

Analysis and control of the electronic motion with TDDFT

Alberto Castro

Introduction:
Atto-second physics:
electrons in real time

TDDFT for the analysis of ultrafast pump-probe experiments

Optimal Control Theory

Optimal control theory

Control of quantum processes

TDDFT + QOCT

Summary

From the femto- to the atto-second time scale: analysis and control of the electronic motion with time-dependent density-functional theory

Alberto Castro

ARAID Foundation and Institute for Biocomputation and Physics of Complex Systems (BIFI), Zaragoza (Spain)

Ψ_k -CECAM Research Conference on Multi-Scale Modeling from First Principles, September 9th-13th, 2013

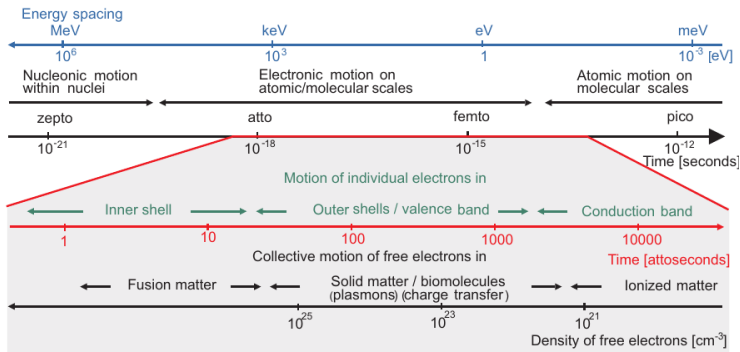


Instituto Universitario de Investigación
**Biocomputación y Física
de Sistemas Complejos**
Universidad Zaragoza



Electronic time-scale

The atto-second is the natural time scale of electrons in atoms and molecules. Atto-second physics allows to observe, manipulate, and control electrons in real time.



From Krausz & Ivanov, Rev. Mod. Phys. **81**, 169 (2009)

Collaborators

- ▶ Jorge Bugadosky
- ▶ Hardy Gross
- ▶ Jan Werschnik
- ▶ Kevin Krieger
- ▶ Esa Räsänen
- ▶ Angel Rubio
- ▶ Umberto de Giovanninni
- ▶ Jessica Walkenhorst
- ▶ Ilya Tokatly

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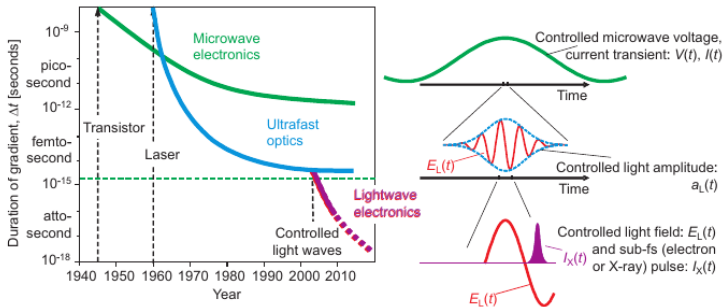
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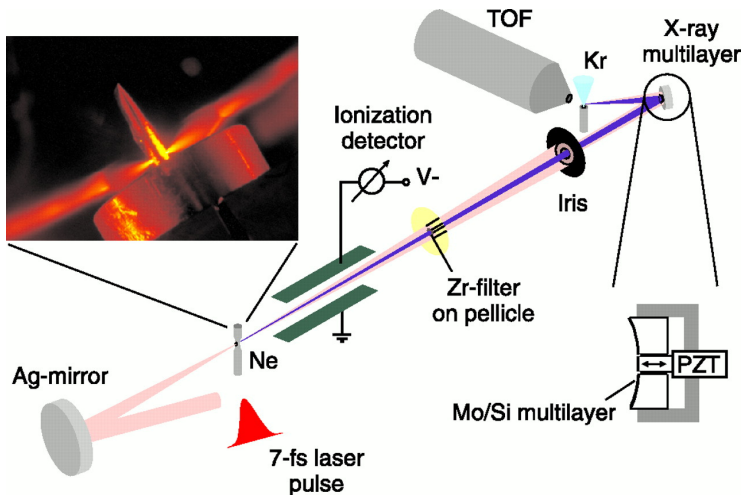
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The road to atto-second physics



From Krausz & Ivanov, Rev. Mod. Phys. **81**, 169 (2009)

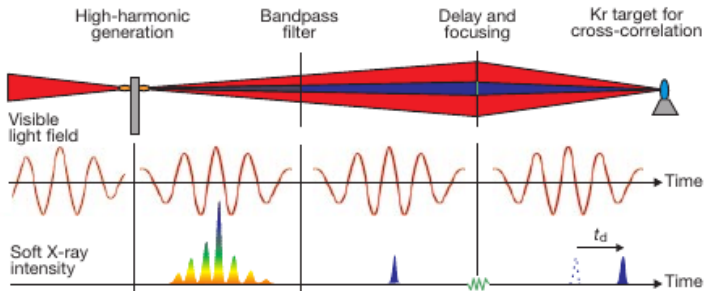
Some initial break-throughs



“X-ray pulses approaching the attosecond frontier”, Drescher *et al.*
Science **291**, 1923 (2001).

First generation of isolated “almost” atto-second pulses.

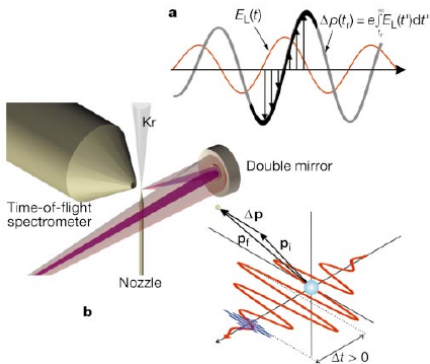
Some initial break-throughs



“Attosecond metrology”, Hentschel *et al*, *Nature* **214**, 509 (2001).

Trace of electronic dynamics with at time resolution of 150 as.

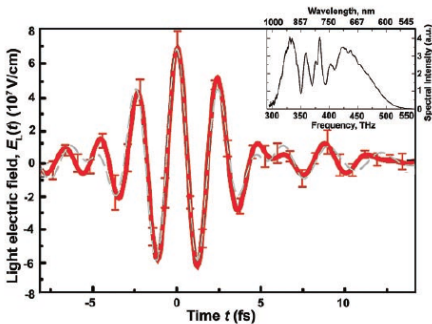
Some initial break-throughs



“Time resolved atomic inner-shell spectroscopy”, Drescher *et al*, *Nature* **419**, 803 (2002).

Measurement of the relaxation dynamics time-constants of core excited atoms with atto-second resolution by means of a pump-probe scheme based on a 7 fs 750 nm pulse and a sub-femtosecond soft X-ray pulse.

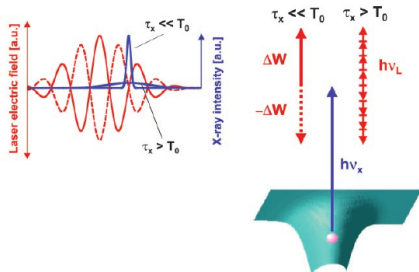
Some initial break-throughs



“Direct measurement of light waves”, Goulielmakis *et al*, *Science* **305**, 1267 (2004).

Full characterization of a visible femto-second pulse by making use of 250 as. electron bursts.

Some initial break-throughs



“Steering atto-second electron wave packets with light”, Kienberger *et al*, *Science* **297**, 1144 (2004).

Control over the velocity of photo-electrons generated by sub-femtosecond XUV pulses, by making use of a synchronized longer femtosecond pulse.

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Pump-probe spectroscopy on the atto-second time-scale with TDDFT

Pump-probe spectroscopy with weak probes can be considered as a generalized out-of equilibrium response theory:

$$\hat{H}(t) = \mathcal{H} + \varepsilon(t)\hat{V} + f(t)\hat{V}$$

$$i\frac{d}{dt}\hat{\rho}(t) = [\hat{H}(t), \hat{\rho}(t)]$$

$$A(t) = \text{Tr}\hat{A}\hat{\rho}(t)$$

$$\delta A(t) = \int_0^T d\tau f(\tau)\chi_{\hat{A},\hat{V}}[\varepsilon](t, \tau)$$

$$\chi_{\hat{A},\hat{V}}(t, \tau) = -i\theta(t - \tau)\text{Tr}\{\hat{\rho}(t_0) [\hat{A}_H[\varepsilon](t), \hat{V}_H[\varepsilon](\tau)]\}$$

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Pump-probe spectroscopy on the atto-second time-scale with TDDFT

- ▶ TDDFT:

$$n(\vec{r}, t) \Leftrightarrow v(\vec{r}, t) \rightarrow \Psi(t)$$

[Runge and Gross, Phys. Rev. Lett. **52**, 997 (1984)]

- ▶ Kohn-Sham substitution: we work with a proxy system of non-interacting electrons whose density is by construction identical to that of the real system:

$$i \frac{\partial \varphi_i}{\partial t}(\vec{r}, t) = \left[-\frac{1}{2} \nabla^2 \varphi_i(\vec{r}, t) + v_{\text{Hartree}}[n](\vec{r}, t) + v_{\text{xc}}[n](\vec{r}, t) + v_{\text{ext}}(\vec{r}, t) \right] \varphi_i(\vec{r}, t),$$
$$n(\vec{r}, t) = \sum_{i=1}^N 2 |\varphi_i(\vec{r}, t)|^2.$$

Observables

- ▶ Dipole moment (for, e.g., absorption spectroscopy):

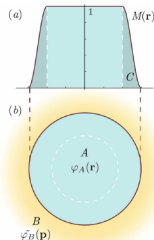
$$\langle \Psi(t) | \hat{D} | \Psi(t) \rangle = \int d^3r n(\vec{r}, t) \vec{r}$$

- ▶ High-harmonic generation

$$H(\omega) = \left| \int_0^T dt \frac{d^2}{dt^2} \langle \hat{D} \rangle(t) e^{-i\omega t} \right|^2$$

- ▶ Photo-electron spectroscopy

$$P(\vec{p}) \approx \lim_{t \rightarrow \infty} \sum_{occ} |\tilde{\varphi}_i^B(\vec{r}, t)|^2$$



Attosecond Transient Absorption Spectroscopy

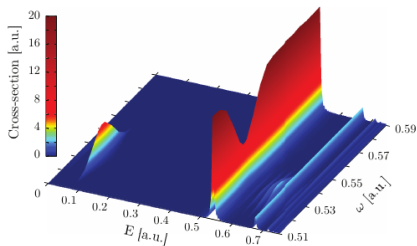


FIG. 2. Out of equilibrium absorption spectrum as function of the pump laser frequency for one-dimensional Helium. The system is driven out of equilibrium by 45 cycle \sin^2 envelope laser pulses of intensity $I = 5.26 \times 10^{11}$ W/cm², at different carrier frequencies and then probed right after. Maximal response is observed for frequencies close to the first optical transition $\omega = 0.533$ a.u..

"Simulating Pump-Probe Photoelectron and Absorption Spectroscopy on the Attosecond Timescale with Time-Dependent Density Functional Theory",

U. de Giovannini, G. Brunetto, AC, J. Walkenhorst, and A. Rubio ChemPhysChem 14, 1363

(2013)

Attosecond Time Resolved Photo-electron Spectroscopy

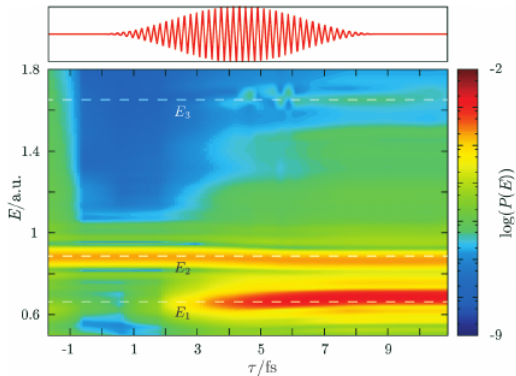


Figure 7. Helium transient photoelectron spectrum on a logarithmic scale. The pump laser (upper panel) is the same as that given in Figure 5 and the probe is a 40 cycle trapezoidal laser pulse with an 8 cycle ramp, $\omega_p = 1.8$ a.u., $I = 5.4 \times 10^9$ W cm⁻² aligned with the pump pulse.

U. de Giovannini, G. Brunetto, AC, J. Walkenhorst, and A. Rubio ChemPhysChem 14, 1363

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Optimal control theory

Typical formulation of a (general) optimal control problem:

- ▶ Dynamical system:

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t), t) \\ x(0) &= x_0\end{aligned}$$

Typically, $u = u(t)$. But it can be a set of parameters whatsoever.

- ▶ Minimize the cost functional:

$$F[x, u] = F^{\text{terminal}}[x(T), u] + \int_0^T dt L(x(t), u(t))$$

- ▶ Since $u \rightarrow x[u]$, it amounts to minimizing

$$G[u] = F[x[u], u]$$

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Essential theoretical results

- ▶ Pontryagin's minimum principle (1956)
[V.G. Boltyanskii, R.V. Gamkrelidze, and L.S. Pontryagin, "Towards a theory of optimal processes", (Russian), Reports Acad. Sci. USSR **110**, 1 (1956)]
It provides a *necessary* condition for the minimum – in practice, typically, an expression for $\nabla G[u]$ so that the equation $\nabla G[u] = 0$ can be posed.
- ▶ Hamilton-Jacobi-Bellman equation (1954)
(Theory of "dynamic programming", Richard Bellman)
[R.E Bellman, "Dynamic Programming and a new formalism in the calculus of variations" Proc. Nat. Acad. Sci. **40**, 231 (1954)]

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Essential theoretical results

- ▶ Simpler approaches: *direct* or *gradient-less* algorithms. They only require a means to compute $G[u]$ (i.e. a method to propagate the dynamical equation and compute the resulting cost or target functional).
 - ▶ The most fashionable, the families of *evolutionary* or *genetic* algorithms.
 - ▶ Our choices:
 - ▶ The *simplex* algorithm [J.A. Nelder and R. Mead, *Computer Journal* **7**, 308 (1965)], and
 - ▶ the NEWUOA algorithm [M. J. D. Powell, *IMA J. Numer. Anal.* **28**, 649 (2008)].

Pontryagin's minimum principle

If we define the “Hamiltonian”

$$H(\lambda(t), x(t), u(t), t) = \lambda^\dagger(t) f(x(t), u(t), t) + L(x(t), u(t))$$

where λ is the “costate”, an object of the same kind of x , the following holds:

1. The optimal control u^0 , trajectory x^0 and costate λ^0 minimize H at all times:

$$H(\lambda^0(t), x^0(t), u^0(t), t) \leq H(\lambda(t), x(t), u(t), t)$$

2. The costate verifies the following equation of motion:

$$\dot{\lambda}^{0\dagger}(t) = \lambda^{0\dagger}(t) \frac{\delta f}{\delta x}(x^0(t), u^0(t)) + \frac{\delta L}{\delta x}(x^0(t), u^0(t))$$

$$\lambda^{0\dagger}(T) = \frac{\delta}{\delta x} F^{\text{terminal}}[x^0(T), u^0(T)]$$

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Application to Hamiltonian systems

$$\begin{aligned}\dot{q}_i(t) &= \frac{\partial H}{\partial p_i}(q(t), p(t), u, t), \\ \dot{p}_i(t) &= -\frac{\partial H}{\partial q_i}(q(t), p(t), u, t).\end{aligned}$$

In condensed vector notation:

$$\begin{aligned}\dot{\mathbf{q}} &= \frac{\partial H}{\partial \mathbf{p}}, \\ \dot{\mathbf{p}} &= -\frac{\partial H}{\partial \mathbf{q}}.\end{aligned}$$

In this case, x is the vector $[\mathbf{q}, \mathbf{p}]^\dagger$.

The cost (or target) functional can be, for example, any expression in the form:

$$F[\mathbf{q}, \mathbf{p}, u] = J_1[\mathbf{q}(T), \mathbf{p}(T)] + J_2[u].$$

Application to Hamiltonian systems

$$\begin{aligned} \frac{\partial G}{\partial u_m} &= \frac{\partial J_2}{\partial u_m} \\ &\quad - \int_0^T dt \tilde{\mathbf{p}}(t) \cdot \frac{\partial}{\partial u_m} \frac{\partial H}{\partial \mathbf{p}}(\mathbf{q}(t), \mathbf{p}(t), u, t) \\ &\quad + \int_0^T dt \tilde{\mathbf{q}}(t) \cdot \frac{\partial}{\partial u_m} \frac{\partial H}{\partial \mathbf{q}}(\mathbf{q}(t), \mathbf{p}(t), u, t) \end{aligned}$$

The “costate” $[\tilde{\mathbf{q}}, \tilde{\mathbf{p}}]^\dagger$ is itself a Hamiltonian system, determined by the quadratic Hamiltonian:

$$\begin{aligned} \tilde{H}(\tilde{q}, \tilde{p}, q, p, u, t) &= \\ &\frac{1}{2} \tilde{\mathbf{q}}^t \mathbf{H}^{qq}(q, p, u, t) \tilde{\mathbf{q}} + \frac{1}{2} \tilde{\mathbf{q}}^t \mathbf{H}^{qp}(q, p, u, t) \tilde{\mathbf{p}} + \\ &+ \frac{1}{2} \tilde{\mathbf{q}}^t \mathbf{H}^{pq}(q, p, u, t) \tilde{\mathbf{p}} + \frac{1}{2} \tilde{\mathbf{q}}^t \mathbf{H}^{pp}(q, p, u, t) \tilde{\mathbf{p}}. \end{aligned}$$

Application to Hamiltonian systems

The matrices \mathbf{H}^{xy} are defined as:

$$H_{ij}^{qq} = \frac{\partial^2 H}{\partial q_i \partial q_j}(q, p, u, t)$$

$$H_{ij}^{qp} = \frac{\partial^2 H}{\partial q_i \partial p_j}(q, p, u, t)$$

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Quantum optimal control theory

$$\hat{H} = \hat{H}[u_1, \dots, u_M; t]$$

$$i \frac{d}{dt} |\Psi(t)\rangle = \hat{H}[u; t] |\Psi(t)\rangle$$
$$|\Psi(t_0)\rangle = |\Psi_0\rangle$$

$$\Psi(t_0) \longrightarrow \Psi[u](t) \longrightarrow \Psi[u](T)$$

Maximize a quantity

$$F = F[\Psi[u](t)],$$

that depends on the system evolution, or final state, or both.

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Is there anything new about QOCT?

Quantum mechanics as a particular case of Hamiltonian systems:

$$|\Psi\rangle = \sum_k c_k |\Psi_k\rangle$$

$$c_k = q_k + ip_k \rightarrow (q_k, p_k) \in \mathbb{R}^{2n}$$

$$\frac{d}{dt} |\Psi(t)\rangle = \hat{H}(t) |\Psi(t)\rangle \quad \equiv \quad \begin{aligned} \dot{\mathbf{q}} &= \frac{\partial H}{\partial \mathbf{p}}, \\ \dot{\mathbf{p}} &= -\frac{\partial H}{\partial \mathbf{q}}. \end{aligned}$$

where

$$H(\mathbf{q}, \mathbf{p}, t) := \langle \Psi(\mathbf{q}, \mathbf{p}) | \hat{H}(t) | \Psi(\mathbf{q}, \mathbf{p}) \rangle$$

is a quadratic expression in \mathbf{q}, \mathbf{p} , i.e. a “generalized harmonic oscillator system”.

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QOCT + TDDFT

PRL **109**, 153603 (2012)

PHYSICAL REVIEW LETTERS

week ending
12 OCTOBER 2012

Controlling the Dynamics of Many-Electron Systems from First Principles: A Combination of Optimal Control and Time-Dependent Density-Functional Theory

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Also in:

AC and E. K. U. Gross, “Quantum Optimal Control”, in “Fundamentals of Time-Dependent Density Functional Theory”, edited by M.A.L. Marques, N. Maitra, F. Nogueira, E.K.U Gross. and Angel Rubio (Springer, Berlin, 2012), pages 265-276.

QOCT + TDDFT

- ▶ We have a system of N electrons, driven by an external potential $v_{\text{ext}}(\vec{r}, t, \mathbf{u})$.
- ▶ The time-dependent density is therefore determined by \mathbf{u} :

$$\mathbf{u} \longrightarrow n[\mathbf{u}](\vec{r}, t) = \langle \Psi[\mathbf{u}](t) | \hat{n}(\vec{r}) | \Psi[\mathbf{u}](t) \rangle$$

- ▶ The objective is to maximize some function G of the *control parameters* \mathbf{u} , defined in terms of a functional of the density:

$$G[\mathbf{u}] = \tilde{F}[n[\mathbf{u}], \mathbf{u}].$$

- ▶ Since the definition is given in terms of the density, everything can be reformulated for the Kohn-Sham system, and the optimization will be equivalent. Since we use the Kohn-Sham substitution, we may use the Kohn-Sham orbitals instead:

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QOCT + TDDFT

- ▶ We have a system of N electrons, driven by an external potential $v_{\text{ext}}(\vec{r}, t, \mathbf{u})$.
- ▶ The time-dependent density is therefore determined by \mathbf{u} :

$$\mathbf{u} \longrightarrow n[\mathbf{u}](\vec{r}, t) = \langle \Psi[\mathbf{u}](t) | \hat{n}(\vec{r}) | \Psi[\mathbf{u}](t) \rangle$$

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QOCT + TDDFT

Optimal control theory equations for TDDFT (terminal target only):

$$\nabla_u G[u] = \nabla_u F[\underline{\varphi}[u], u] + 2\text{Im} \left[\sum_{i=1}^N \int_0^T dt \langle \lambda_i[u](t) | \nabla_u \hat{H}[n[u](t), u, t] | \varphi_i[u](t) \rangle \right]$$

$$\underline{\dot{\varphi}}[u](t) = -i \underline{\hat{H}}[n(t), u, t] \underline{\varphi}[u](t),$$

$$\underline{\varphi}_u(0) = \underline{\varphi}_0,$$

$$\underline{\dot{\lambda}}[u](t) = -i \left[\underline{\hat{H}}[n(t), u, t] + \underline{\hat{K}}[\underline{\varphi}[u](t)] \right] \underline{\lambda}[u](t),$$

$$\underline{\lambda}[u](T) = \frac{\delta F}{\delta \underline{\varphi}^*} [\underline{\varphi}[u](T), u].$$

QOCT + TDDFT

Analysis and control of the electronic motion with TDDFT

Alberto Castro

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$$\dot{\underline{\lambda}}[u](t) = -i \left[\underline{\hat{H}}^\dagger [n[u](t), u, t] + \underline{\hat{K}}[\underline{\varphi}[u](t)] \right] \underline{\lambda}[u](t),$$

$$\dot{\lambda}_i[u](t) = -i \hat{H}^\dagger [n[u](t), u, t] \lambda_i[u](t) - i \sum_{j=1}^N \hat{K}_{ij}[\underline{\varphi}[u](t)] \lambda_j[u](t)$$

$$\langle \vec{r}' | \hat{K}_{ij}[\underline{\varphi}[u](t)] | \lambda_j[u](t) \rangle = -2i \varphi_i[u](\vec{r}, t) \text{Im} \left[\int d^3 r' \lambda_j[u]^*(\vec{r}', t) f_{\text{Hxc}}[n[u](t)](\vec{r}, \vec{r}') \varphi_j[u](\vec{r}', t) \right]$$

$$f_{\text{Hxc}}[n[u](t)](\vec{r}, \vec{r}') = \frac{1}{|\vec{r} - \vec{r}'|} + f_{\text{xc}}[n[u](t)](\vec{r}, \vec{r}')$$

QOCT + TDDFT

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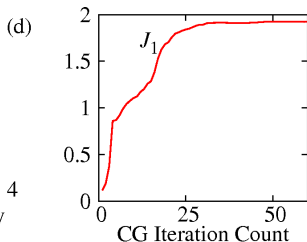
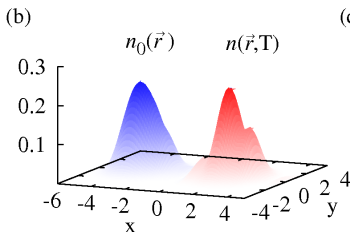
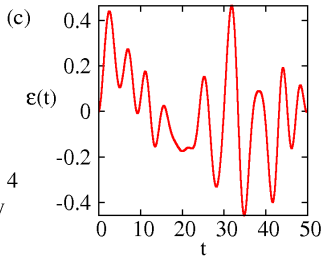
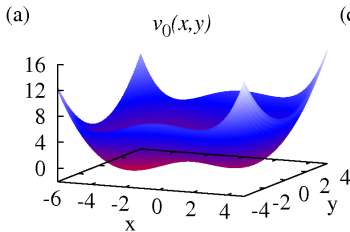
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Summary



QOCT + TDDFT

Most important theoretical difficulty: many targets are easily formulated in terms of states (or projectors), e.g.:

$$F[\Psi(T)] = |\langle \Psi_I | \Psi(T) \rangle|^2$$

However, those states are the true many-body states, and we only have access to the time-dependent density, and to the Kohn-Sham orbitals.

Femtosecond laser pulse shaping for enhanced ionization

[AC, E. Räsänen, A. Rubio, and E. K. U. Gross, EPL **87**, 53001 (2009)]

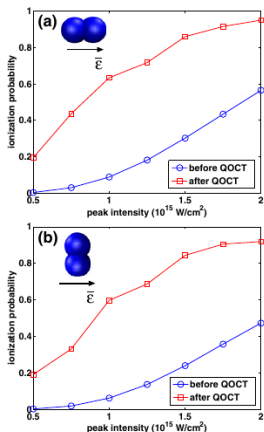


Fig. 1: (Color online) Ionization probability for the initial pulse (circles) and for the optimized pulse (squares) as a function of the peak intensity of the initial pulse. The polarization of the pulse is (a) parallel and (b) perpendicular to the molecule.

- ▶ Target: Maximal ionization of H₂⁺ molecule (clamped nuclei).
- ▶ $F[\Psi(T)] = \langle \Psi(T) | \Psi(T) \rangle - \sum_{\text{bound}} |\langle \Psi | \Psi_I \rangle|^2$
- ▶ Use of absorbing boundary conditions
- ▶ Use of *direct* optimization algorithm.
- ▶ Expansion of control field into a Fourier series \Rightarrow automatic existence of a frequency constraint.
- ▶ Further constraints: total length (5fs) and total fluence.

Femtosecond laser pulse shaping for enhanced ionization

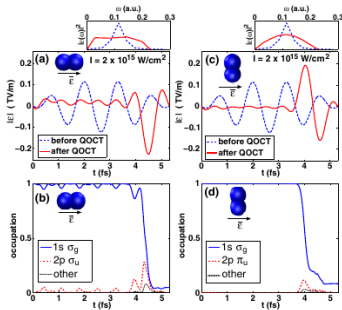


Fig. 2: (Color online) (a) Initial and optimized pulses (parallel polarization) and their power spectra (in arbitrary units) and (b) the occupation of selected single-electron states in the optimized ionization process, when $I = 2 \times 10^{15} \text{ W/cm}^2$. (c), (d) Same as (a), (b) but for perpendicular polarization.

- ▶ Using a stringent frequency cut-off, the optimization attempts to build a peak with maximum intensity. With short, intense pulses, most ionization occurs during the maximum.
- ▶ With parallel orientation, zero carrier envelope phase (half-cycle pulse), and $\pi/2$ with perpendicular orientation.

Femtosecond laser pulse shaping for enhanced ionization

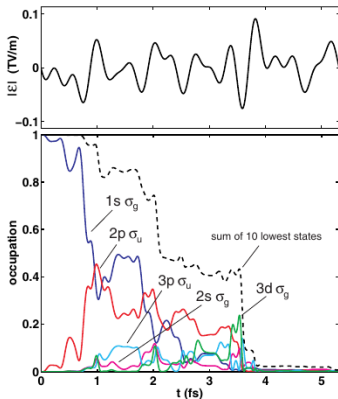


Fig. 4: (Color online) Upper panel: optimized laser pulse for the ionization when the cutoff frequency is $4\omega_0$ (see text) and the intensity is fixed to $0.5 \times 10^{15} \text{ W/cm}^2$. Lower panel: occupation of a few lowest states during the pulse interaction.

- ▶ Higher cut-off frequency implies more complicated structure for the optimal pulse.
- ▶ Ionization is not a direct ground-state to continuum step.

Optimal Control of Quantum Rings by Terahertz Laser Pulses

[E. Räsänen, AC, J. Werschnik, A. Rubio, and E. K. U. Gross, Phys. Rev. Lett. **98**, 157404 (2007)]

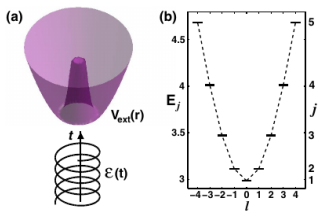


FIG. 1 (color online). (a) Shape of the external confining potential for a quantum ring and an example of a circularly polarized laser field. (b) Energy-level spectrum of a quantum ring. The transitions are allowed along the dashed line so that $\Delta l = \pm 1$.

- ▶ Electron trapped in a ring edged into a 2D semiconductor heterostructure (2D electron gas).
- ▶ Levels are coupled in a consecutive fashion, ordered by angular momentum.
- ▶ Use of a two-component laser pulse.
- ▶ The target is the population of any of the levels, from any of the other levels (precise control over the electronic current).

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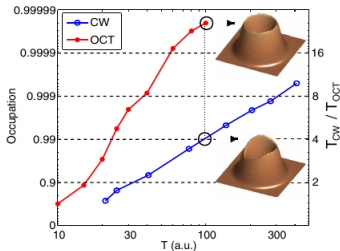


FIG. 3 (color online). Maximum occupation of the target state in transition $|1\rangle \rightarrow |2\rangle$ as a function of the pulse length. The open (blue) circles correspond to continuous waves and the filled (red) circles to the optimal-control result. The insets show the densities $|\Psi(T=100)|^2$ when the corresponding achieved occupations are 0.99 and 0.99998 for these pulse types, respectively.

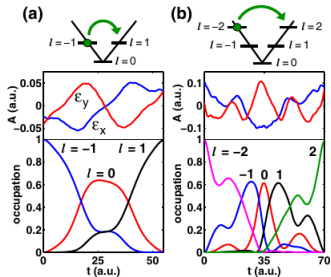
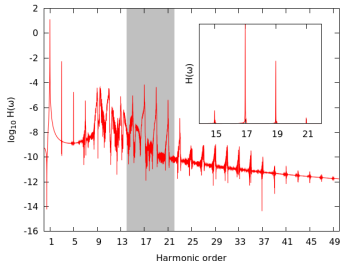


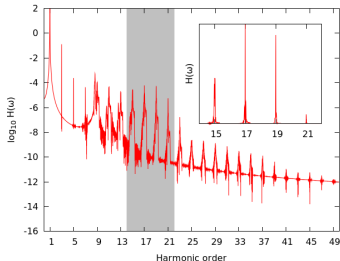
FIG. 4 (color online). Schematic picture of transitions from $l = -1$ to $l = 1$ (a) and from $l = -2$ to $l = 2$ (b) (upper panel), optimized fields for these transitions (middle panel), and the occupations of the states (lower panel).

Optimal laser control of the harmonic generation

He atom, EXX:



He atom, EXX, frozen H+xc:



- ▶ Target: selective enhancement or quenching of harmonics:

$$F[\varphi] = \sum_k \alpha_k \max_{\omega \approx k\omega_0} \{ \log_{10} H[\varphi](\omega) \}$$

$$H(\omega) = \left| \int_0^T dt \frac{d^2}{dt^2} \langle \hat{\mu} \rangle(t) e^{-i\omega t} \right|^2$$

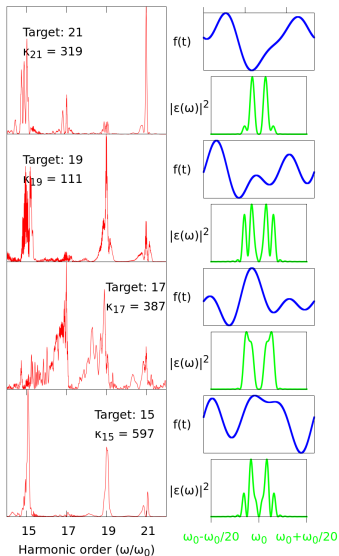
- ▶ Time-dependent target, it depends on the full evolution of the system.
- ▶ “TDDFT-friendly” target: it only depends on the time-dependent density.

Optimal laser control of the harmonic generation

He atom, EXX:

Harmonic order (ω/ω_0)

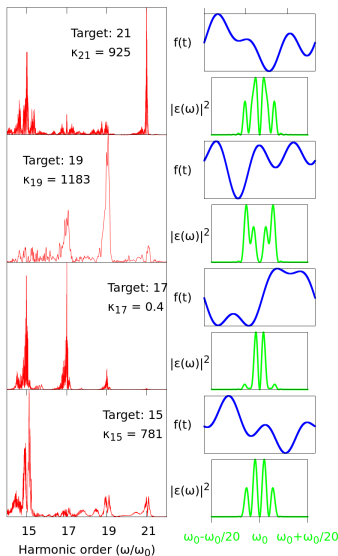
15 17 19 21



He atom, froze H+xc

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Optimal laser control of the harmonic generation: Cut-off extension

$$\begin{aligned} F[\varphi] &= \int d\omega \alpha(\omega) H[\varphi](\omega) \\ &= \int d\omega \alpha(\omega) |\vec{f}[\varphi](\omega)|^2, \end{aligned}$$

$$\vec{f}(t) = \int d^3r n(\vec{r}, t) \nabla v(\vec{r}) + N \varepsilon(t) \vec{\pi}.$$

The functional derivative of F , needed for the propagation of the co-state, is:

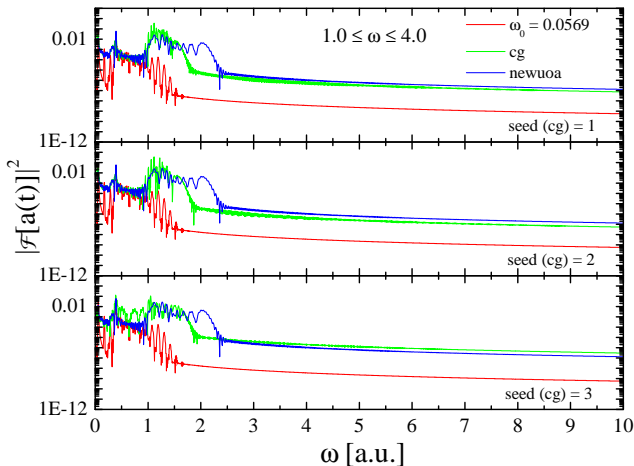
$$\frac{\delta F}{\delta \varphi^*(\vec{r}, t)} = \vec{g}[\varphi](t) \cdot \nabla v(\vec{r}) \varphi(\vec{r}, t),$$

where

$$\vec{g}[\varphi](t) = 2\mu \int d\omega \alpha(\omega) \operatorname{Re} \left[\vec{f}[\varphi](\omega) e^{-i\omega t} \right].$$

Optimal laser control of the harmonic generation: Cut-off extension

$$\alpha(\omega) = \text{step}(\omega - \omega_c)$$



Optimization Schemes for Selective Molecular Cleavage with Tailored Ultrashort Laser Pulses

Coupled electron-ion model: Ehrenfest dynamics:

$$\hat{H}[q, p, u, t] = H_{\text{clas}}[q, p, u, t]\hat{I} + \hat{H}_{\text{quantum}}[q, p, u, t].$$

$$\dot{q}_a(t) = \frac{\partial H_{\text{clas}}}{\partial p_a}[q(t), p(t), u, t] + \langle \Psi(t) | \frac{\partial \hat{H}_{\text{quantum}}}{\partial p_a}[q(t), p(t), u, t] | \Psi(t) \rangle$$

$$\dot{p}_a(t) = -\frac{\partial \hat{H}_{\text{clas}}}{\partial q_a}[q(t), p(t), u, t] - \langle \Psi(t) | \frac{\partial \hat{H}_{\text{quantum}}}{\partial q_a}[q(t), p(t), u, t] | \Psi(t) \rangle$$

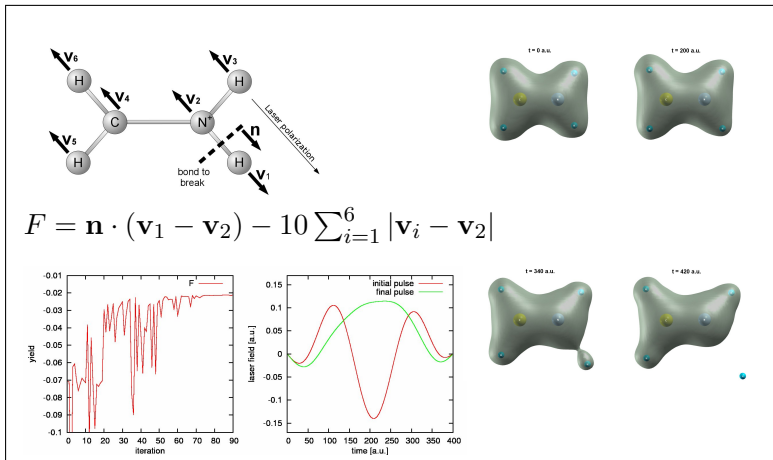
$$\dot{\Psi}(x, t) = -i\hat{H}_{\text{quantum}}[q(t), p(t), u, t]\Psi(x, t),$$

Optimization Schemes for Selective Molecular Cleavage with Tailored Ultrashort Laser Pulses

Several options:

- ▶ Assume the dynamical system is the electronic system only: the ions are clamped during the action of the laser pulse. One can then use the usual QOCT. Valid only for very short pulses.
- ▶ The dynamical system is the Ehrenfest system:
 - ▶ Use a *direct*, or *gradient-less* optimization scheme. No need to develop new theory, since the only operation required is the forward propagation of the Ehrenfest system.
 - ▶ Full use of the QOCT equations, that require the gradient.

Optimization Schemes for Selective Molecular Cleavage with Tailored Ultrashort Laser Pulses



[K. Krieger, AC, and E. K. U. Gross, Chem. Phys. **391**, 50 (2011)]

Ultrafast photo-dissociation of the Hydrogen molecule

- ▶ During the laser irradiation, the nuclei do not have time to move, but the electrons are shaken, and communicate a sudden dissociative momentum to the nuclei.
- ▶ Hybrid quantum-classical model (Ehrenfest model): definition of the targets in terms of the classical forces on the nuclei:

$$\vec{P}_\alpha = \int_0^T dt \vec{F}_\alpha(t),$$

$$\vec{F}_\alpha = \vec{F}_\alpha[n(t)] = - \int d^3r n(\vec{r}, t) \vec{\nabla}_\alpha v_0(\vec{r}, \{\vec{R}_\alpha\}).$$

- ▶ Therefore

$$\vec{P}_\alpha[n] = \int_0^T dt \int d^3r n(\vec{r}, t) \vec{\nabla}_\alpha v_0(\vec{r}, \{\vec{R}_\alpha\}).$$

Definition of the target

- ▶ The target is then defined in terms of a function of these momenta:

$$F[\underline{\varphi}] = \tilde{F}[n] = T(P[n]),$$

for example:

$$\tilde{F}[n] = P_{1x}[n] - P_{2x}[n].$$

- ▶ The remaining ingredient in order to compute the gradient is the functional derivative:

$$\frac{\delta F[\underline{\varphi}]}{\delta \varphi_i^*(\vec{r}, t)} = \sum_{\alpha=1}^K \sum_{\mu=x,y,z} 2 \frac{\partial T}{\partial P_{\alpha\mu}}(p[n]) \frac{\partial v_0(\vec{r}; \vec{R})}{\partial R_{\alpha\mu}} \varphi_i(\vec{r}, t).$$

Results

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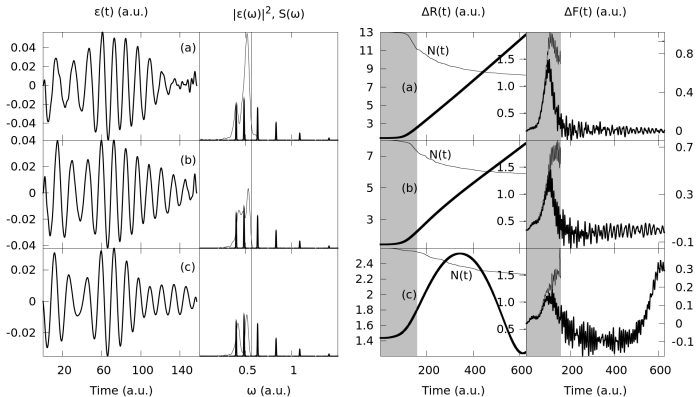
Optimal Control Theory

Optimal control theory

Control of quantum processes

TDDFT + QOCT

Summary



[AC, ChemPhysChem **14**, 1488 (2013)]

QOCT+TDDFT+Ehrenfest

[AC and E. K. U. Gross, "Optimal control theory for quantum-classical systems: Ehrenfest Molecular Dynamics based on TDDFT"

arXiv:1308.4162]

$$\dot{q}_a(t) = \frac{\partial H_{\text{clas}}}{\partial p_a}[q(t), p(t), u, t] + \langle \Psi(t) | \frac{\partial \hat{H}_{\text{quantum}}}{\partial p_a}[q(t), p(t), u, t] | \Psi(t) \rangle$$

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Outline

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- ▶ QOCT and TDDFT can be successfully merged in a numerically tractable way. The targets must be formulated in terms of the density, in order to be consistent with TDDFT. This is not always possible.
- ▶ Very fast photo-dissociation can be tackled in a *static* manner: by optimizing the electron system only, and placing the system in a state that leads to photo-dissociation after the pulse.
- ▶ However, a general treatment would require the construction of a QOCT+TDDFT+Ehrenfest methodology.

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Laser control

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Optimal Control
Theory

Optimal control
theory
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quantum
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TDDFT +
QOCT

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Rederivation in the language of linear response

(A) Generalized, out of equilibrium linear response:

$$\hat{H}(T) = \hat{H}_0(t) + f(t)\hat{V}$$

$$i\frac{d}{dt}\hat{\rho}(t) = [\hat{H}(t), \hat{\rho}(t)]$$

$$A(t) = \text{Tr}\hat{A}\hat{\rho}(t)$$

$$\delta A(t) = \int_0^T d\tau f(\tau)\chi_{\hat{A},\hat{V}}(t,\tau)$$

$$\chi_{\hat{A},\hat{V}}(t,\tau) = -i\theta(t-\tau)\text{Tr}\{\hat{\rho}(t_0) [\hat{A}_H(t), \hat{V}_H(\tau)]\}$$

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Rederivation in the language of linear response

(A) Corollarium: QOCT equations

$$\hat{H}[u_1, \dots, u_m + \delta u_m, \dots; t] = \hat{H}[u; t] + \delta u \frac{\partial \hat{H}}{\partial u_m}[u; t] \\ \hat{H}_0(t) + f(t)\hat{V}$$

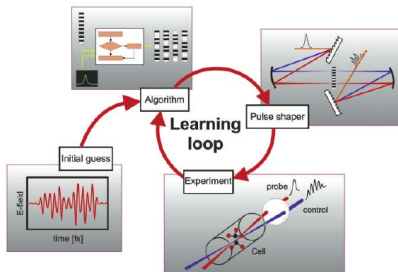
Assuming $\hat{H}[u] = \hat{\mathcal{H}} + \varepsilon(u, t)$,

$$\frac{\partial G}{\partial u_m} = \frac{\partial J_2}{\partial u_m} + \int_0^T dt \frac{\partial \varepsilon}{\partial u_m} \chi_{\hat{A}, \hat{V}}(T, t)$$

[AC and I. Tokatly, Phys. Rev. A **84**, 033410 (2011)]

Laser control

Adaptive feedback control:



“Whither the future of controlling quantum phenomena?”, Rabitz *et al*, *Science* **288**, 824 (2000).

Laser control

Analysis and control of the electronic motion with TDDFT

Alberto Castro

Introduction: Atto-second physics: electrons in real time

TDDFT for the analysis of ultrafast pump-probe experiments

Optimal Control Theory

Optimal control theory

Control of quantum processes

TDDFT + QOCT

Summary

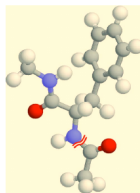
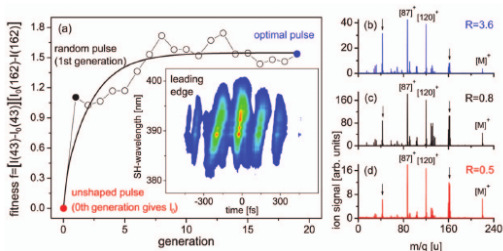


FIG. 1. (Color online) Optimal tailoring of intense femtosecond light can be used to preferentially break peptide bonds, such as the indicated N1-C3 bond in the amino acid complex Ac-Phe-NHMe.



“Coherent control of bond breaking in amino acid complexes with tailored femtosecond pulses”, Laarmann *et al*, *J. Chem. Phys.* **127**, 201101 (2007).