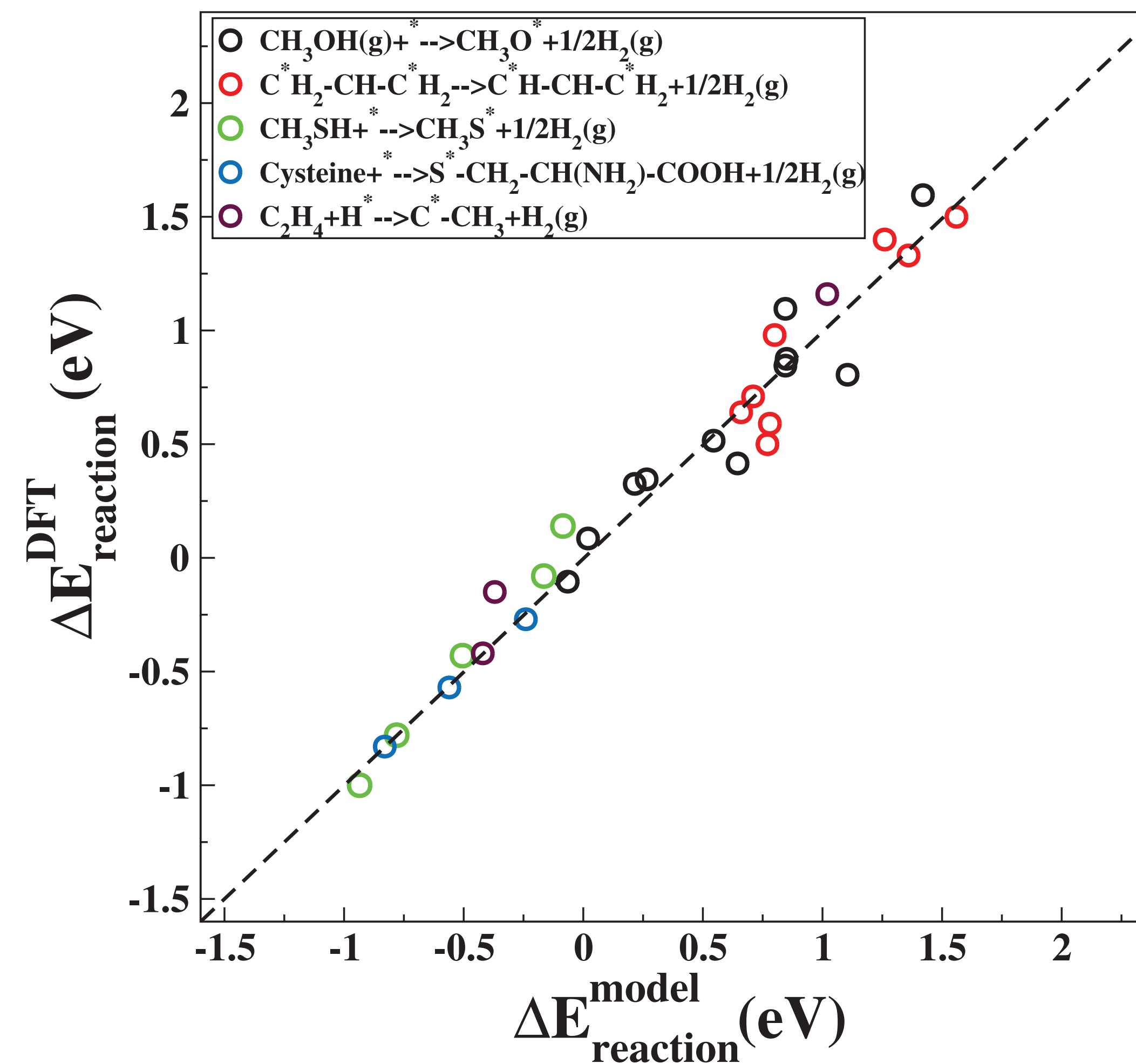
$$\Delta E_d \propto V_{ad}^2 \propto n_{surf}(x) \propto \frac{x_{max} - x}{x_{max}} = \gamma(x)$$





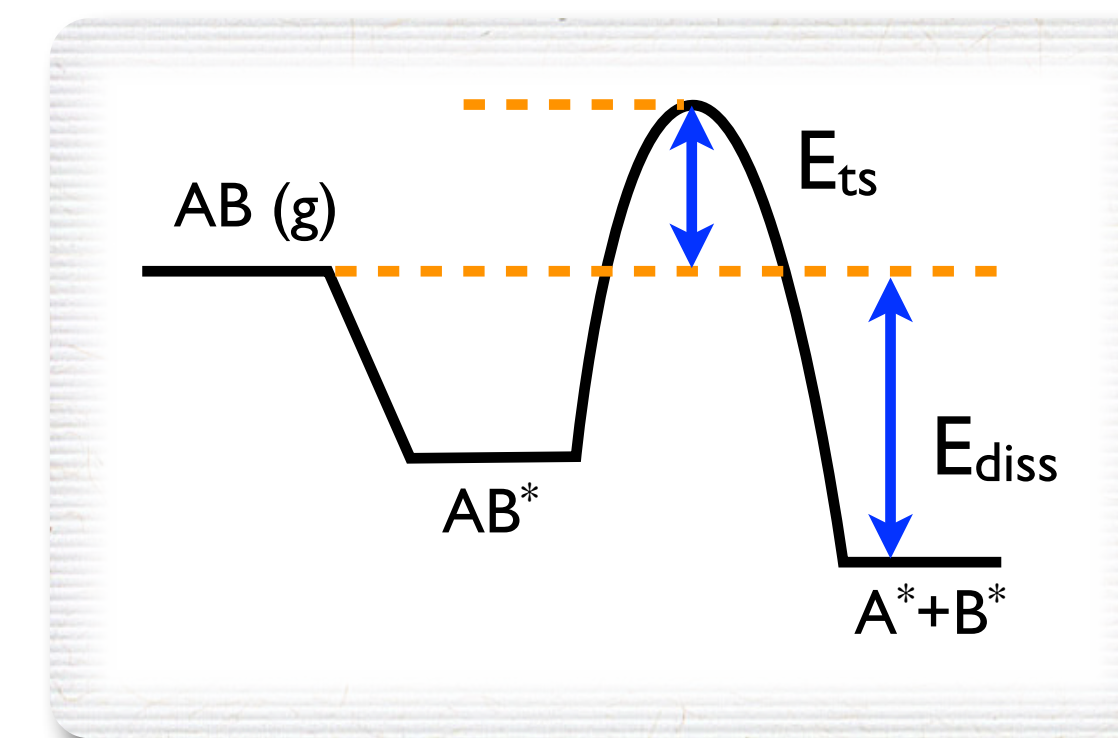
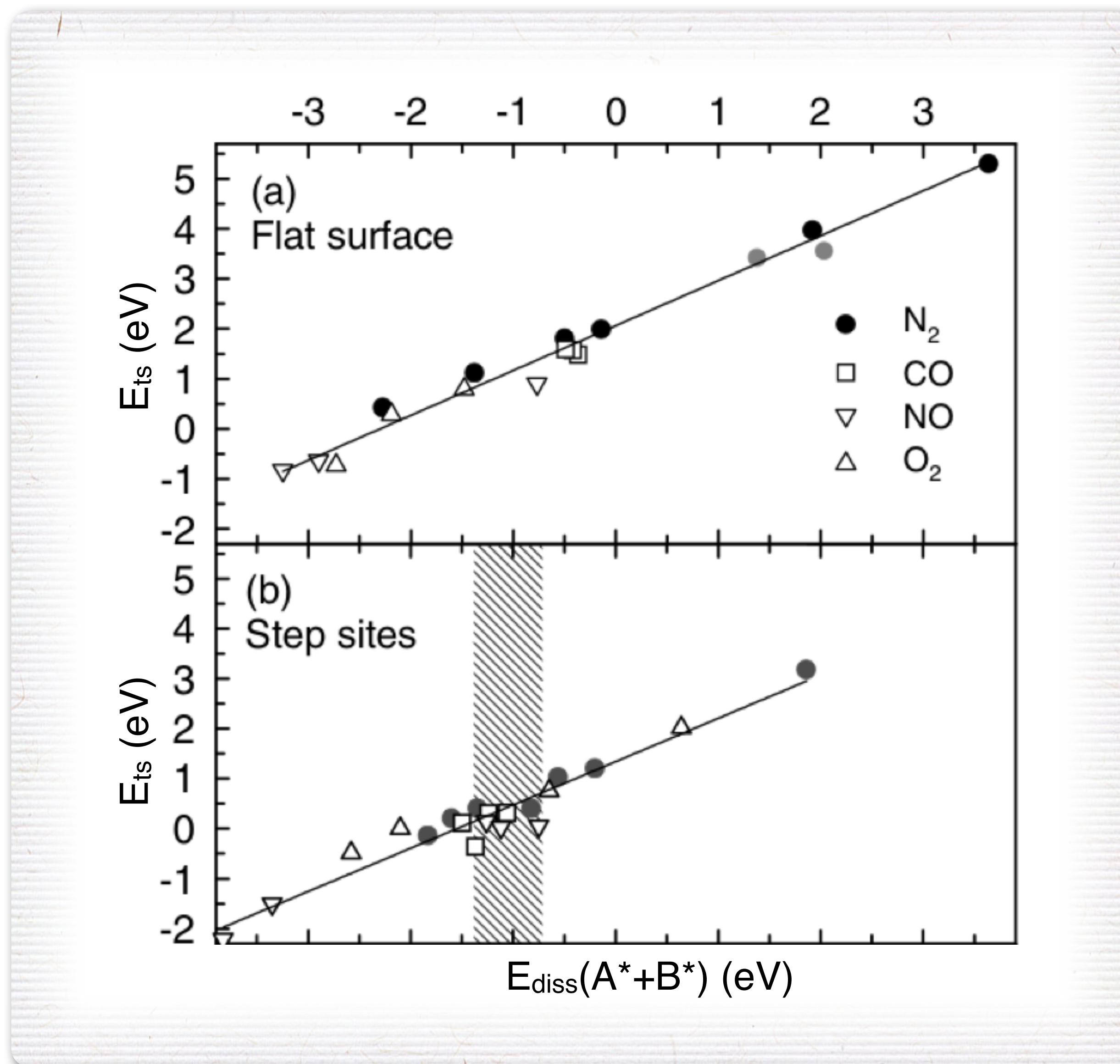
For any (de)hydrogenation reaction of molecules bonding to a TM surface via C, H, O or S atoms estimate the reaction energy  $\Delta E$  for all TM's just by knowing  $\Delta E$  for one TM.

$$\Delta E = \sum_{i=1}^N (\gamma_i \Delta E^{A_i}) + \Delta \xi$$



F. Abild-Pedersen, *et al.*, Phys. Rev. Lett. **99**, 016105 (2007).



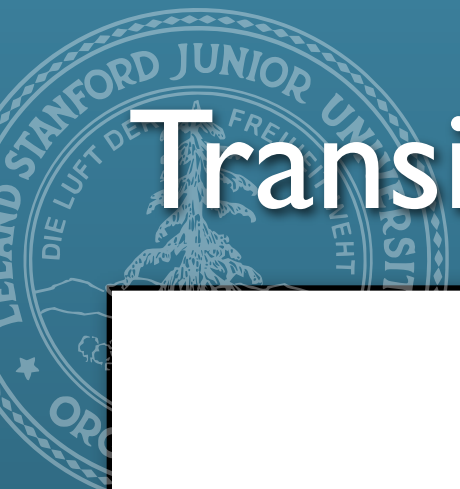


$$E_{ts} = \gamma \Delta E_{diss} + \xi$$

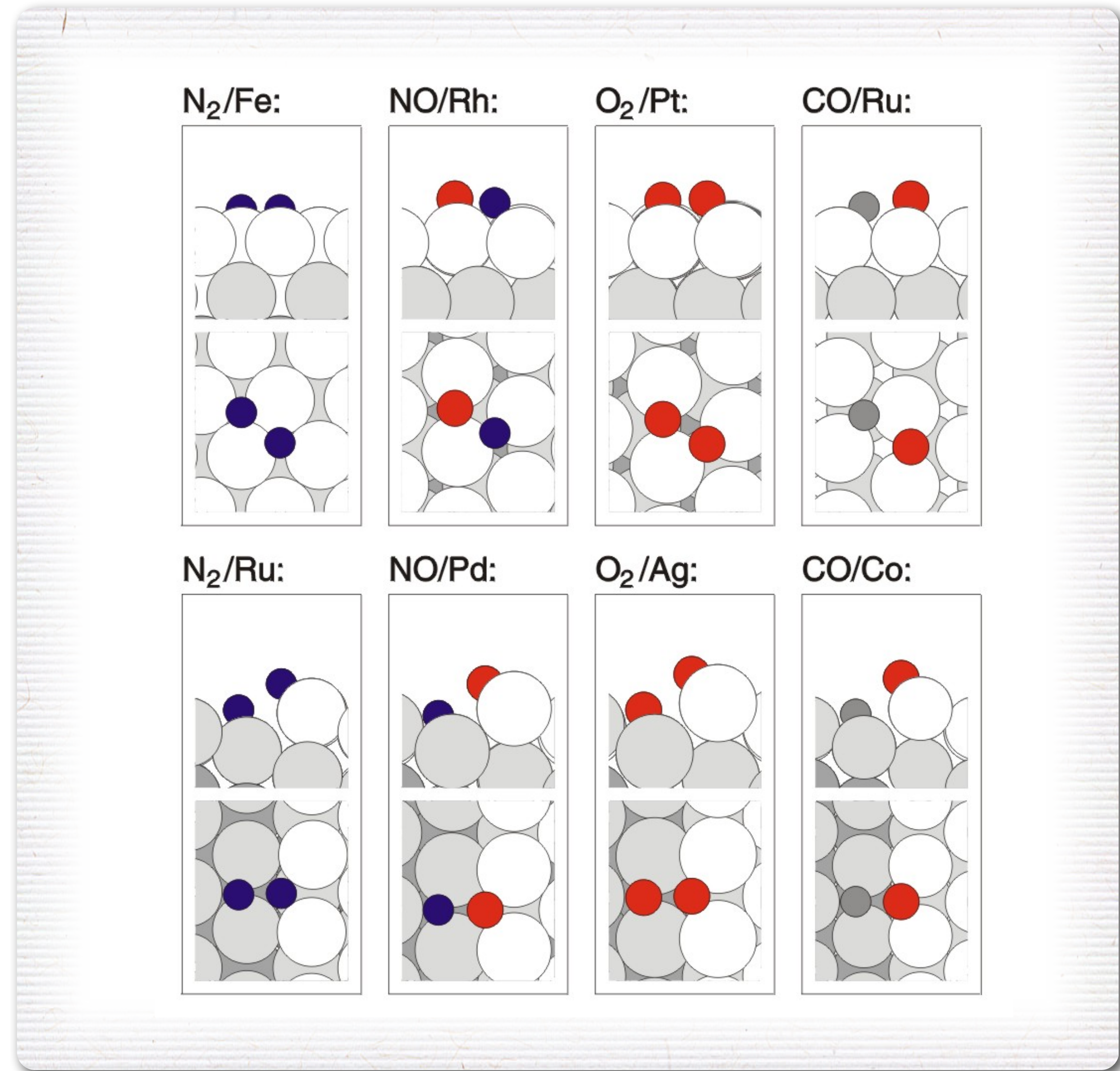
Relations are active site dependent

J. K. Nørskov et al., J. Catal. **209**, 275 (2002).



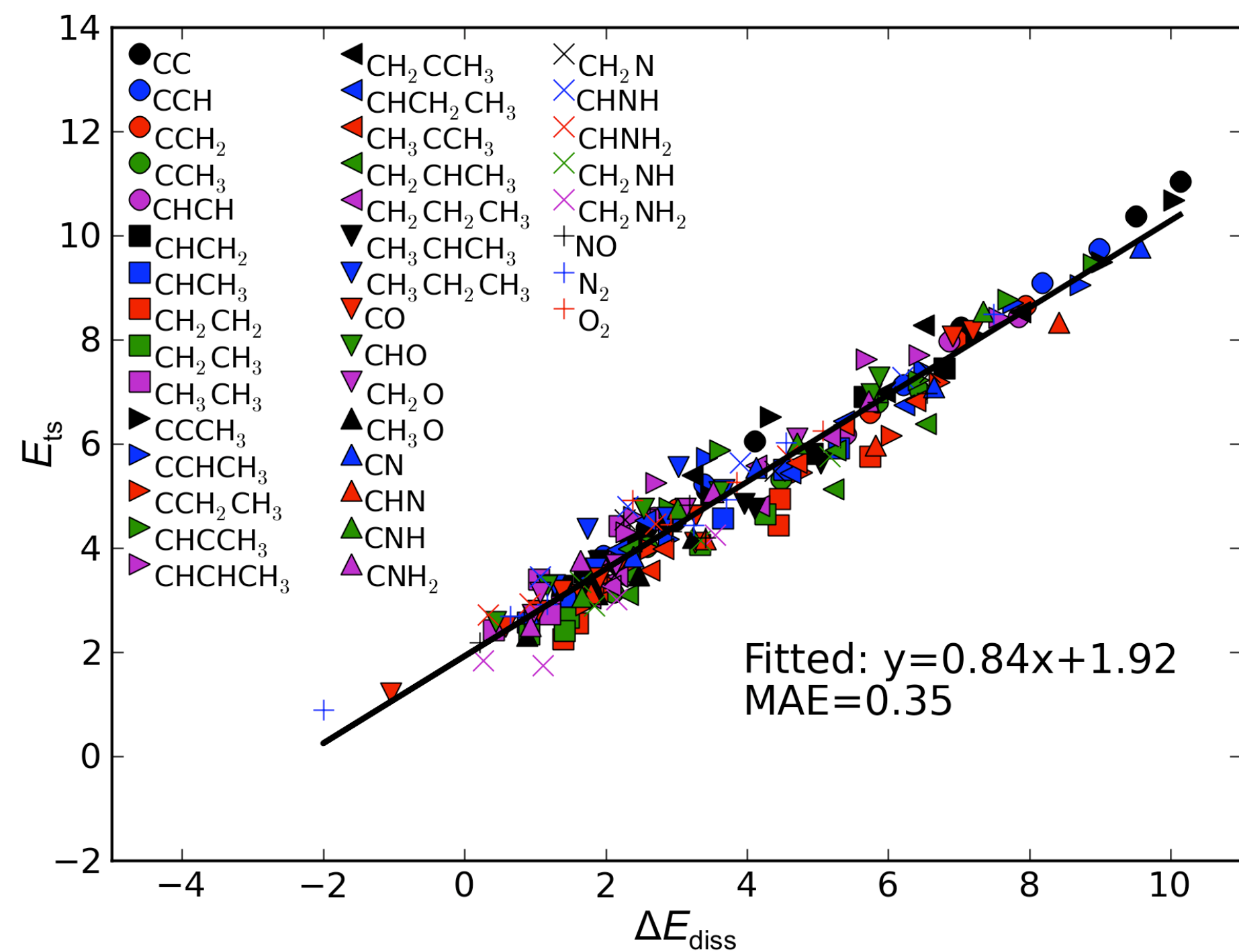


# Transition State Structures

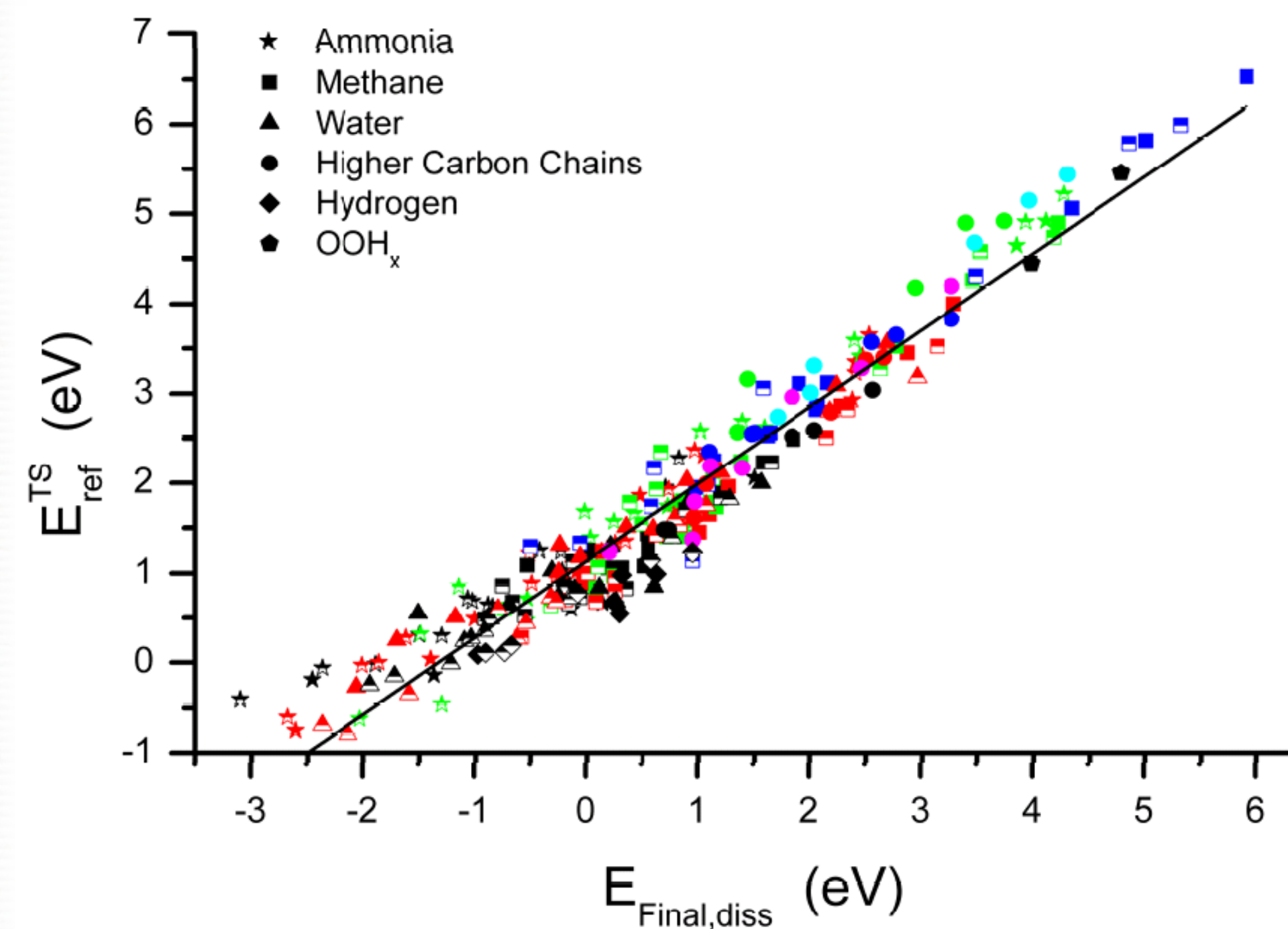




## C-C, C-O, C-N ... bond breaking



## C-H, O-H, N-H ... bond breaking

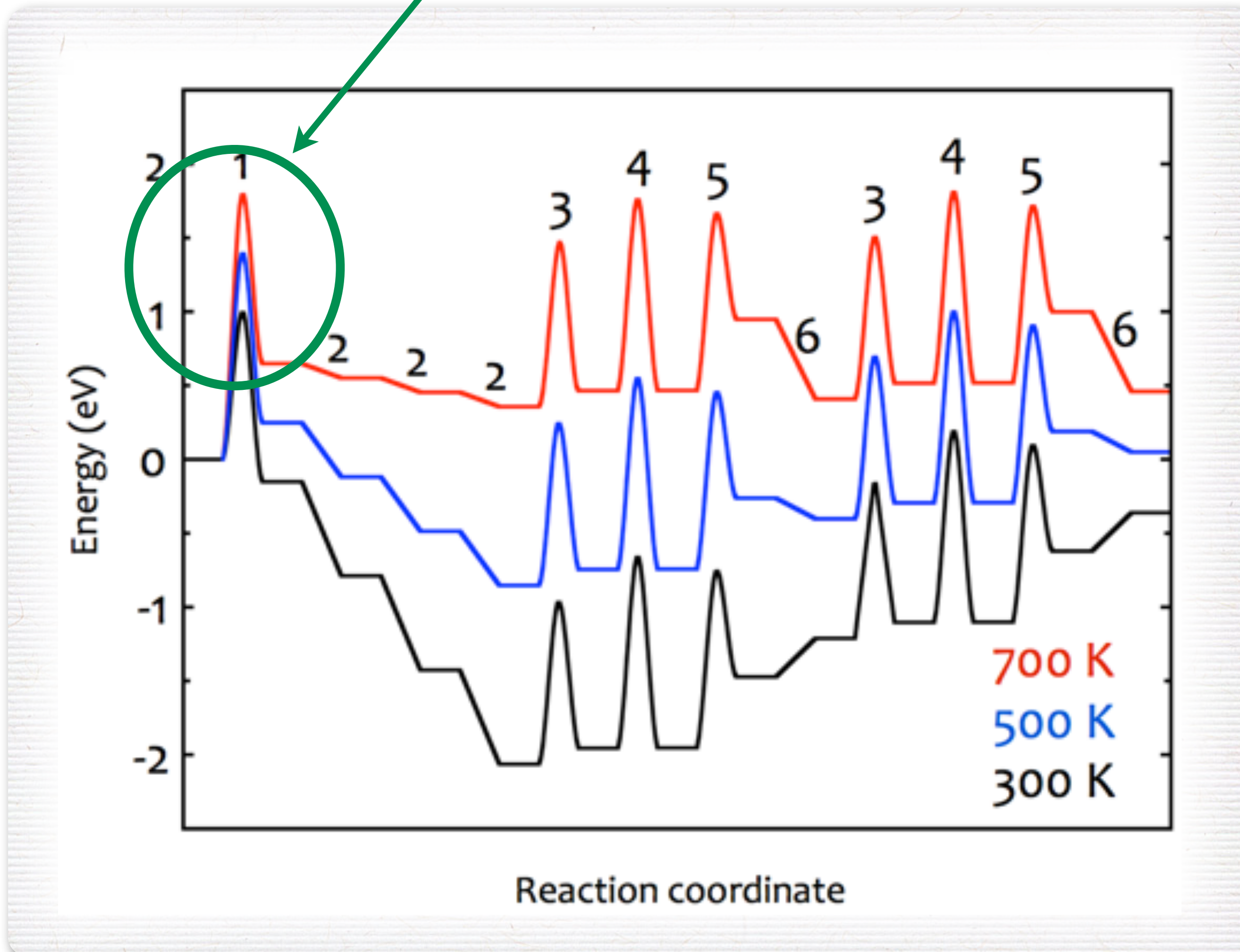


S.Wang, et al, Catal. Lett. **141**, 370 (2011).

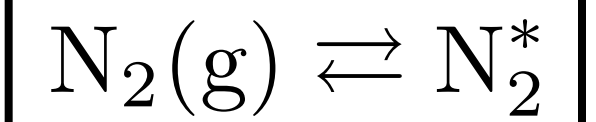
S.Wang, et al, Phys. Chem. Chem. Phys. **13**, 20760 (2011).



**N<sub>2</sub> dissociation is rate determining step**



1.  $\text{N}_2(\text{g}) + 2^* \rightleftharpoons 2\text{N}^*$
2.  $\text{H}_2(\text{g}) + 2^* \rightleftharpoons 2\text{H}^*$
3.  $\text{N}^* + \text{H}^* \rightleftharpoons \text{NH}^*$
4.  $\text{NH}^* + \text{H}^* \rightleftharpoons \text{NH}_2^*$
5.  $\text{NH}_2^* + \text{H}^* \rightleftharpoons \text{NH}_3^*$
6.  $\text{NH}_3^* \rightleftharpoons \text{NH}_3(\text{g})$



• How many energetic variables are there?

6 intermediates

+

6 transition states

=

12 different energy parameters



# Dissociation of A<sub>2</sub> is Rate Determining Step

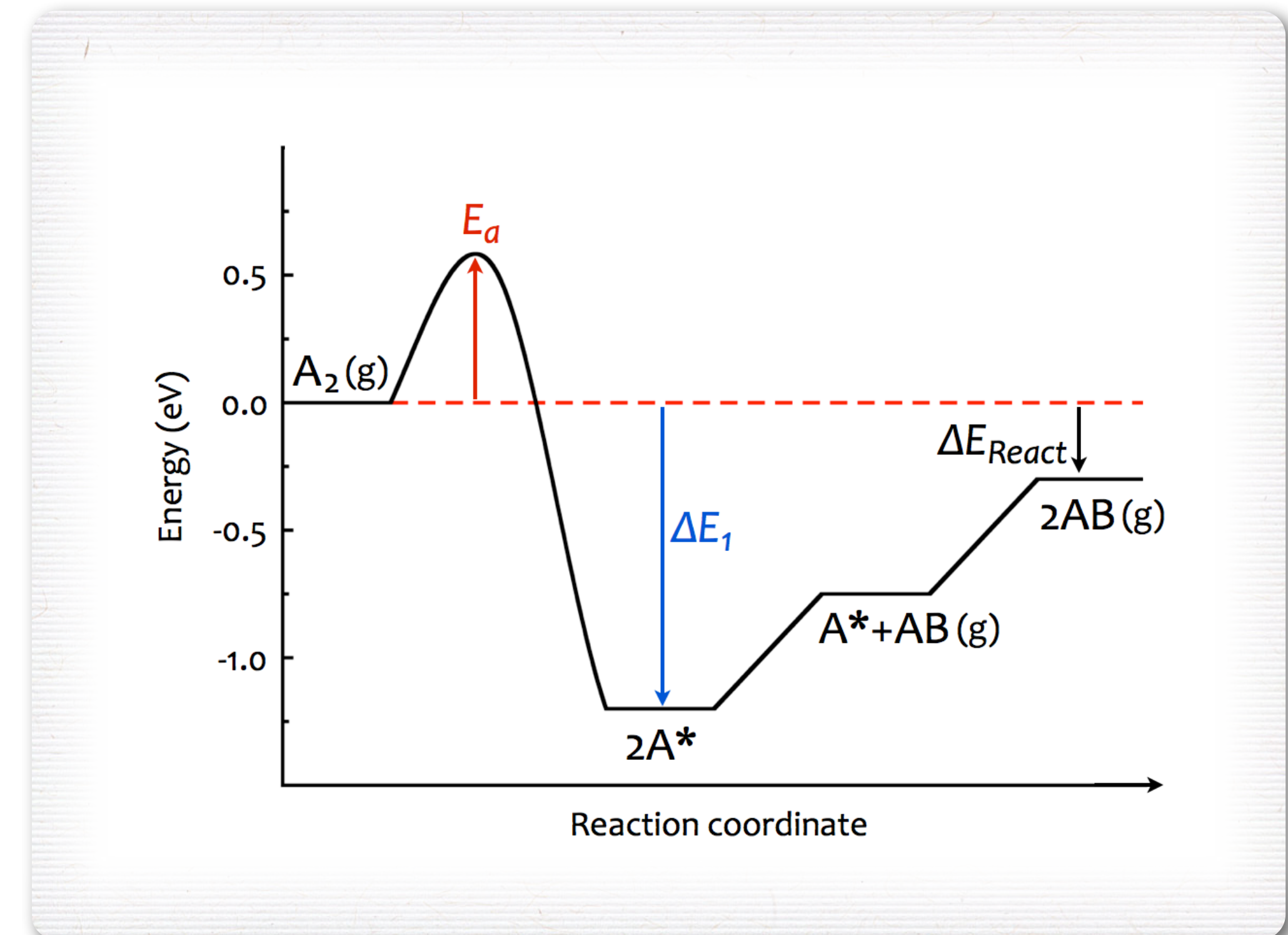
$$R(T, p) = k_1 p_{A_2} \theta_*^2 (1 - \gamma)$$

$$k_1 = \frac{k_B T}{h} e^{-\Delta S_a / k_B} e^{-E_a / k_B T}$$

$$\gamma = \frac{p_{AB}^2}{K_{eq} p_{A_2} p_B^2}$$

$$\theta_* = \frac{1}{1 + \sqrt{K_1 p_{A_2} \gamma}}$$

$$K_1 = e^{-\Delta G_1^\circ / k_B T}$$





$$E_{a_1} = \gamma \Delta E_1 + \xi \equiv E_a(2E_N)$$

$$R(T, p) = k_1 p_{A_2} \theta_*^2 (1 - \gamma)$$

$$k_1 = \frac{k_B T}{h} e^{-\Delta S_a / k_B} e^{-E_a / k_B T} = k_1(\Delta E_1)$$

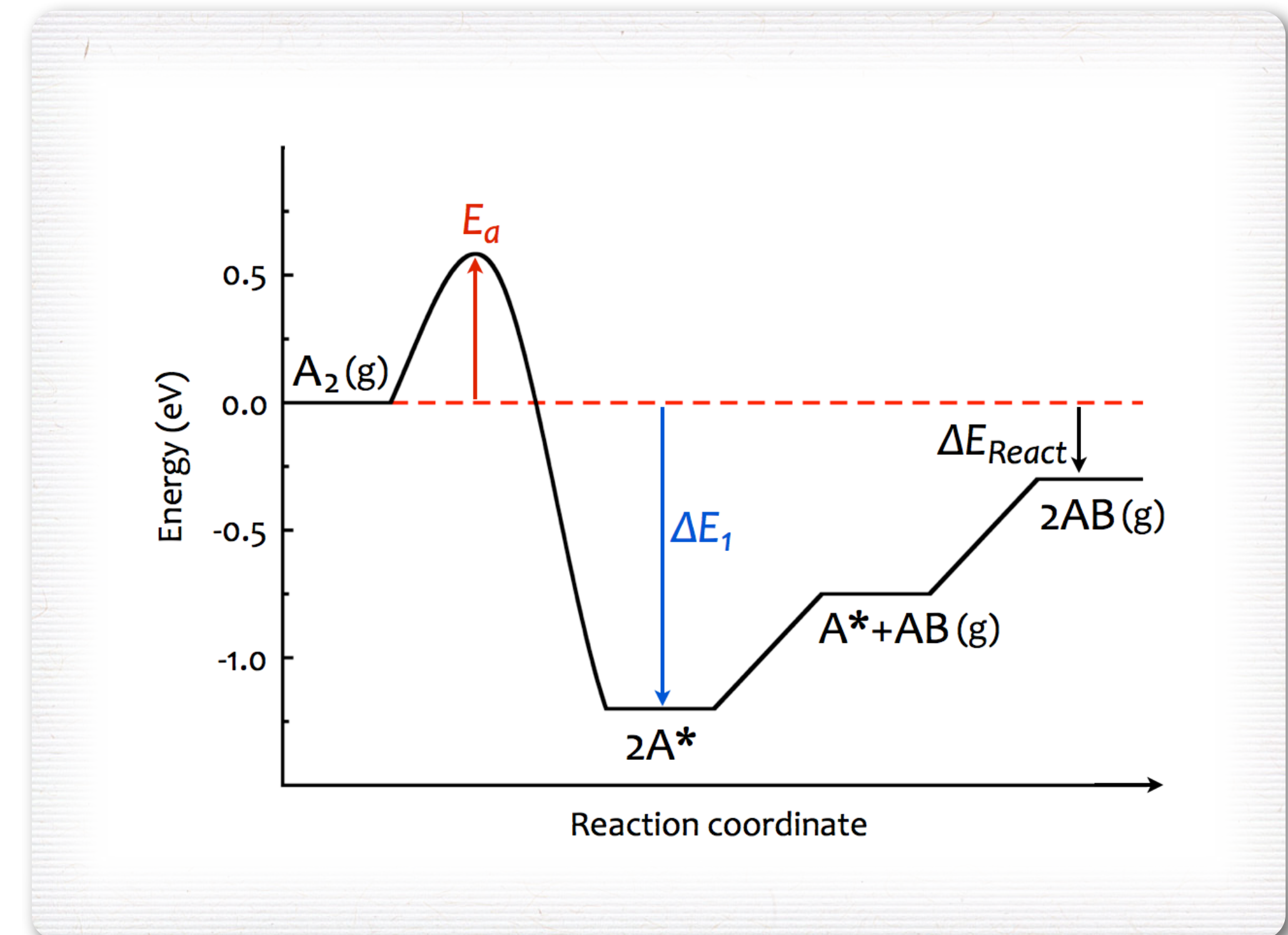
$$\gamma = \frac{p_{AB}^2}{K_{eq} p_{A_2} p_B^2}$$

$$\theta_* = \frac{1}{1 + \sqrt{K_1 p_{A_2} \gamma}}$$

$$K_1 = e^{-\Delta G_1^\circ / k_B T} = K_1(\Delta E_1)$$

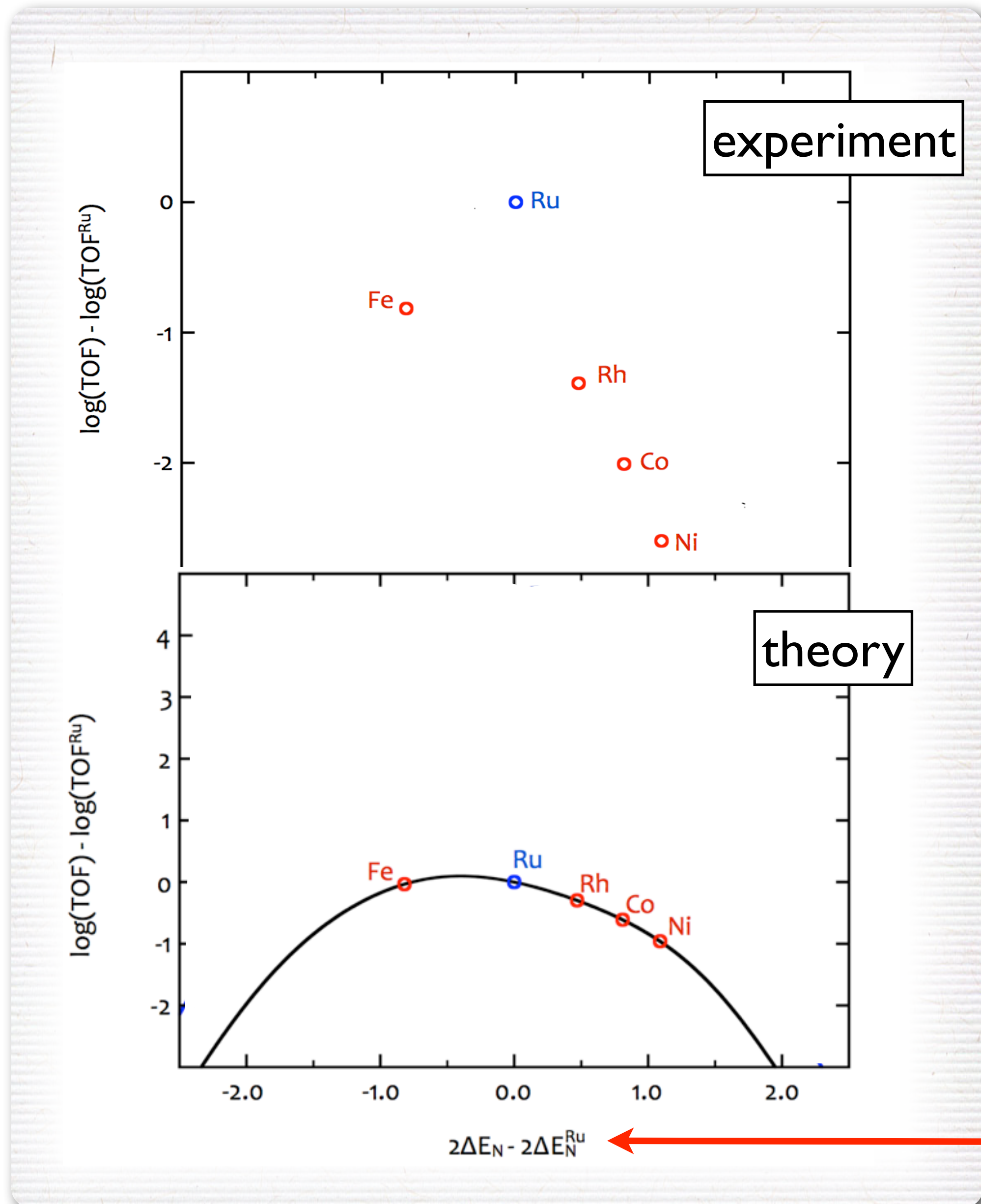
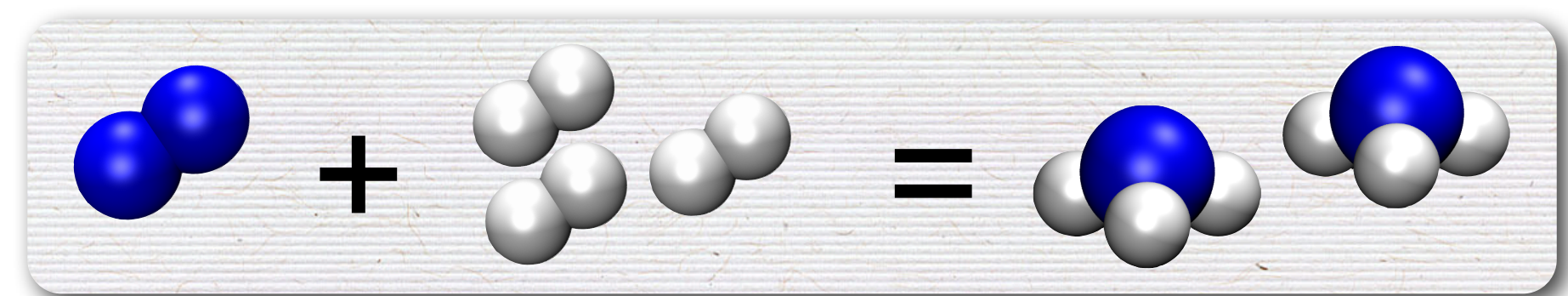
descriptor

$$\Delta E_1 = 2E_N$$





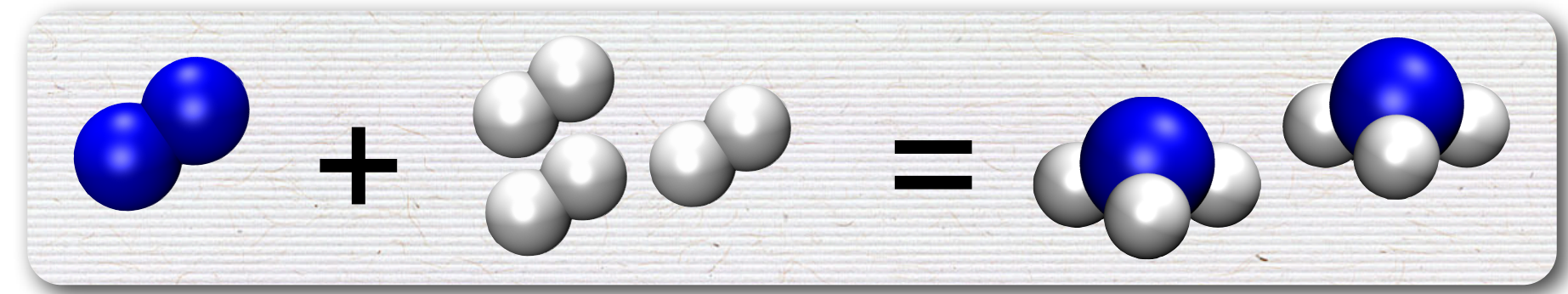
# Volcano for Ammonia Synthesis



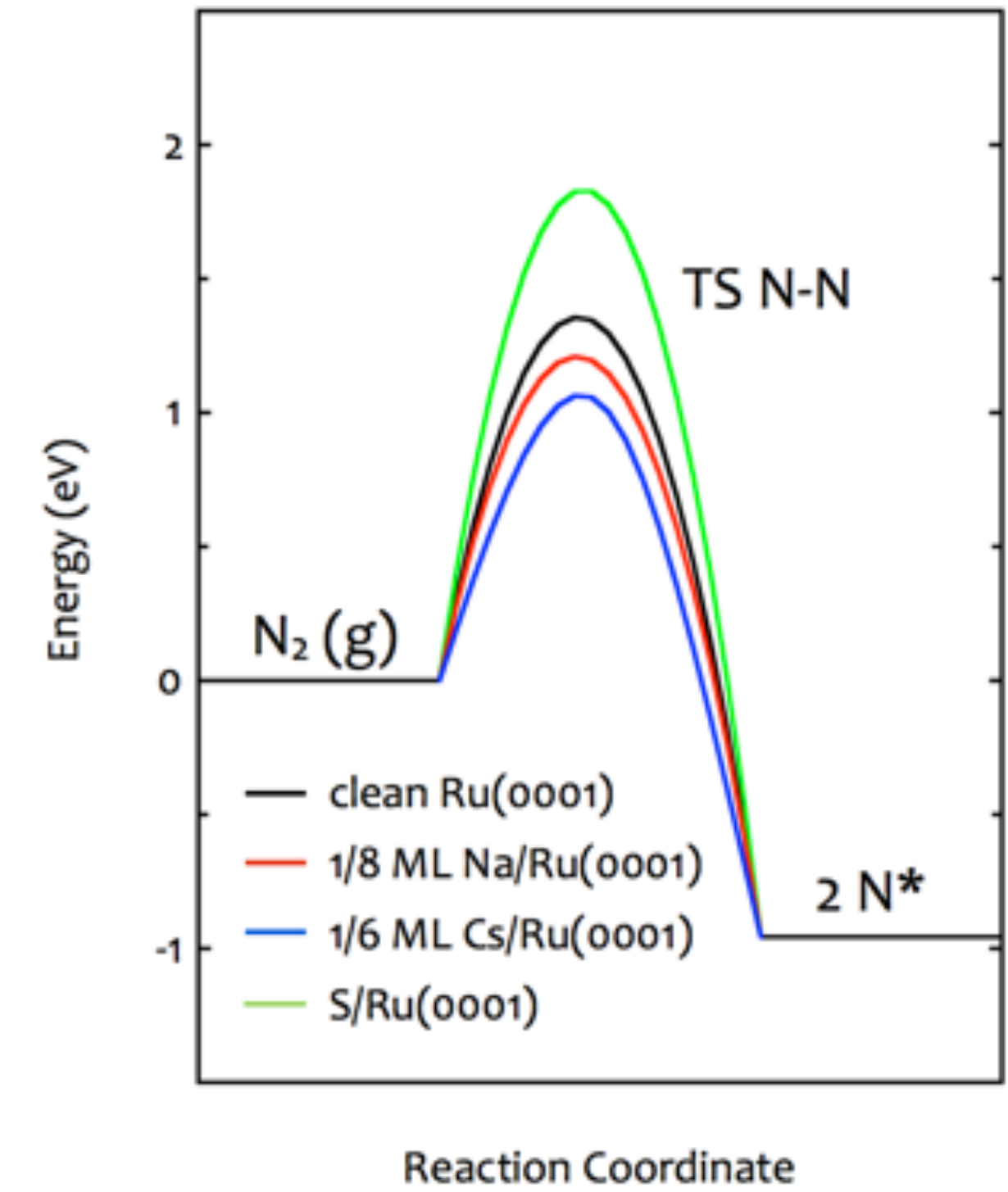
Descriptor  $\Delta E_1 = 2E_N$



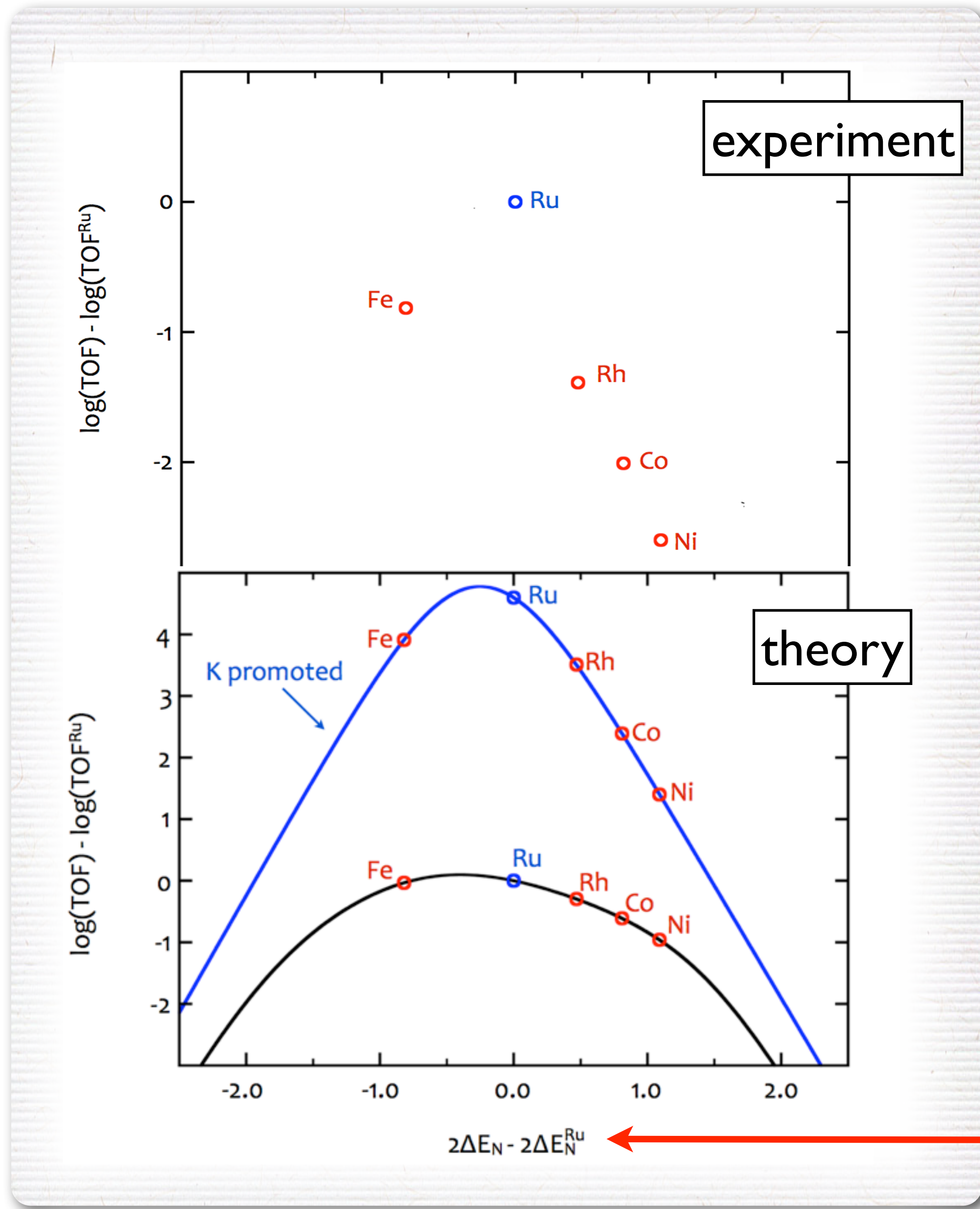
# Volcano for Ammonia Synthesis



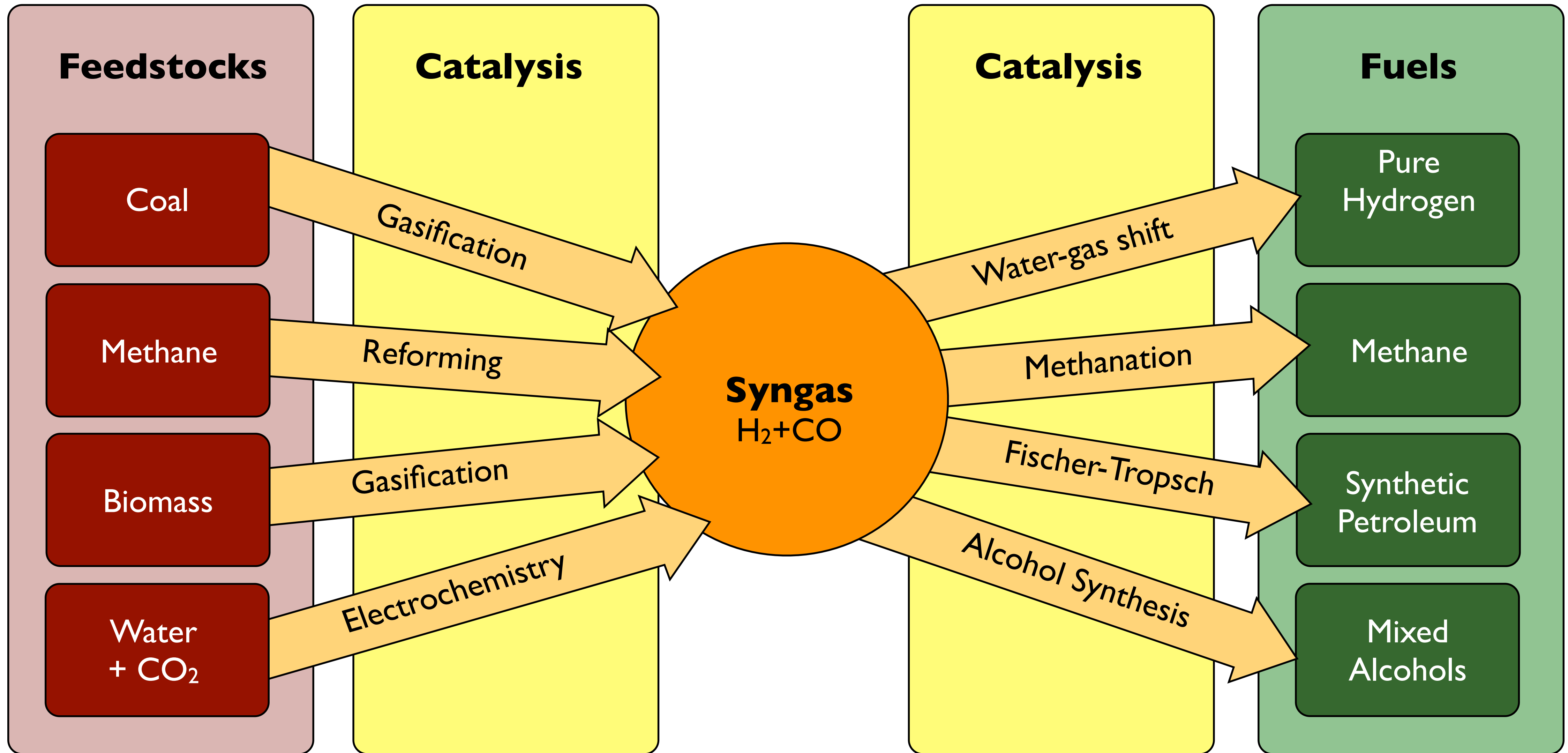
effect of alkali promotion



Descriptor  $\Delta E_1 = 2E_N$



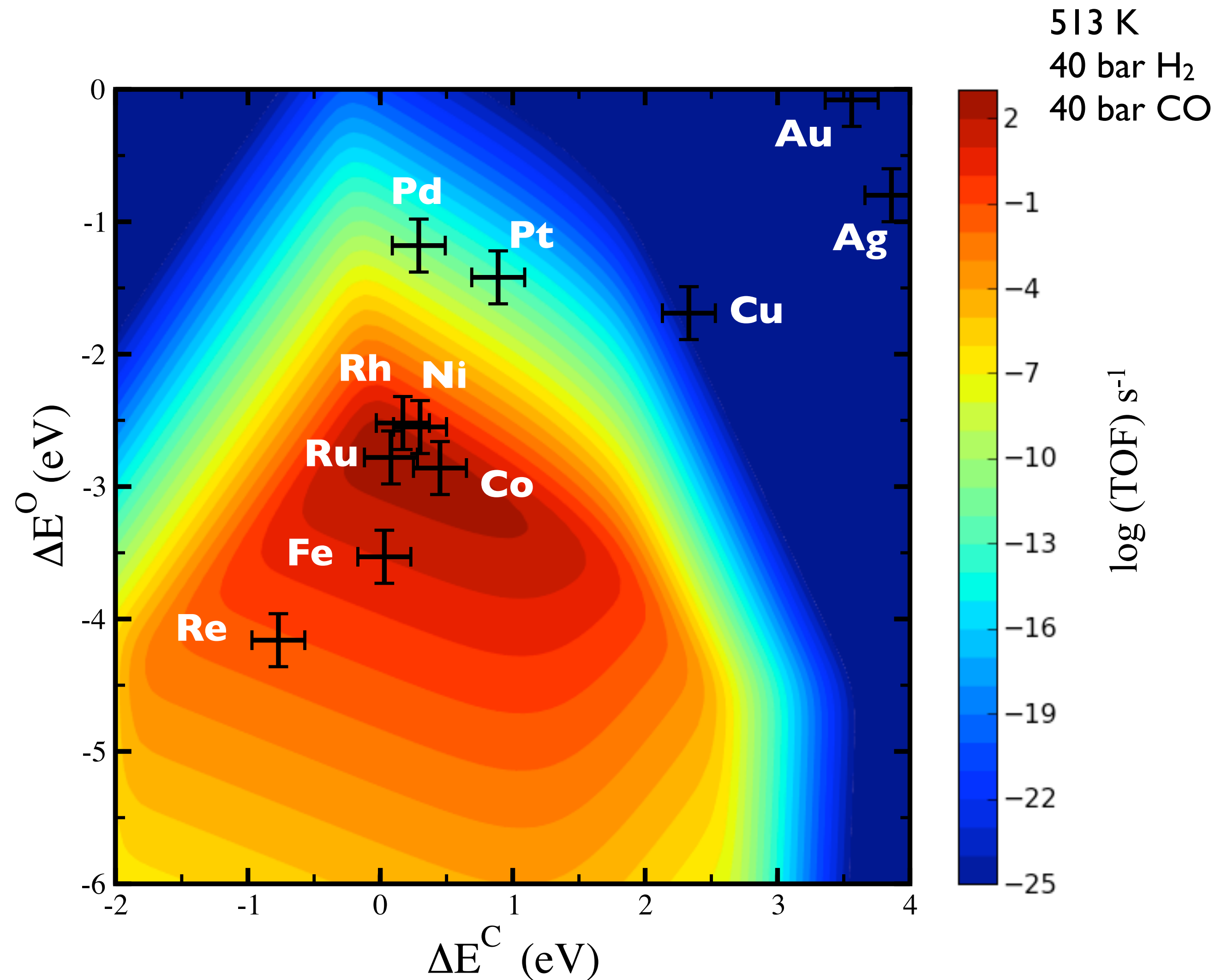




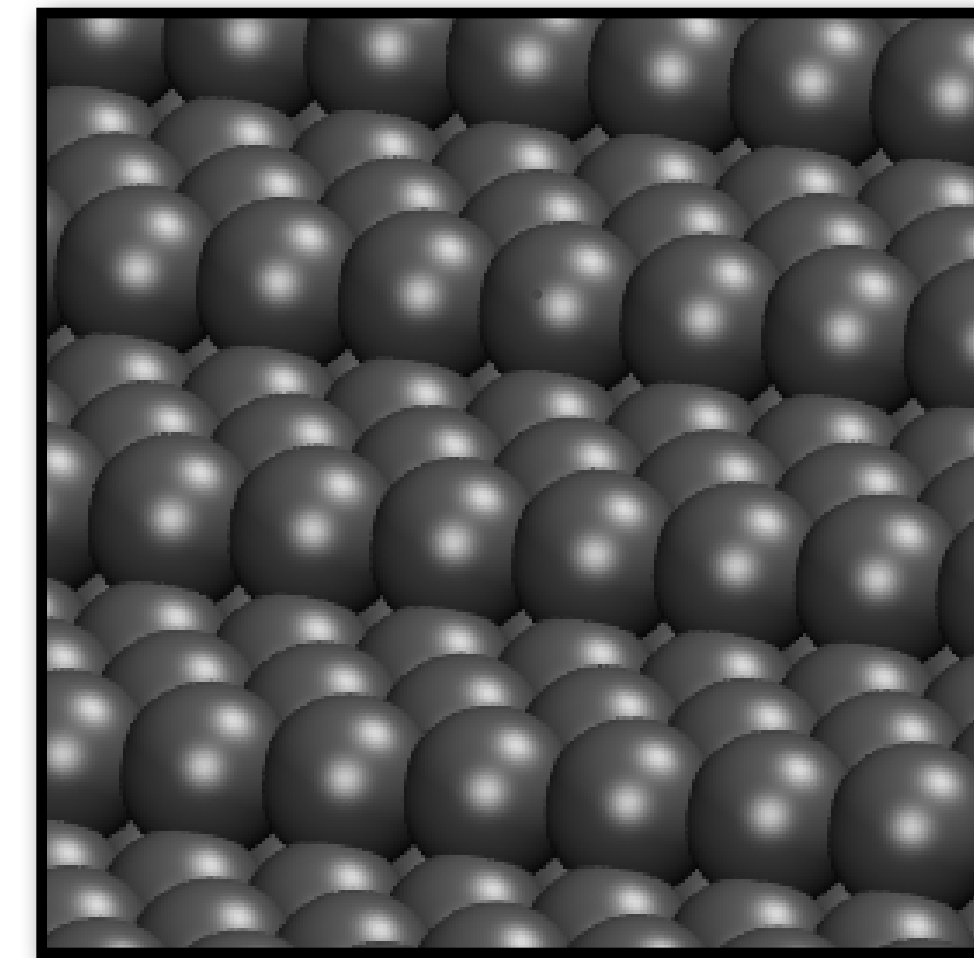
“Anything can be gasified”



# Methanation Volcano ( $\text{CO} + 3\text{H}_2 \Rightarrow \text{CH}_4 + \text{H}_2\text{O}$ )



211 monometallic surface

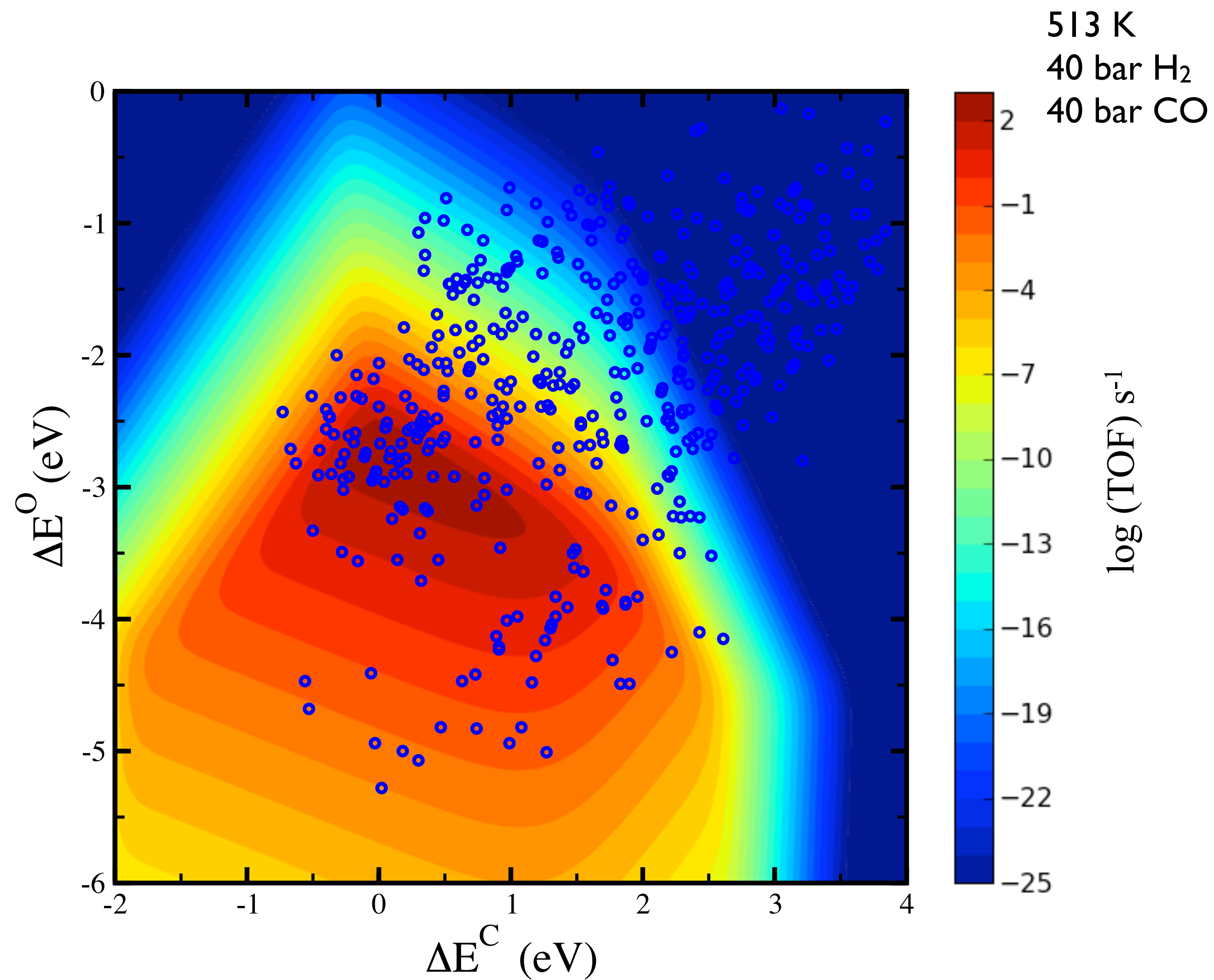


- C & O adsorption energies are descriptors
- CO dissociation = RDS
- Ru, Co highest activity
- Rh, Ni, Fe also have high activity

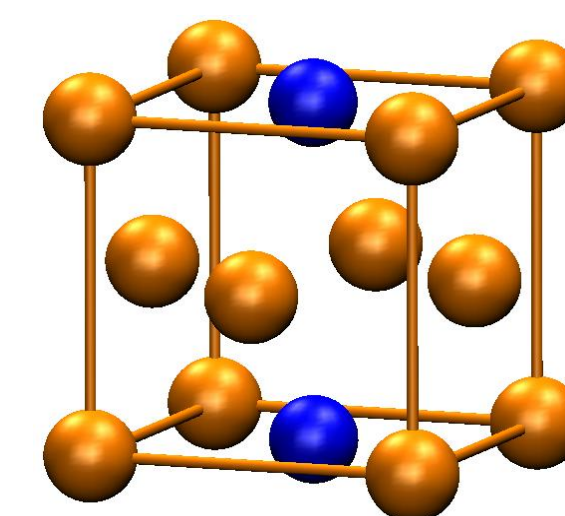
J. K. Nørskov, et. al. Proc. Nat. Acad. Sci. USA **108**, 937 (2011).



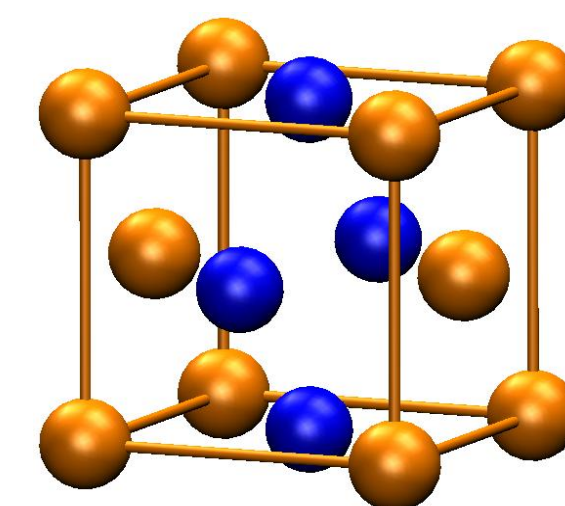
# Finding Methanation Catalyst Leads



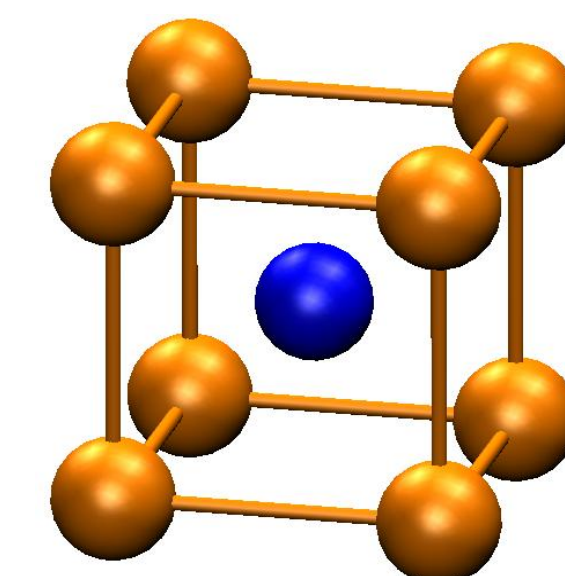
## 211 bimetallic surfaces



A<sub>3</sub>B L<sub>12</sub>



A<sub>5</sub>B<sub>3</sub> L<sub>10</sub>

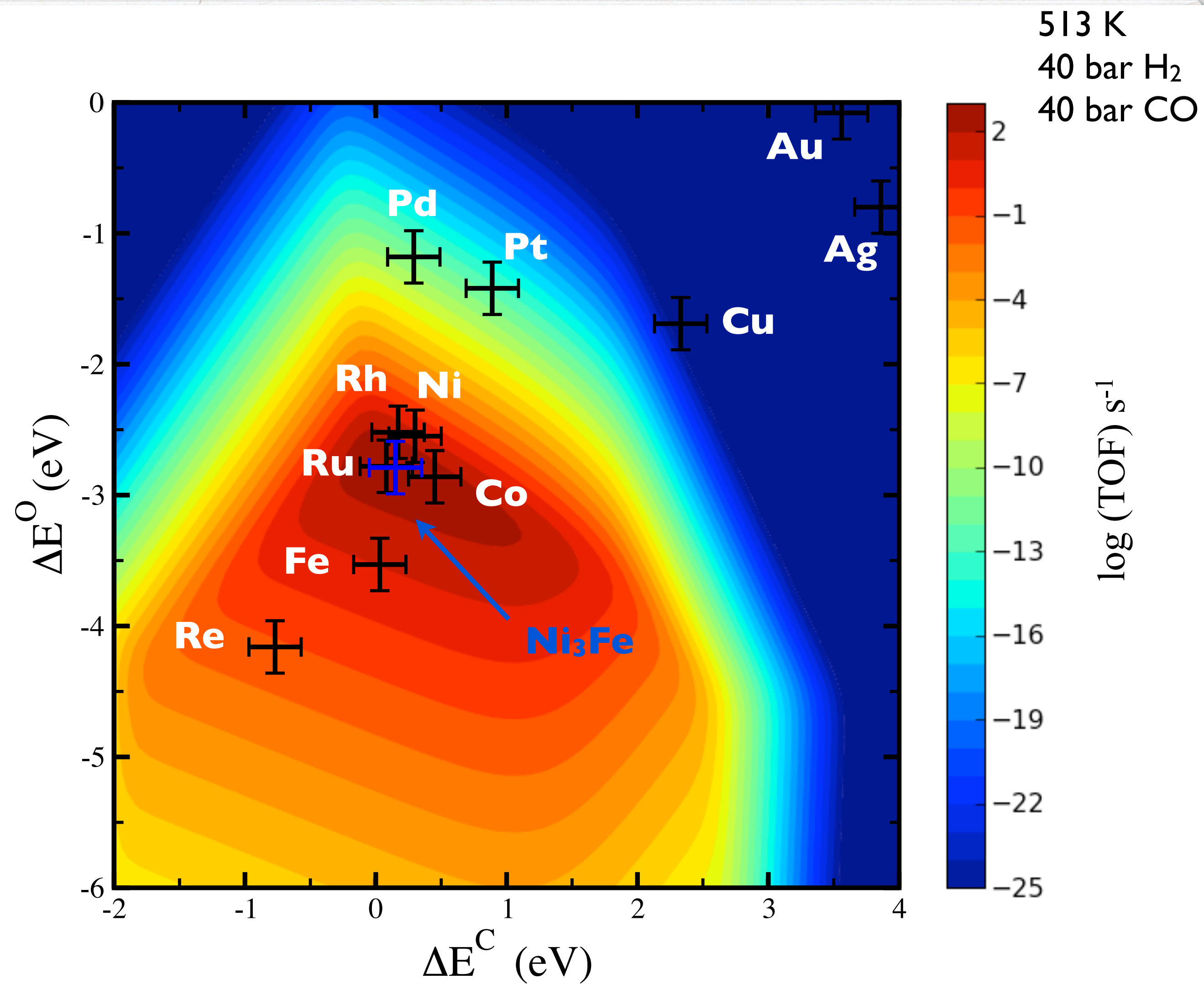


AB BCC-B<sub>2</sub>

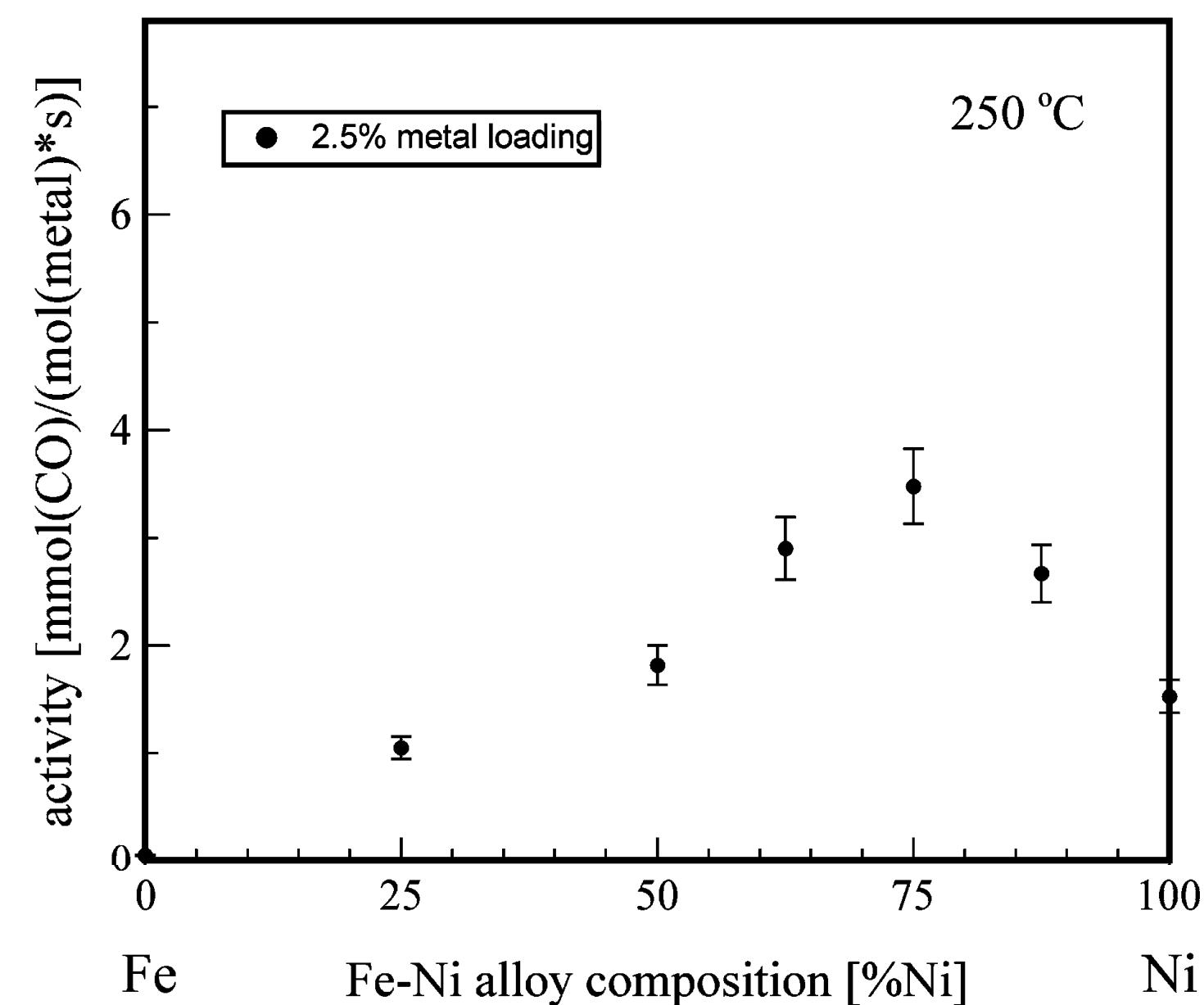
screening of 4400 systems



# Identification of Ni<sub>3</sub>Fe as Methanation Catalyst



Screening + Pareto analysis => Ni<sub>3</sub>Fe

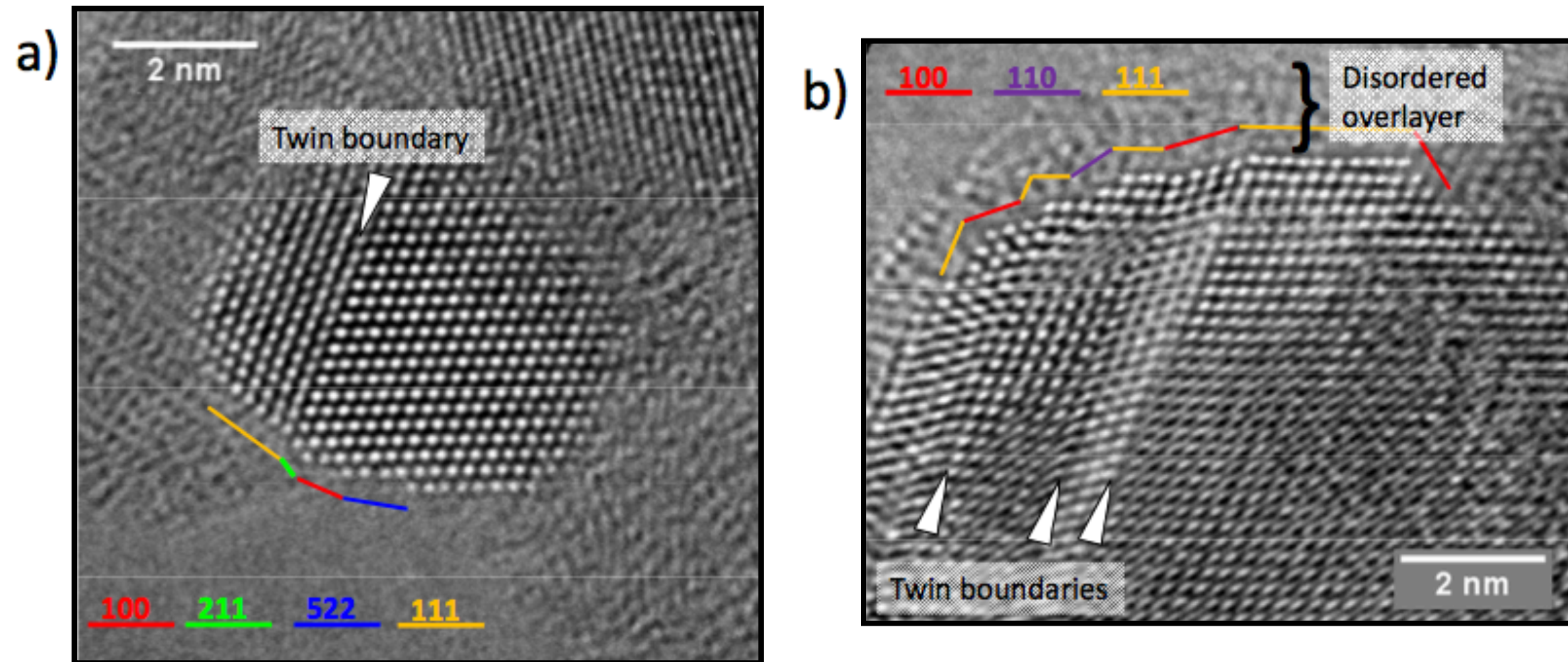


J. K. Nørskov, et. al. Proc. Nat. Acad. Sci. USA **108**, 937 (2011).

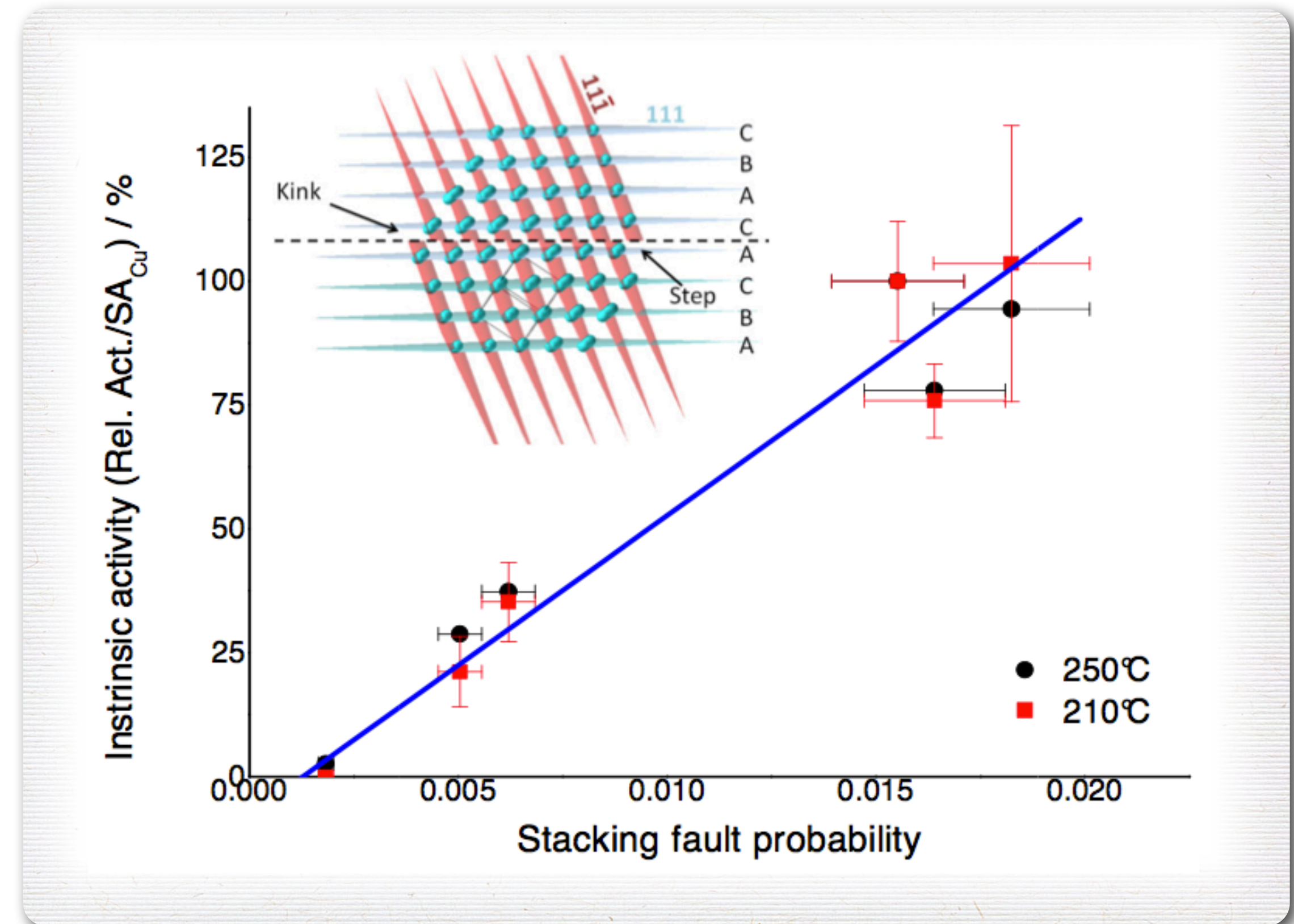
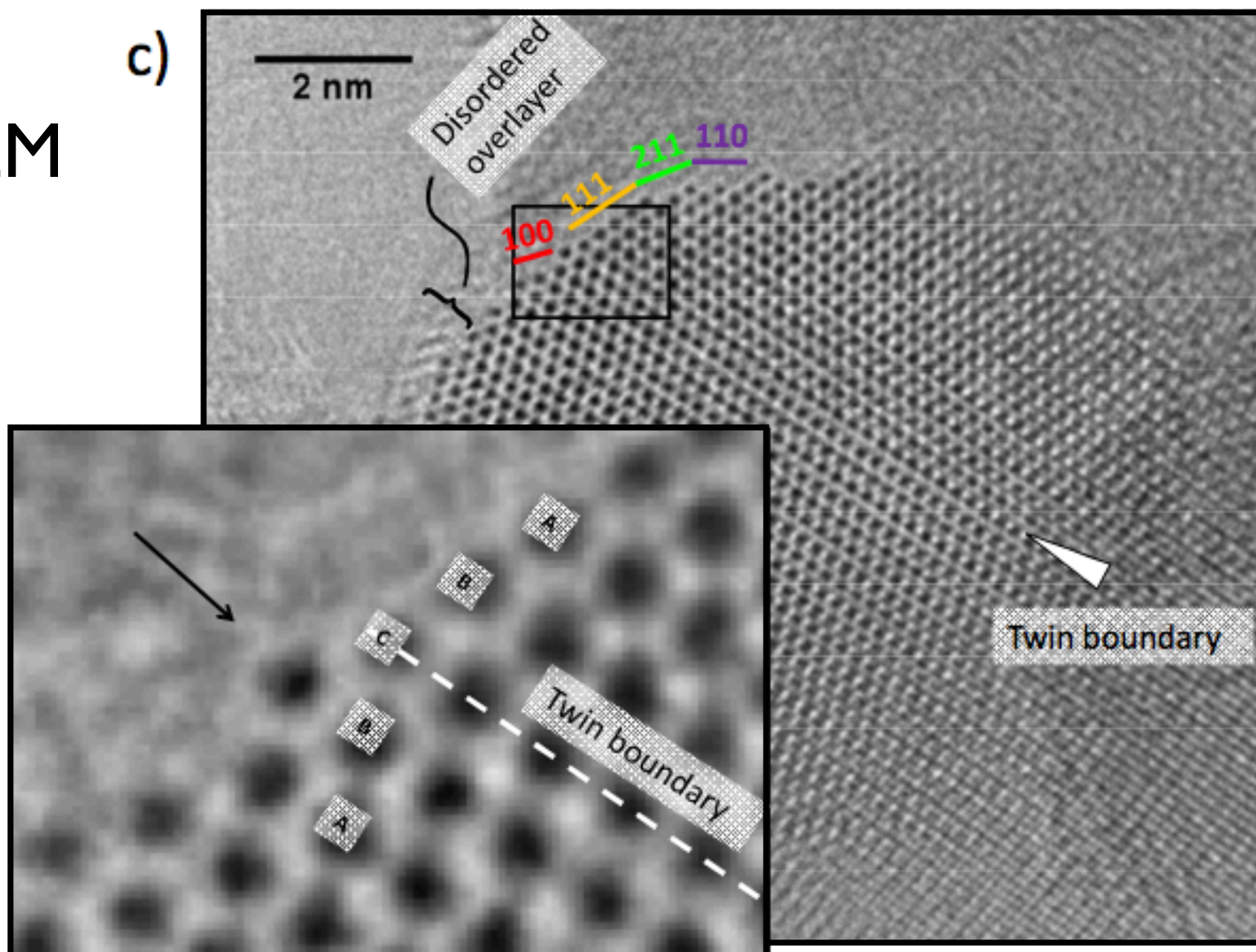
M. Andersson, et. al. J. Catal. **239**, 501 (2006).



# The Industrial Cu/ZnO/Al<sub>2</sub>O<sub>3</sub> Methanol Catalyst



HRTEM

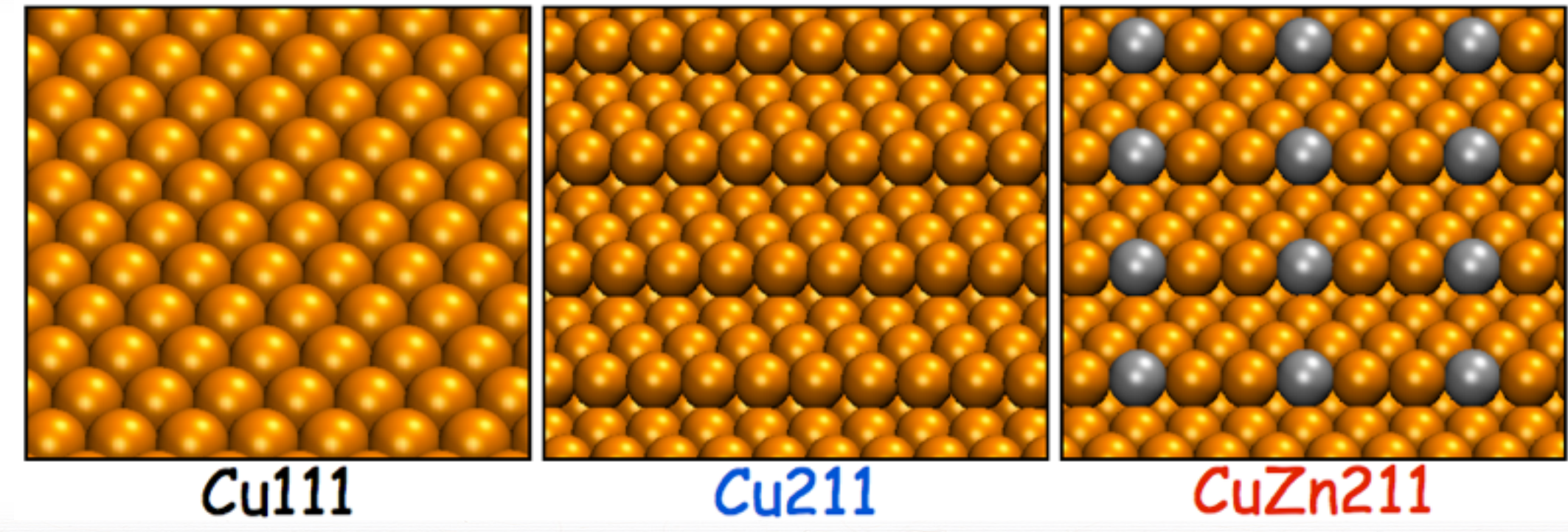
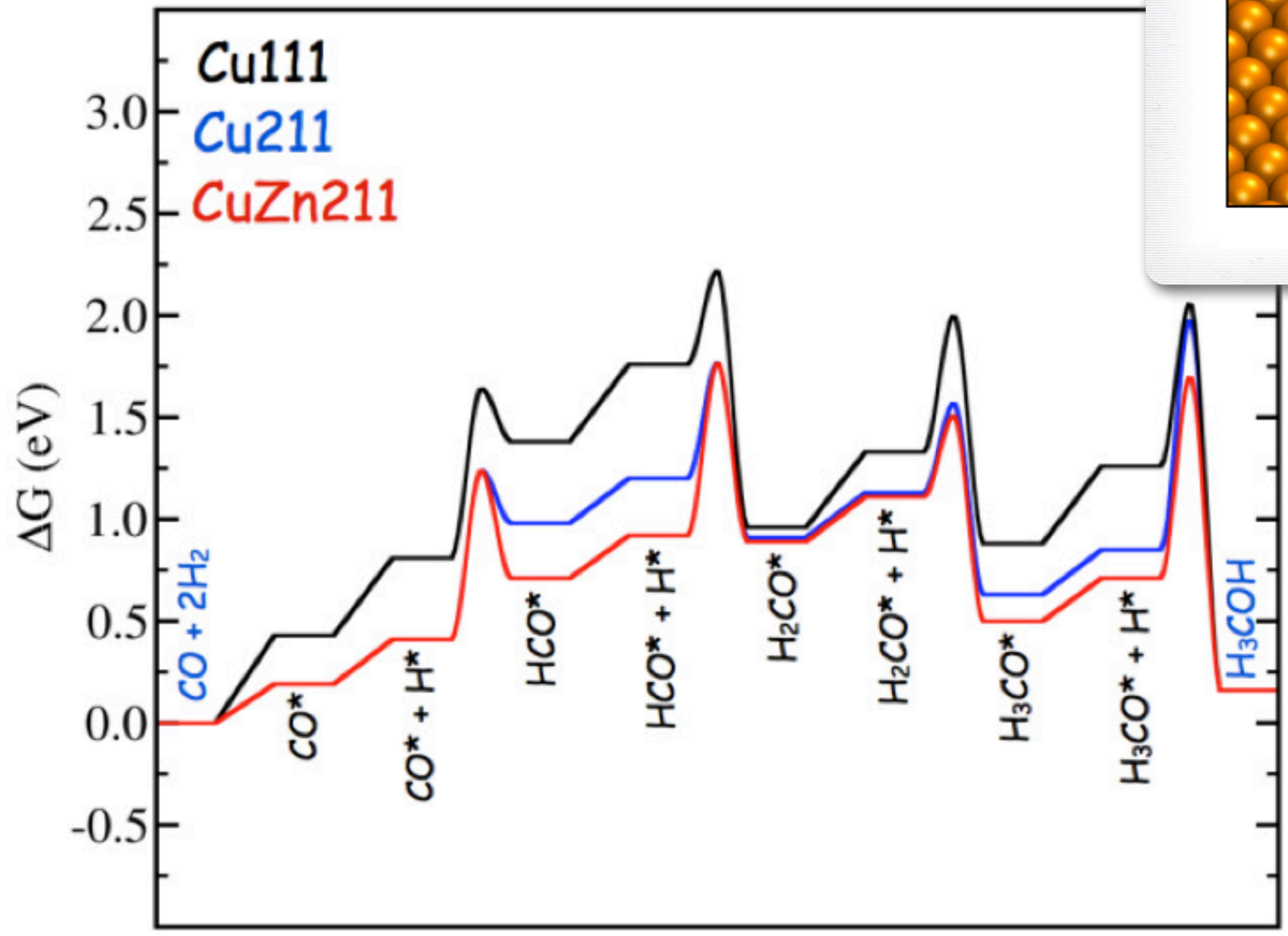


- no correlation btw Cu surface area and activity
- only Cu has low activity => need ZnO
- stacking faults => surface defects (steps/kinks ...)
- correlation btw amount of defects and activity

M. Behrens, *et al.* Science **336**, 893 (2012).



# Reaction Path for $\text{CO} + 2\text{H}_2 \Rightarrow \text{H}_3\text{COH}$

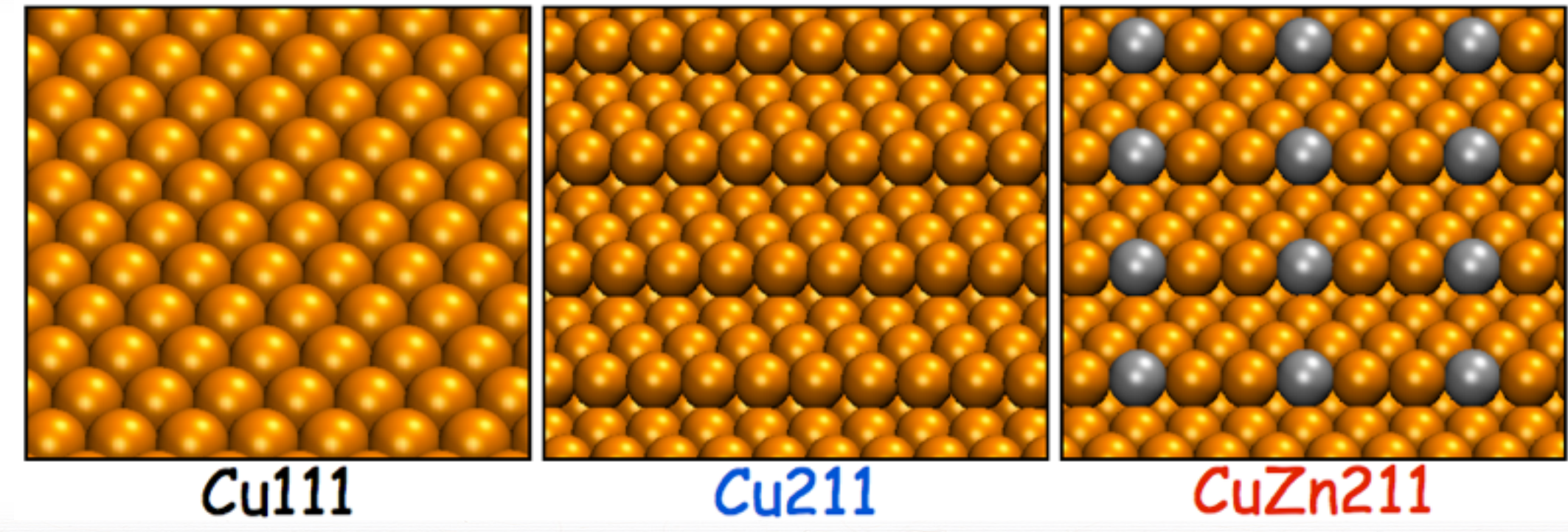
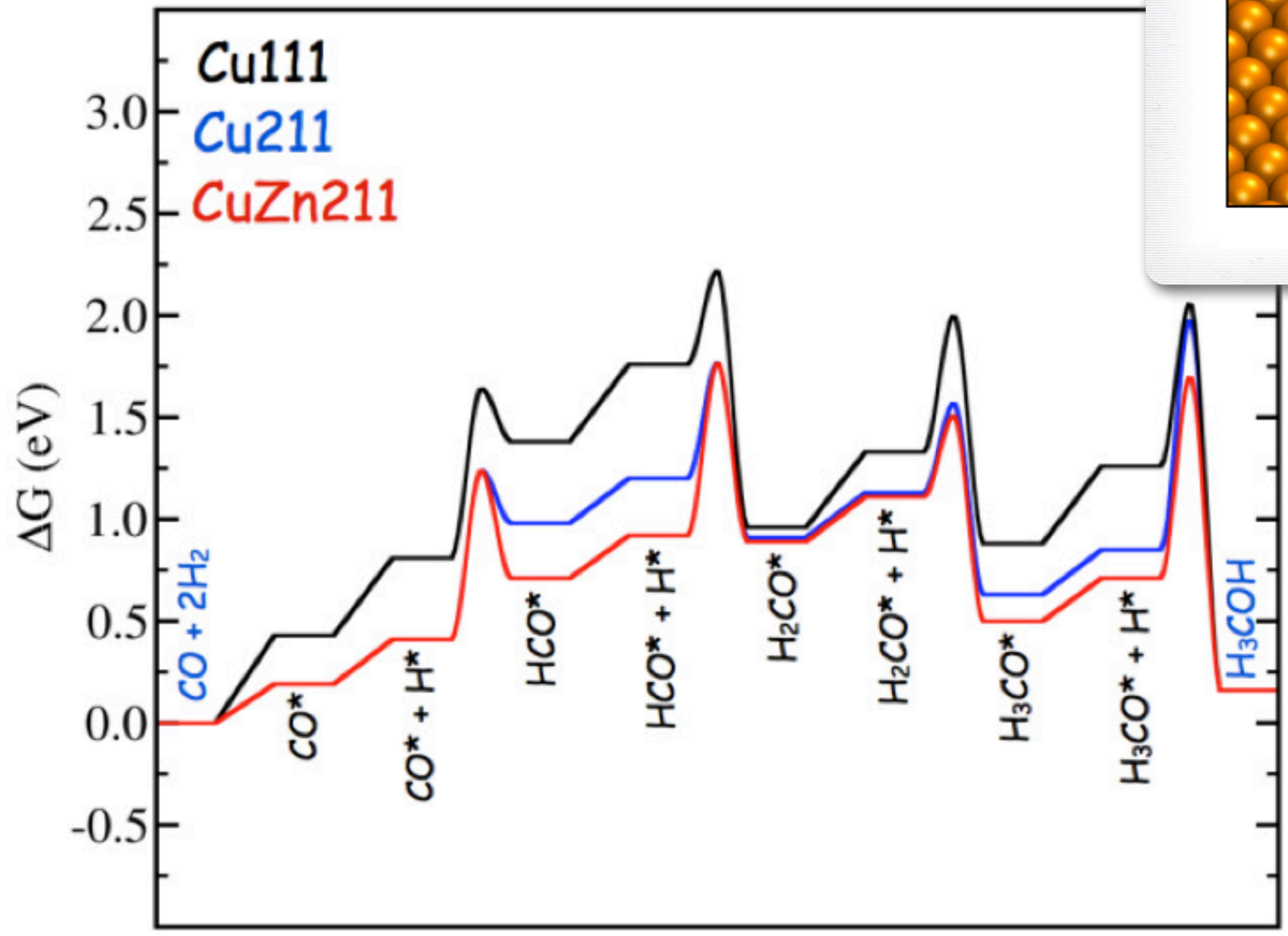


order of activity  
 $\text{CuZn}(211) > \text{Cu}(211) > \text{Cu}(111)$ .

M. Behrens, et. al. Science **336**, 893 (2012).



# Reaction Path for $\text{CO} + 2\text{H}_2 \Rightarrow \text{H}_3\text{COH}$



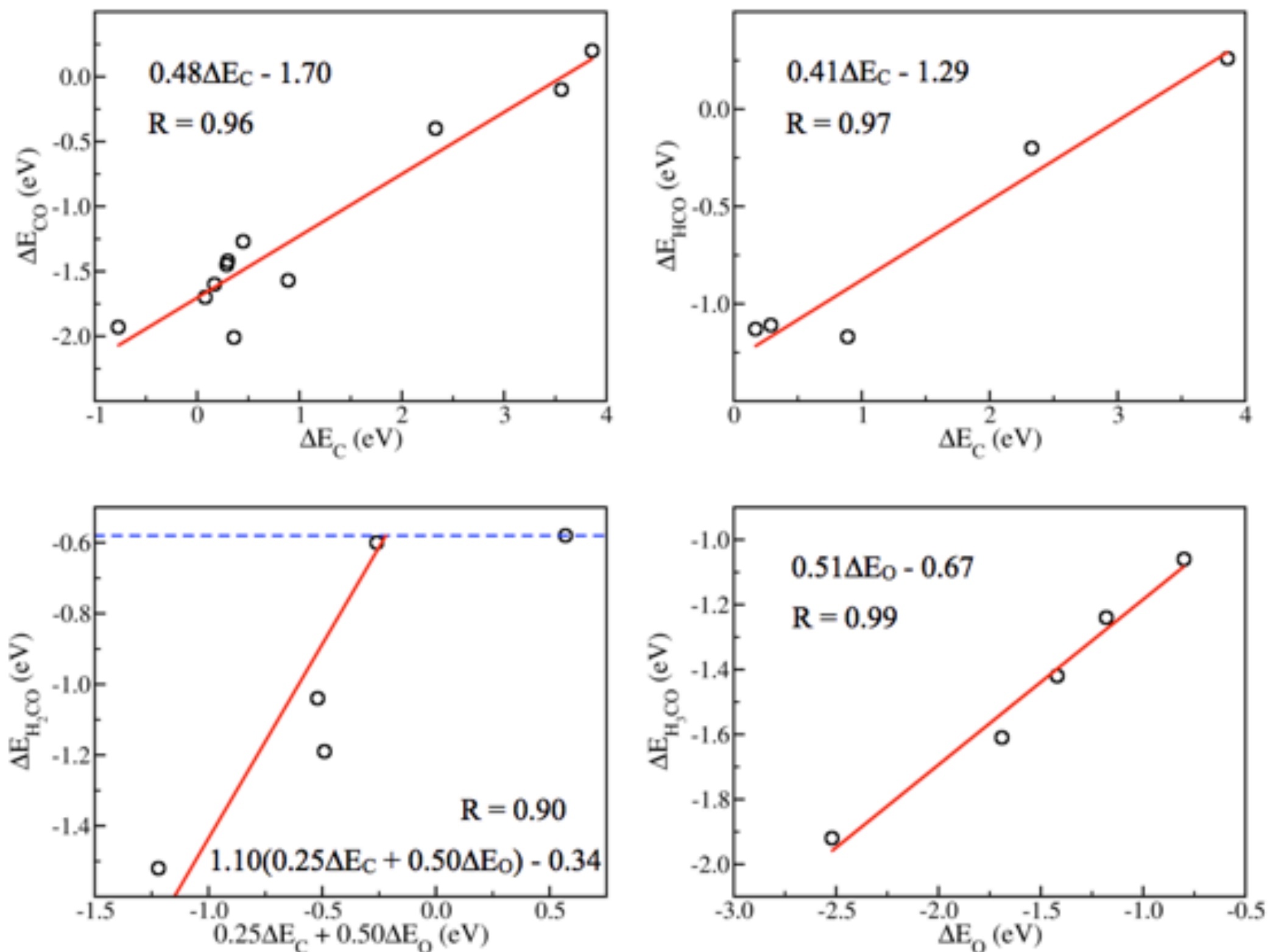
order of activity  
 $\text{CuZn}(211) > \text{Cu}(211) > \text{Cu}(111)$ .

Extend to other metal surfaces => correlations

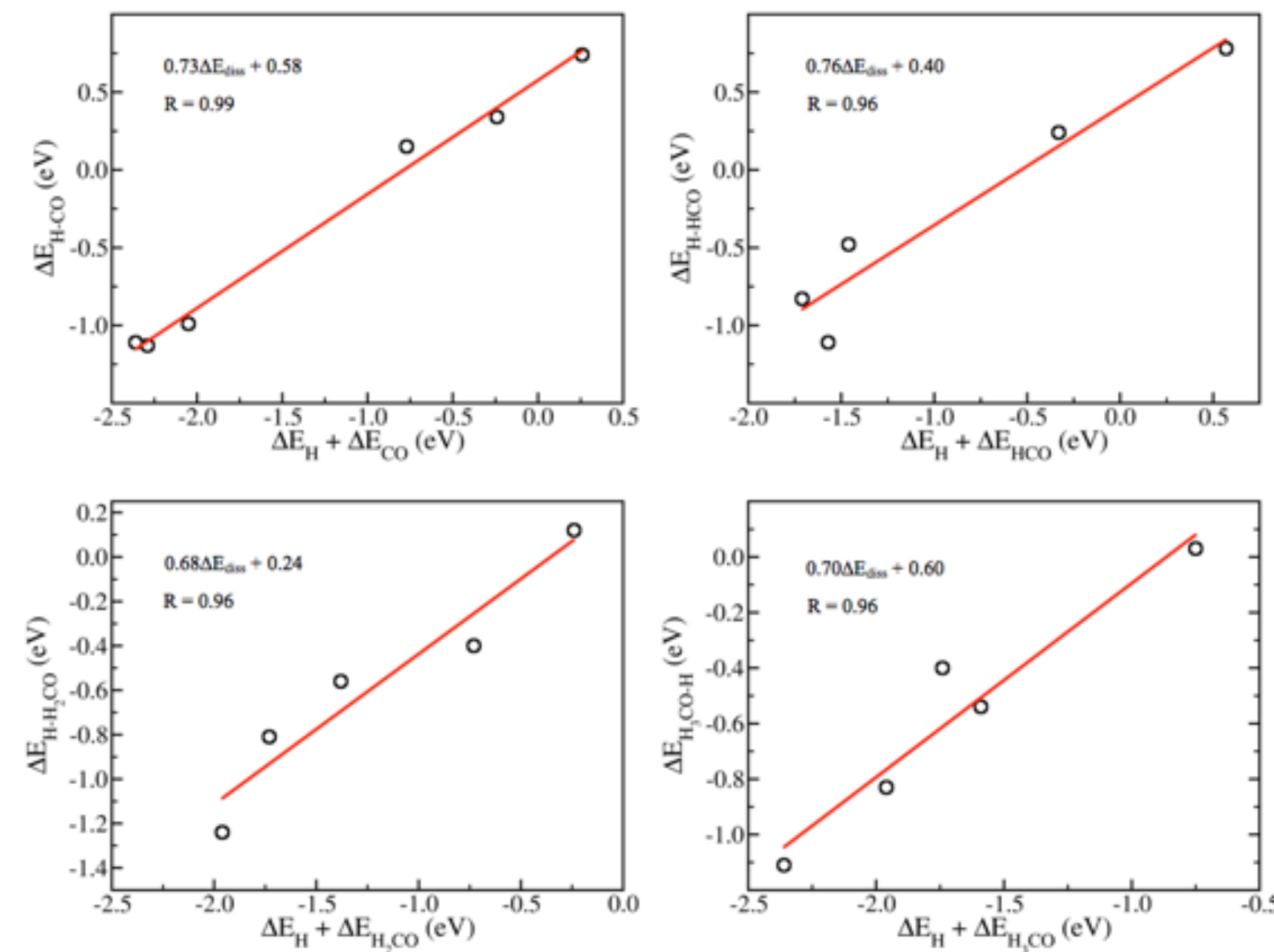
M. Behrens, et. al. Science **336**, 893 (2012).



## adsorption energies



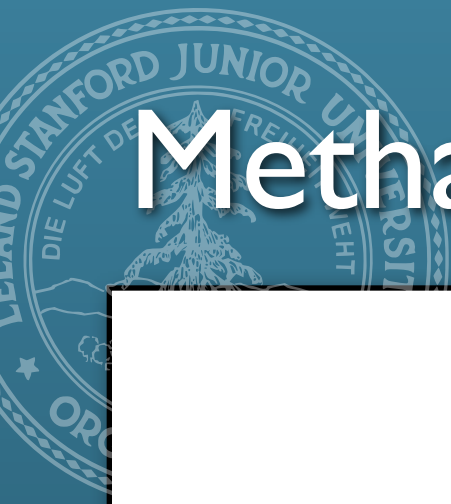
## transition state energies



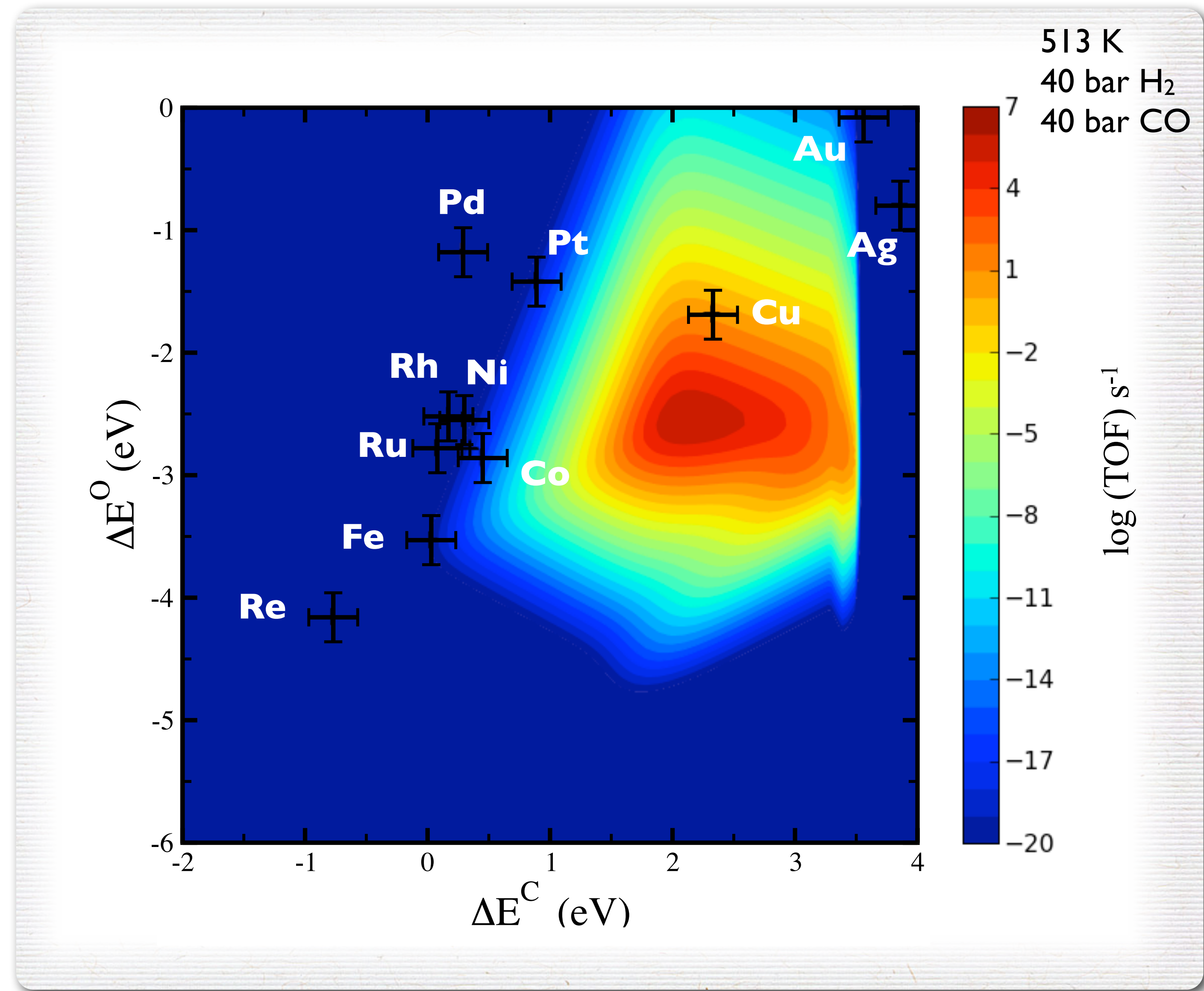
C & O adsorption energies only descriptors

F. Studt, et. al. J. Catal. **293**, 51 (2012).



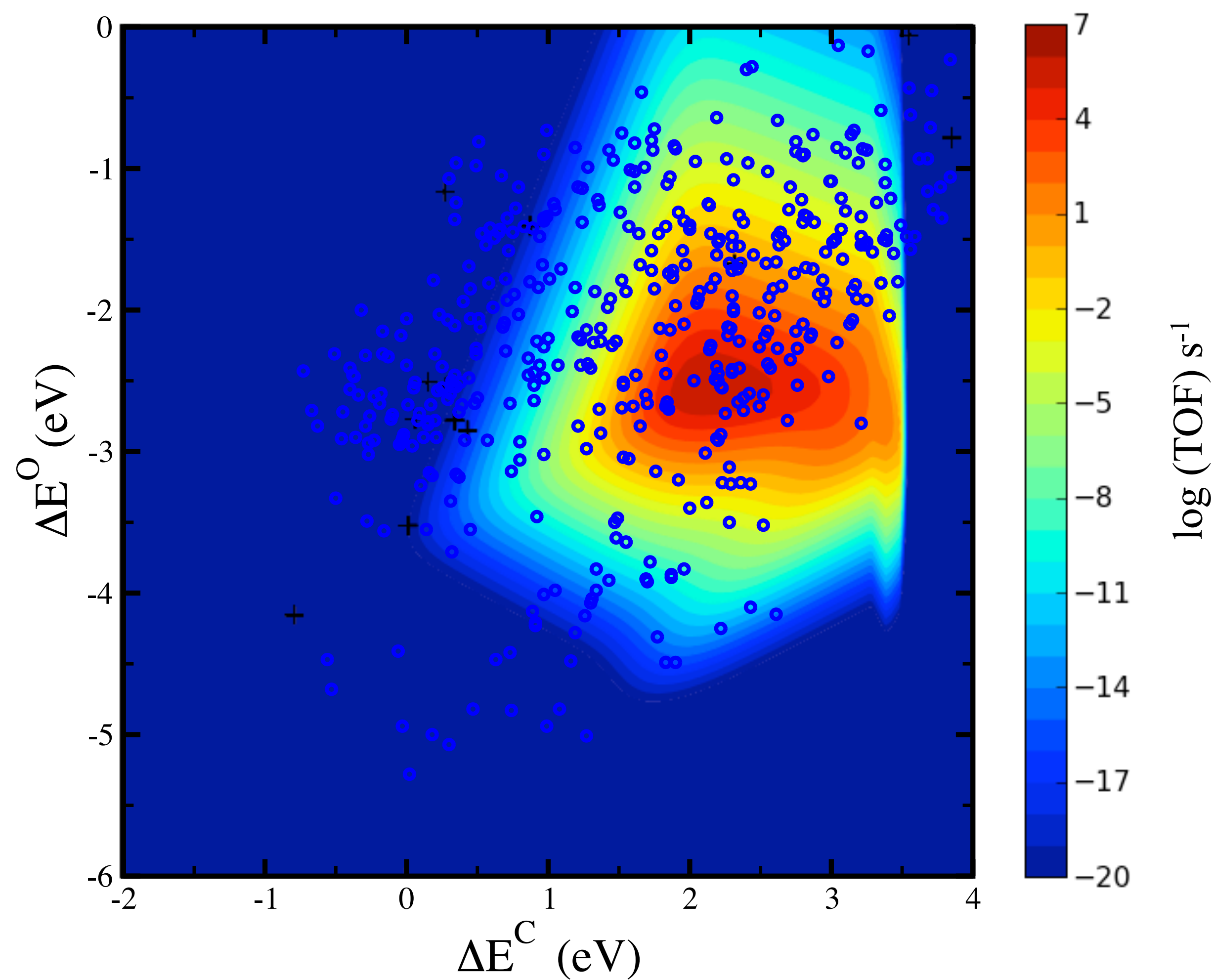


# Methanol Volcano ( $\text{CO} + 2\text{H}_2 \Rightarrow \text{CH}_3\text{OH}$ )



F. Studt, et. al. J. Catal. **293**, 51 (2012).





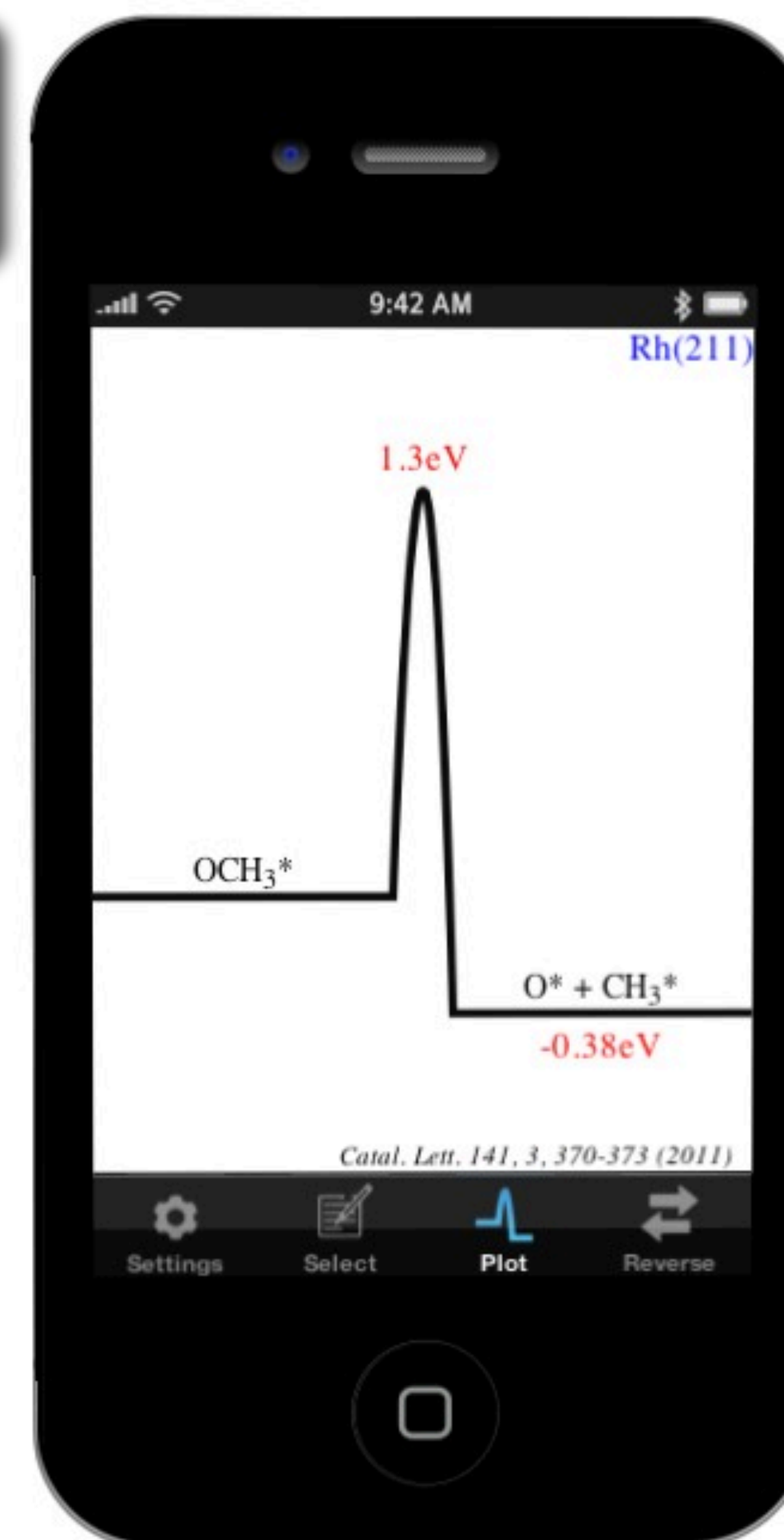
- same data points as for methanation
- screening can be reused for many reactions

Database



<http://suncat.stanford.edu/catapp/>

- ~1500 DFT calculated adsorption and activation energies
- Published data
- Consistent sets of results
- Free web application tool
- Runs in any modern web browser without plug-ins





<http://suncat.stanford.edu/catapp/>

*Angewandte  
And Finally*

*Web Applications*

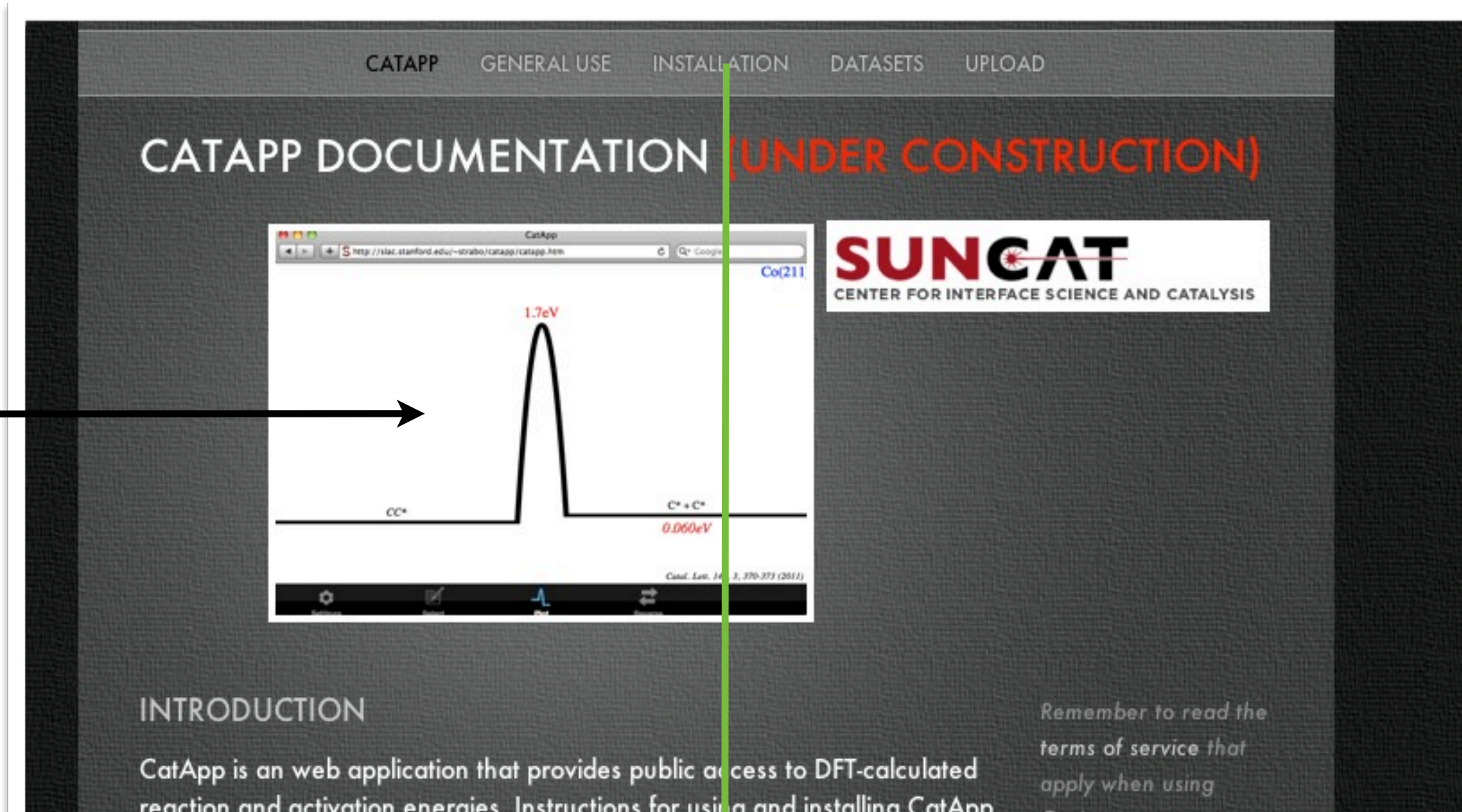
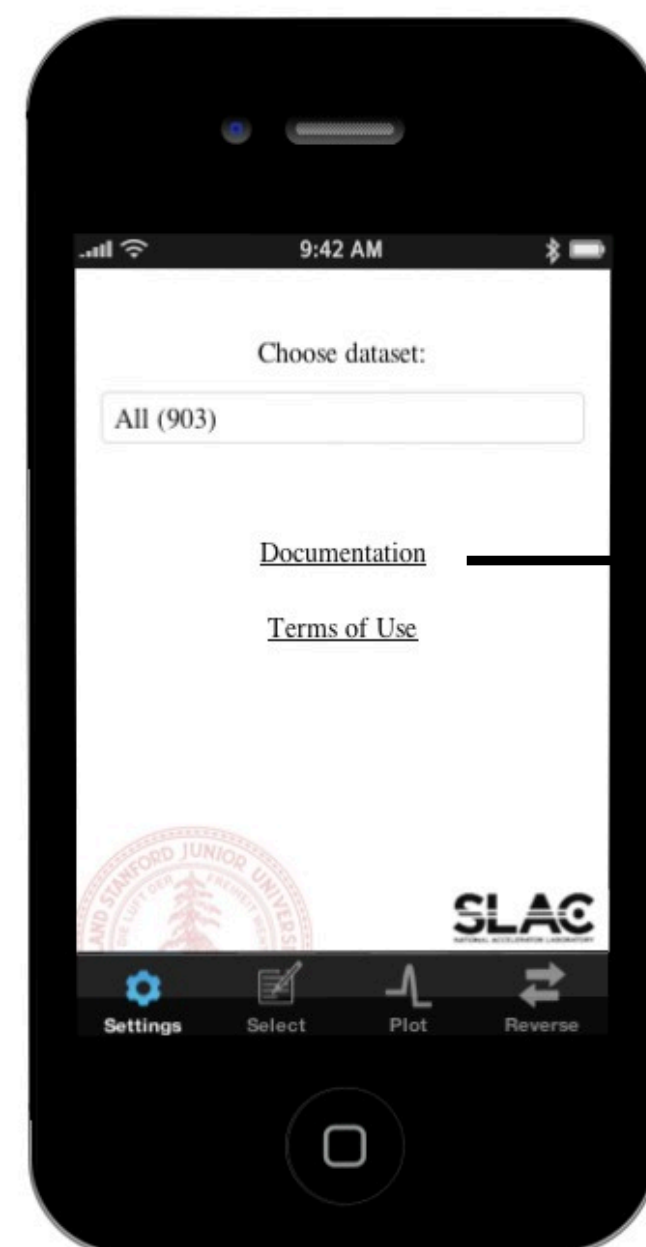
DOI: 10.1002/anie.201107947

## **CatApp: A Web Application for Surface Chemistry and Heterogeneous Catalysis\*\***

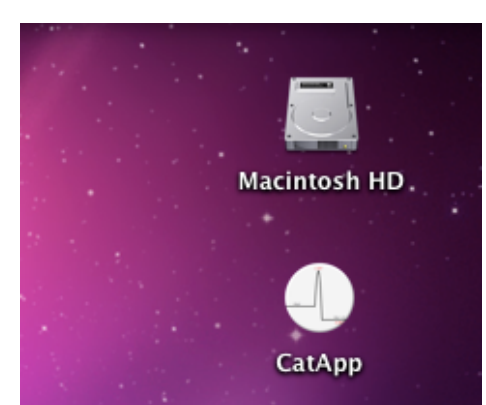
*Jens S. Hummelshøj, Frank Abild-Pedersen, Felix Studt, Thomas Bligaard, and Jens K. Nørskov\**

J. Hummelshøj, Frank Abild-Pedersen, Felix Studt, Thomas Bligaard, and Jens K. Nørskov, *Angew. Chem. Int. Ed.* **51**, 272 (2012).





<http://suncat.stanford.edu/catapp/>





<http://suncat.stanford.edu/catapp/>

SUNCAT

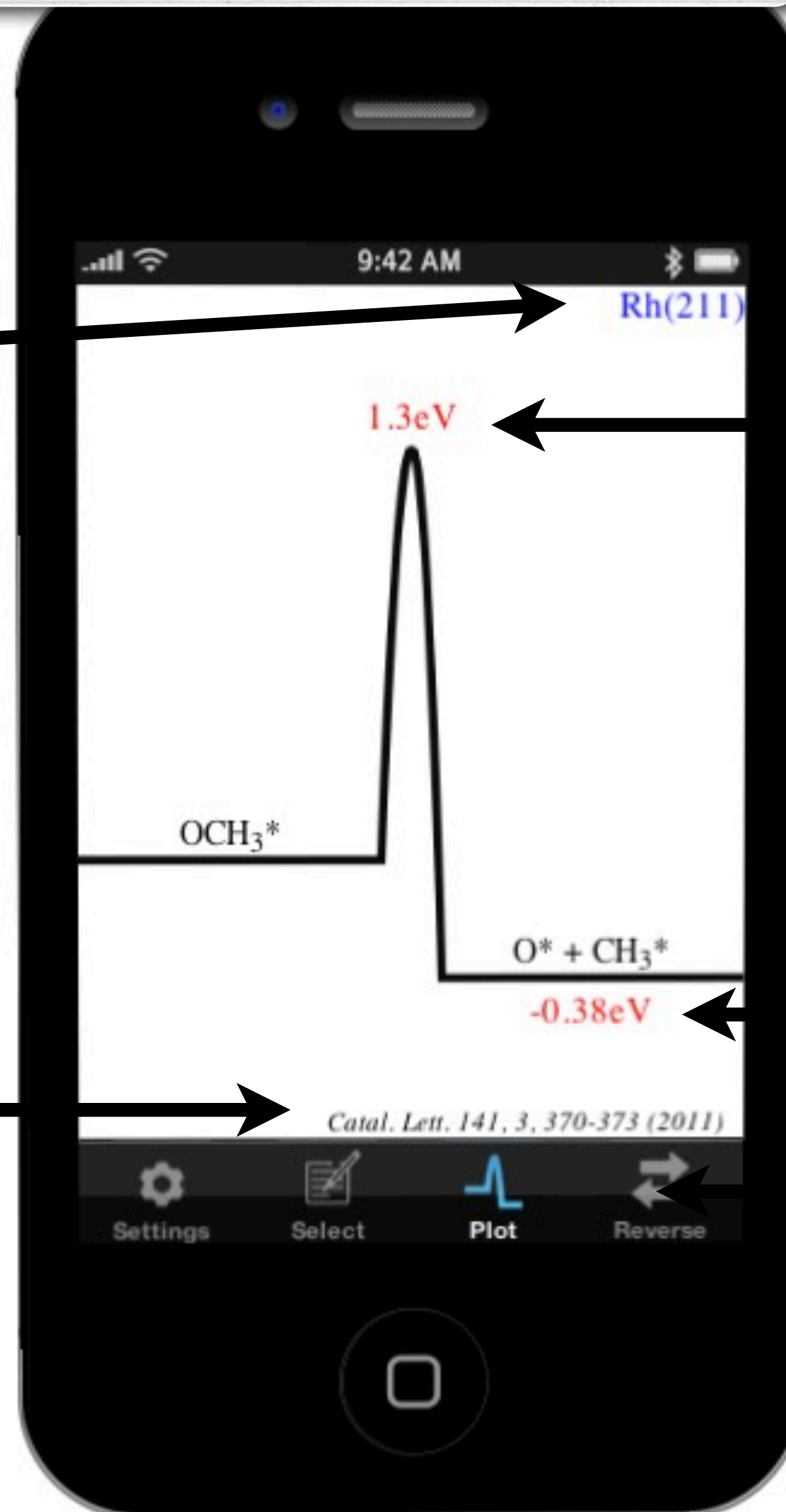
Change surface

Activation energy

Link to publication

Reaction energy

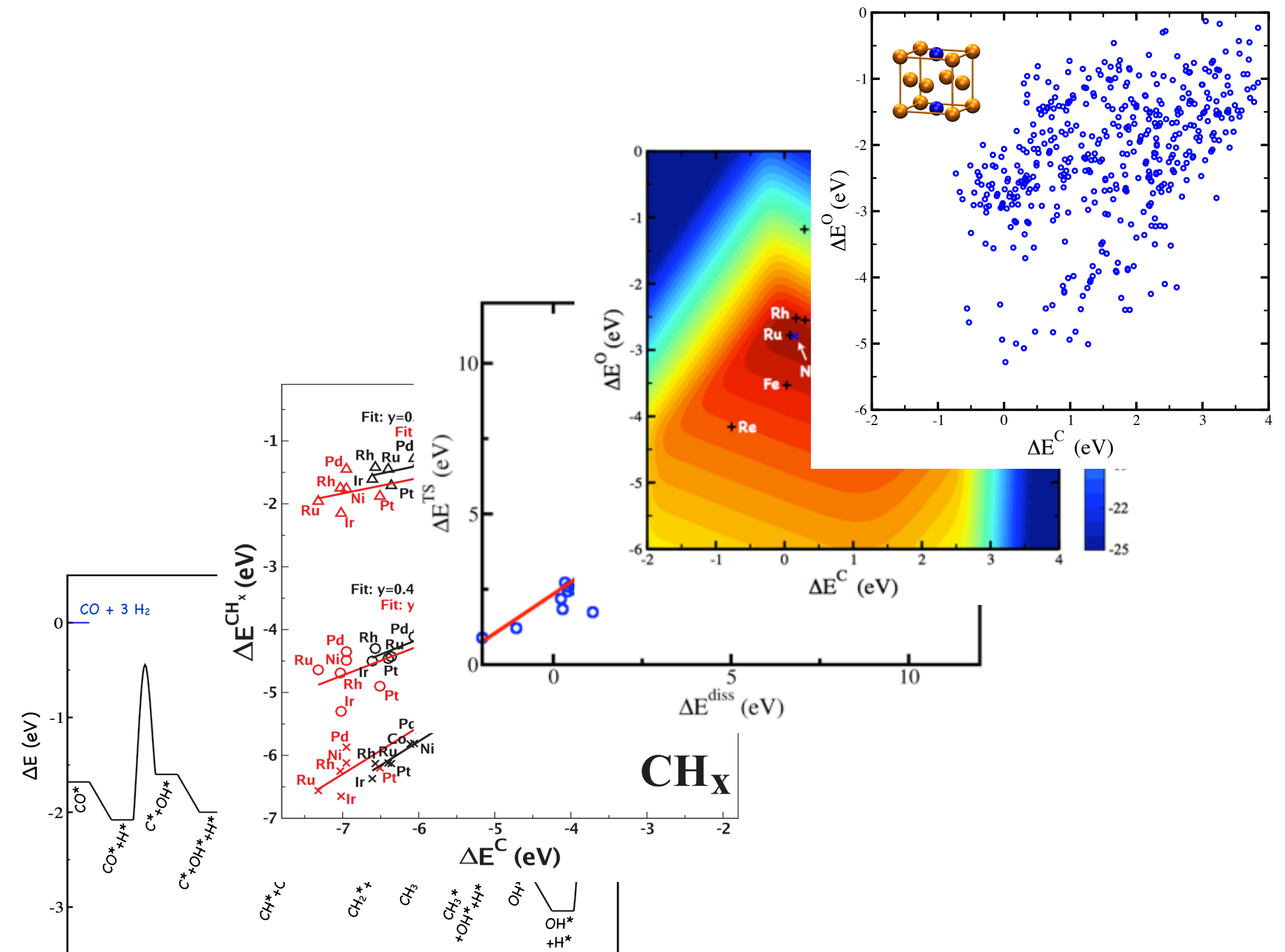
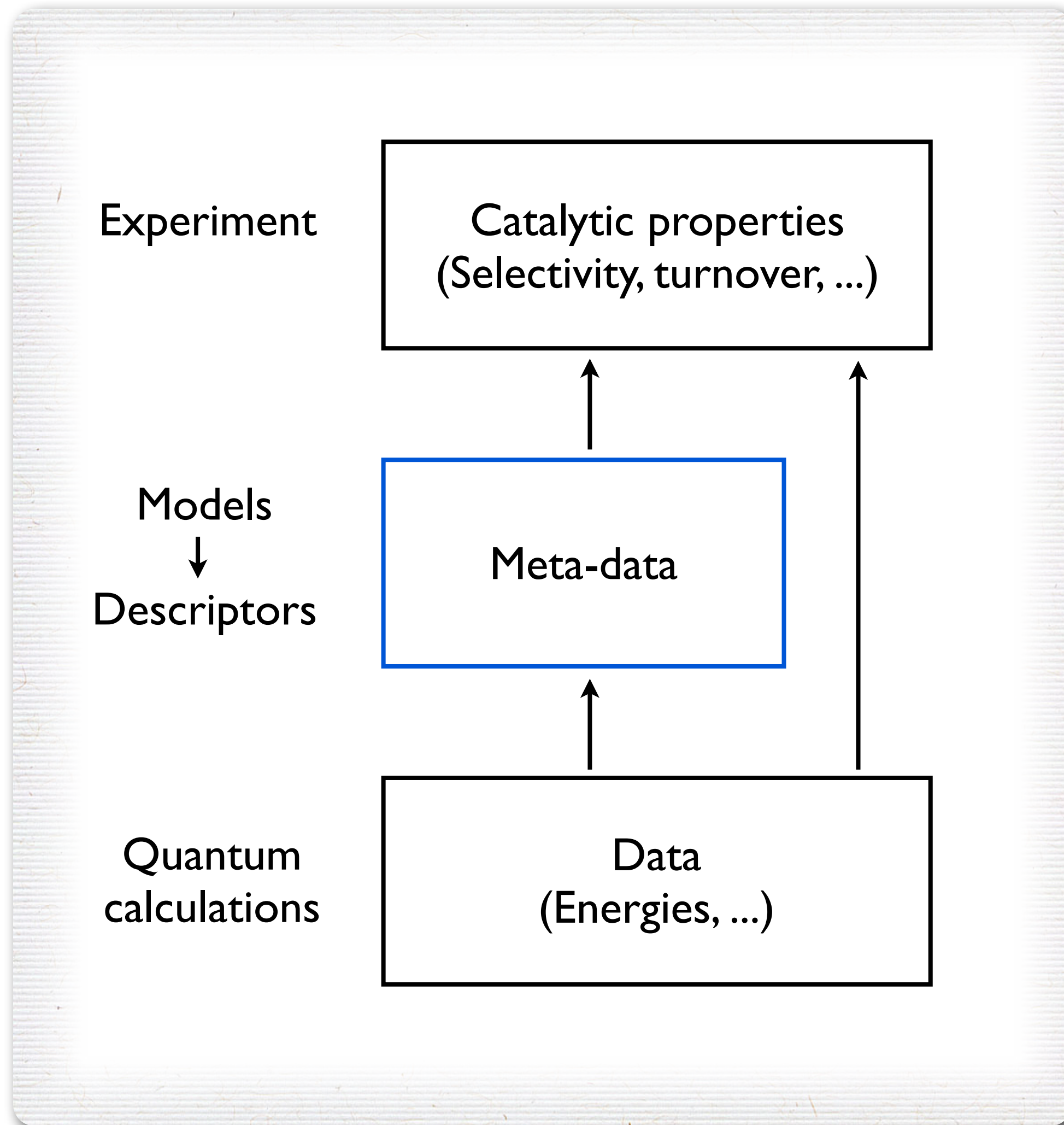
Reverse reaction







# Design Principle





- Concepts of Modern Catalysis and Kinetics, I.Chorkendorff, J.W. Niemantsverdriet
- Chemical Bonding on Surface and Interfaces, Eds A. Nilsson, L. Pettersson, J. K. Nørskov
- Handbook of Surface Science Vol 2, Electronic Structure, Eds, K. Horn, M. Scheffler

A word cloud featuring the phrase "thank you" in numerous languages. The words are arranged in a roughly rectangular shape, with the largest words being "thank you" in English, "Хвалатак" in Macedonian, "matondo" in Zulu, "gracias" in Spanish, "merci" in French, "rahmat" in Indonesian, "tack" in Swedish, and "Vielen Dank" in German. Other visible words include "salamat", "kasih", "terima", "grazie", "dank", "muruu", "hvala", "matando", "gracias", "merci", "tack", "Vielen Dank", "muruu", "hvala", "matando", "gracias", "merci", "tack", "Vielen Dank".