Bayesian Inference in Machine Learning: an example

Wang Weiqi

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Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayesian Inference and Prediction

$$p(\theta|X,\alpha) = \frac{p(X|\theta,\alpha)p(\theta|\alpha)}{p(X|\alpha)} \propto p(X|\theta,\alpha)p(\theta|\alpha)$$

$p(\theta\|\alpha)$	prior distribution
$p(X\|\alpha)$	marginal likelihood
$p(X\|\theta,\alpha)$	likelihood
$p(\theta\|X,\alpha)$	posterior distribution

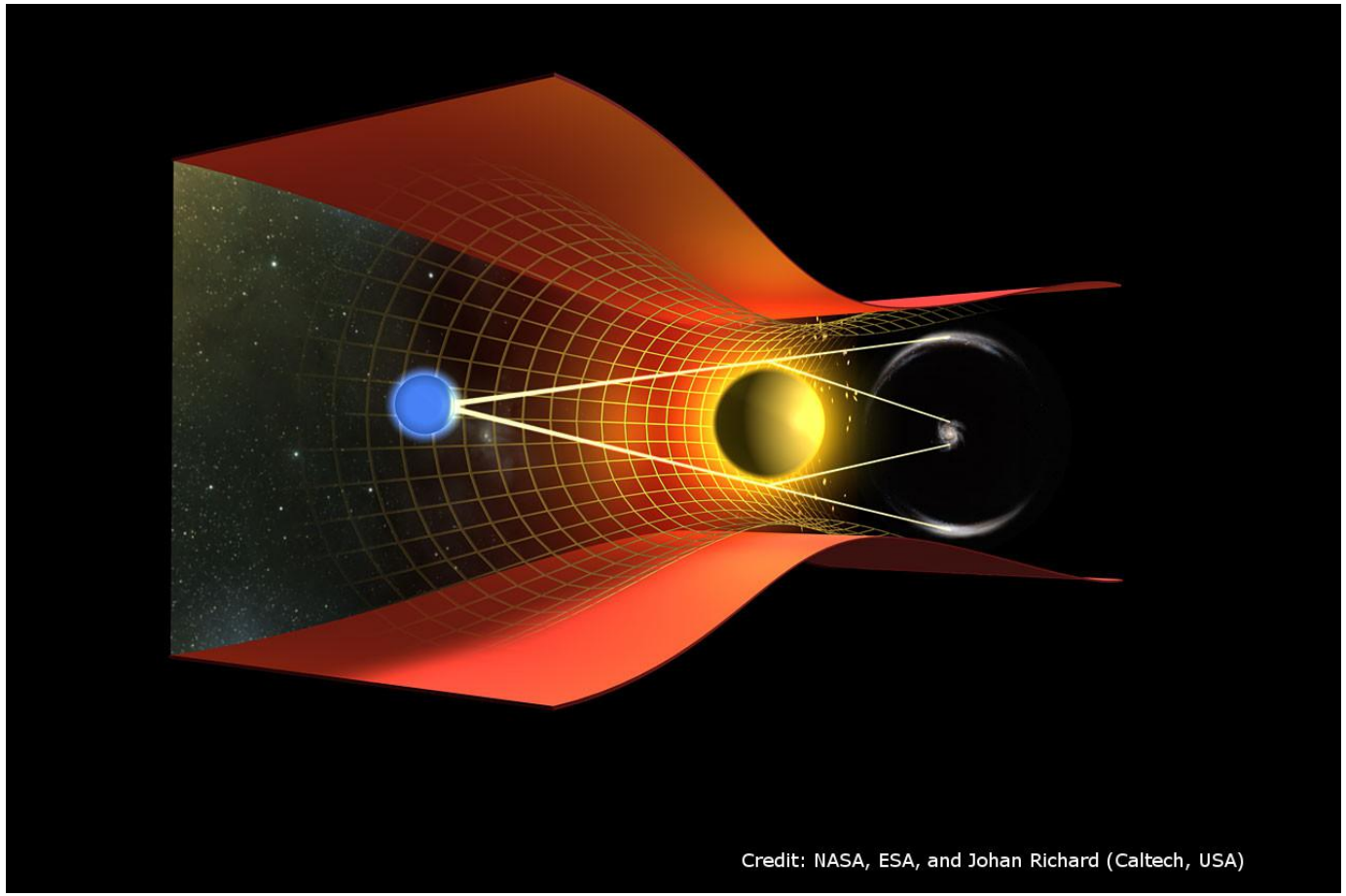
posterior predictive distribution:

$$p(\tilde{x}|X,\alpha) = \int p(\tilde{x}|\theta)p(\theta|X,\alpha)d\theta$$

prior predictive distribution:

$$p(\tilde{x}|\alpha) = \int p(\tilde{x}|\theta)p(\theta|\alpha)d\theta$$

Example: Observing Dark World



https://www.kaggle.com/c/DarkWorlds

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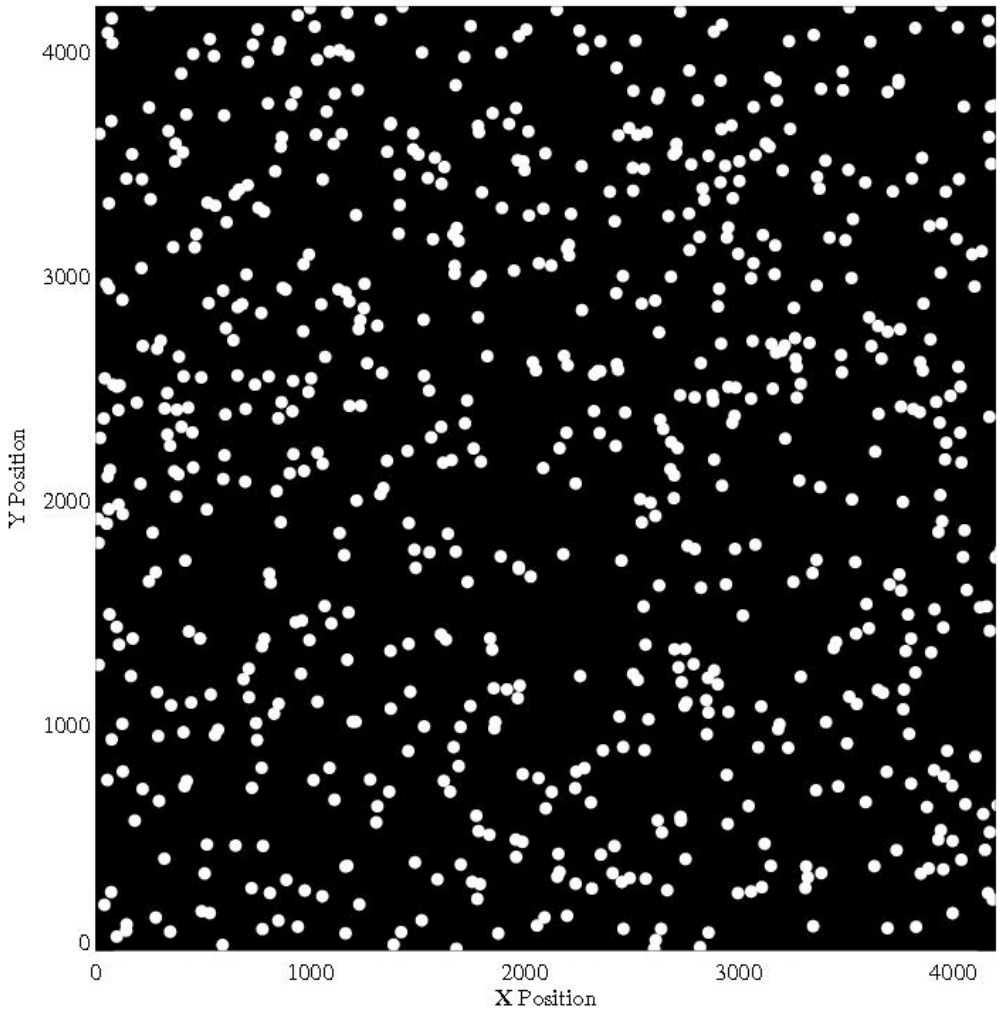

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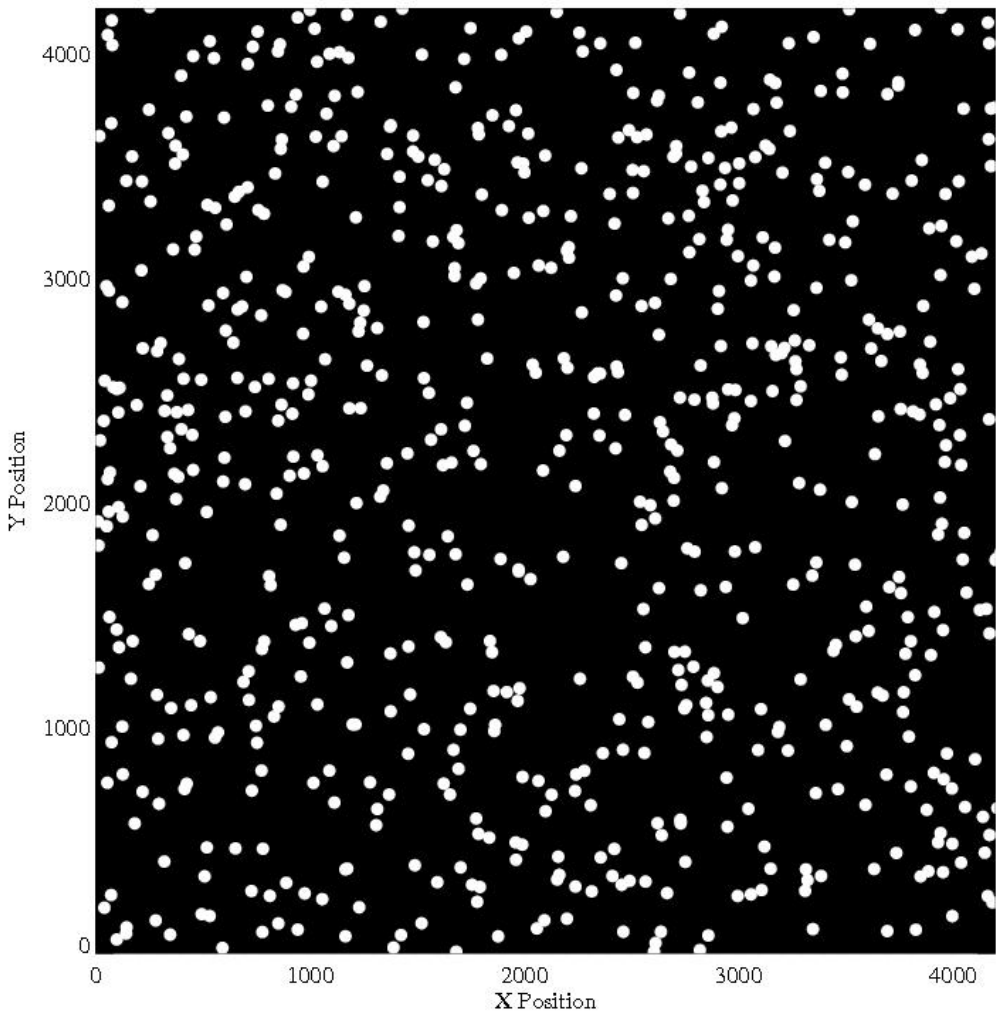

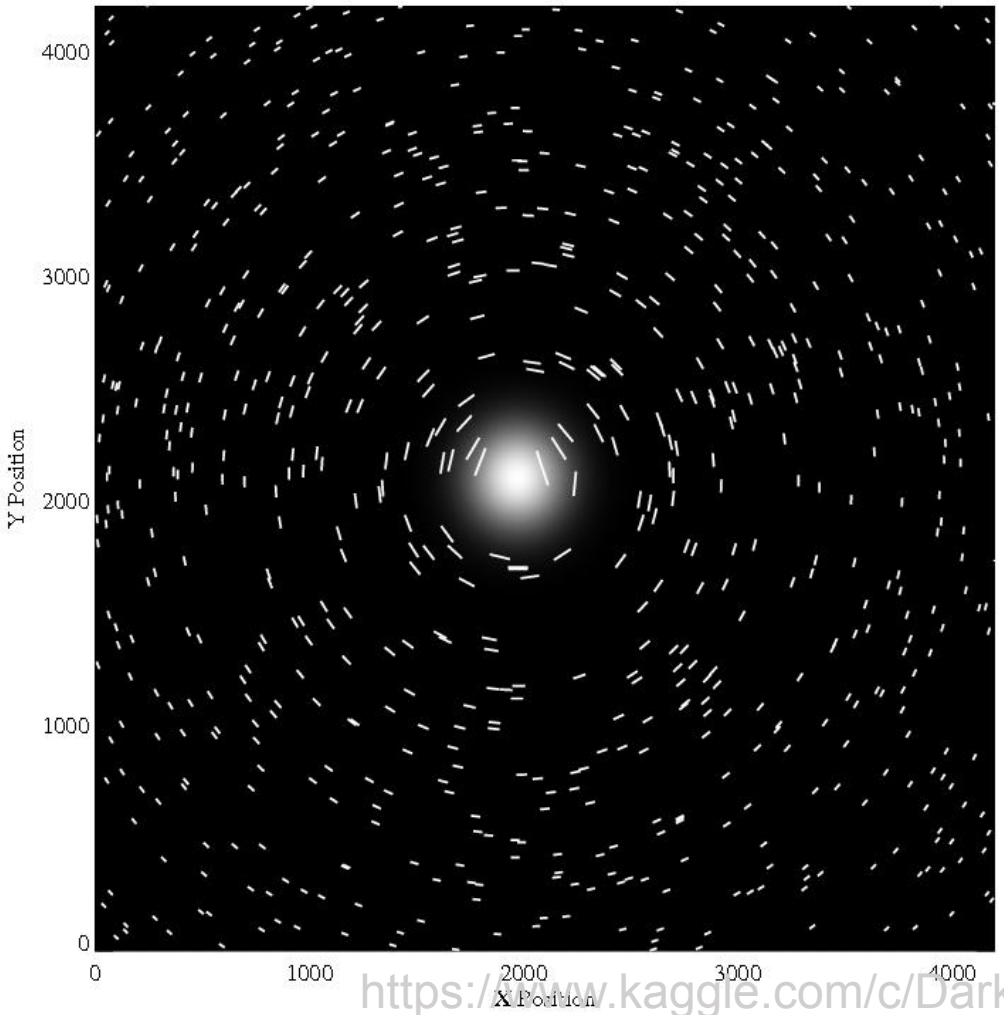

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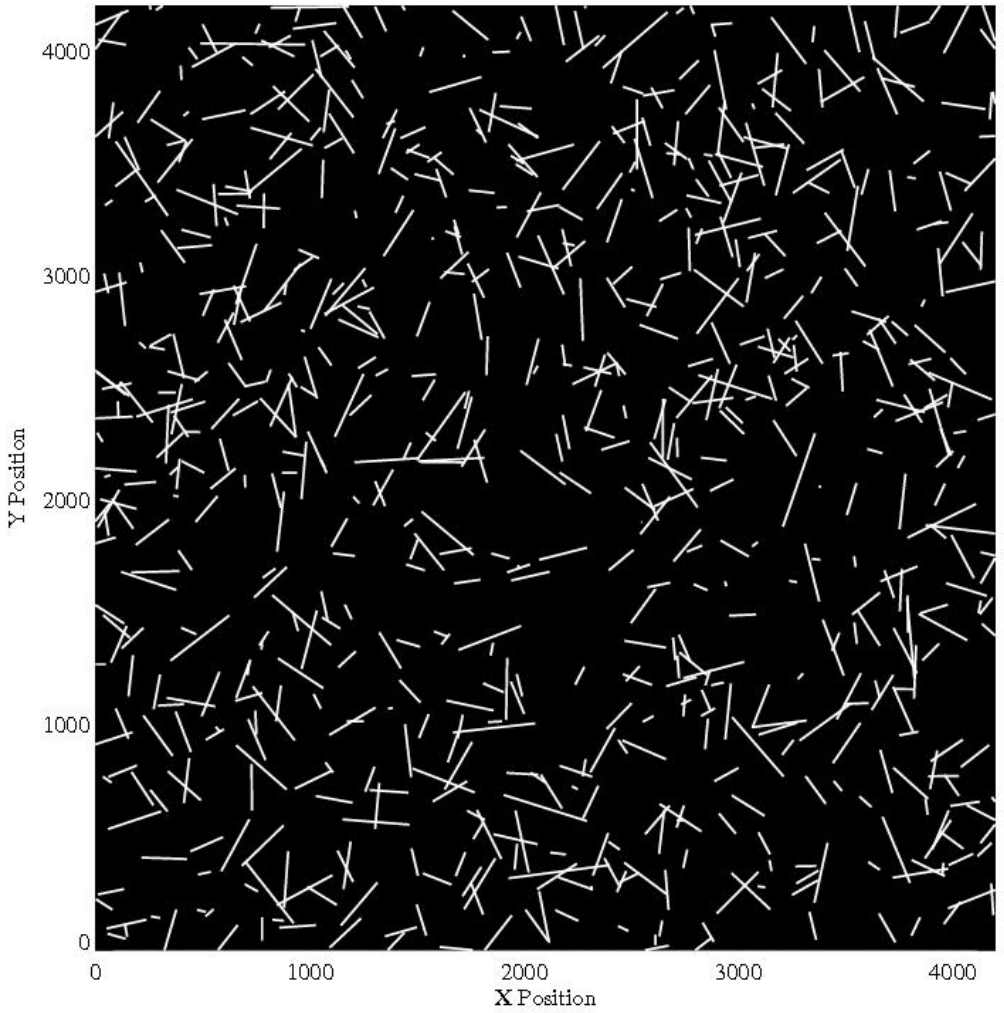

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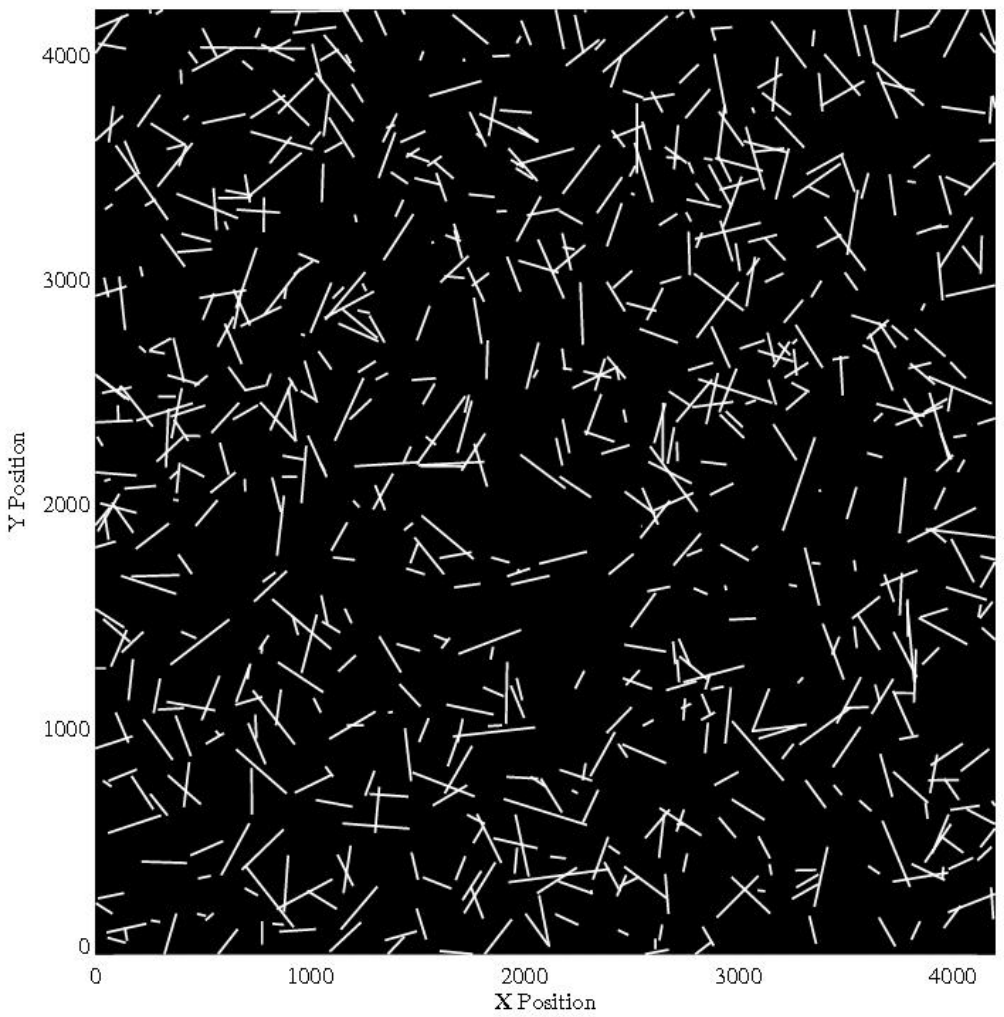

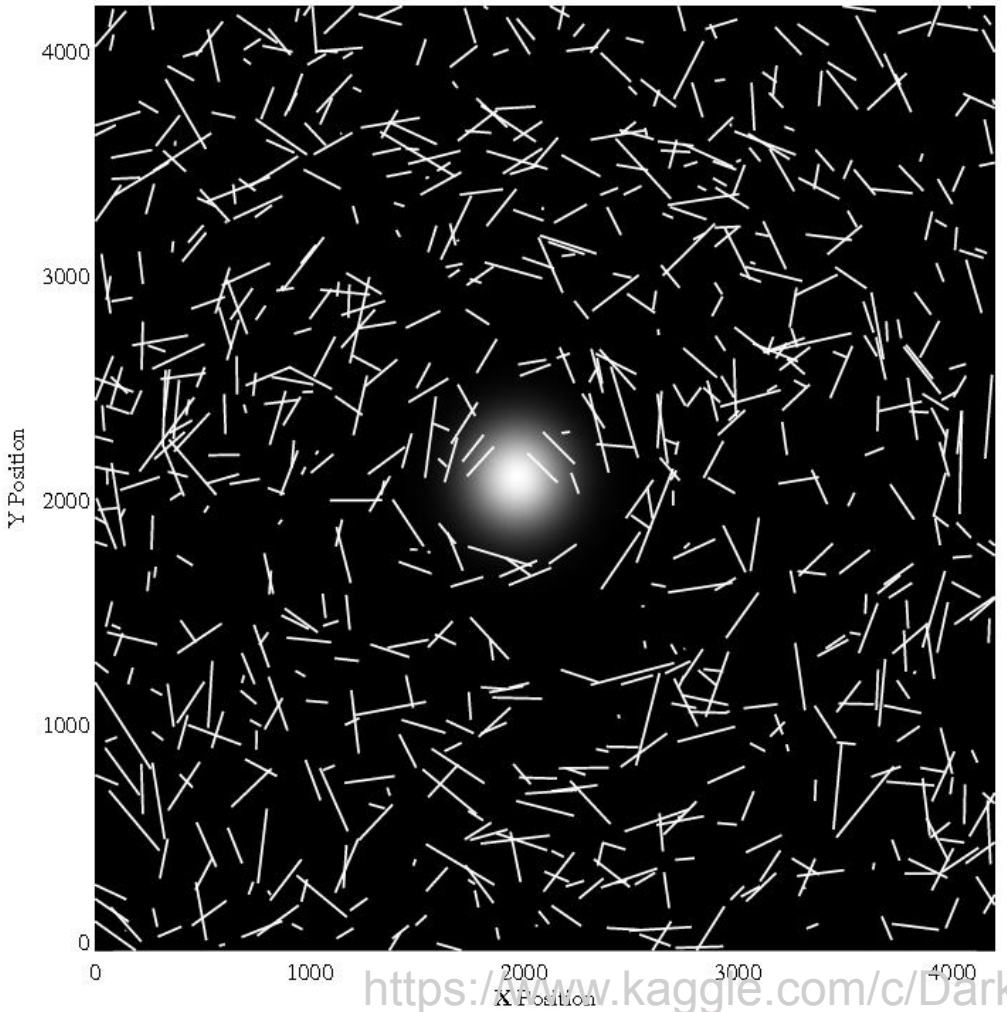

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Winner's Strategy

1. Construct a prior distribution for the halo positions $p(x)$

2. Construct a probabilistic model for the data $p(e|x)$

3. Use Bayes' rule to get the posterior distribution of the halo positions

4. Minimize the expected loss with respect to the posterior distribution

Prior distribution

Prior distribution of halo positions was uniform:

$$x_i \sim Uniform(0, 4200)$$

$$y_i \sim Uniform(0, 4200) \quad i = 1, 2, 3$$

Model

Mass distribution:

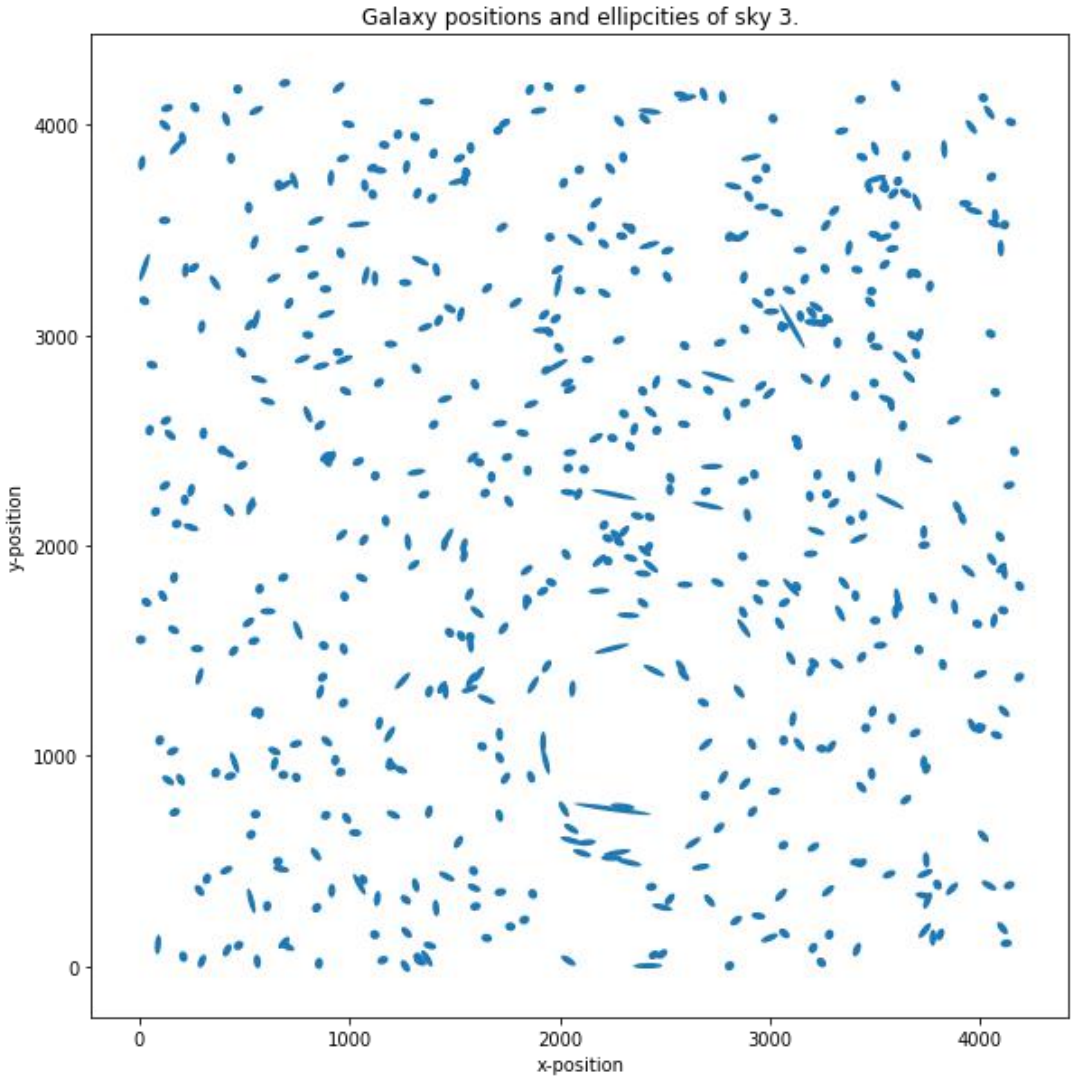
$$m_{large} = logUniform(40, 180)$$

The relationship to connect ellipticity and positions:

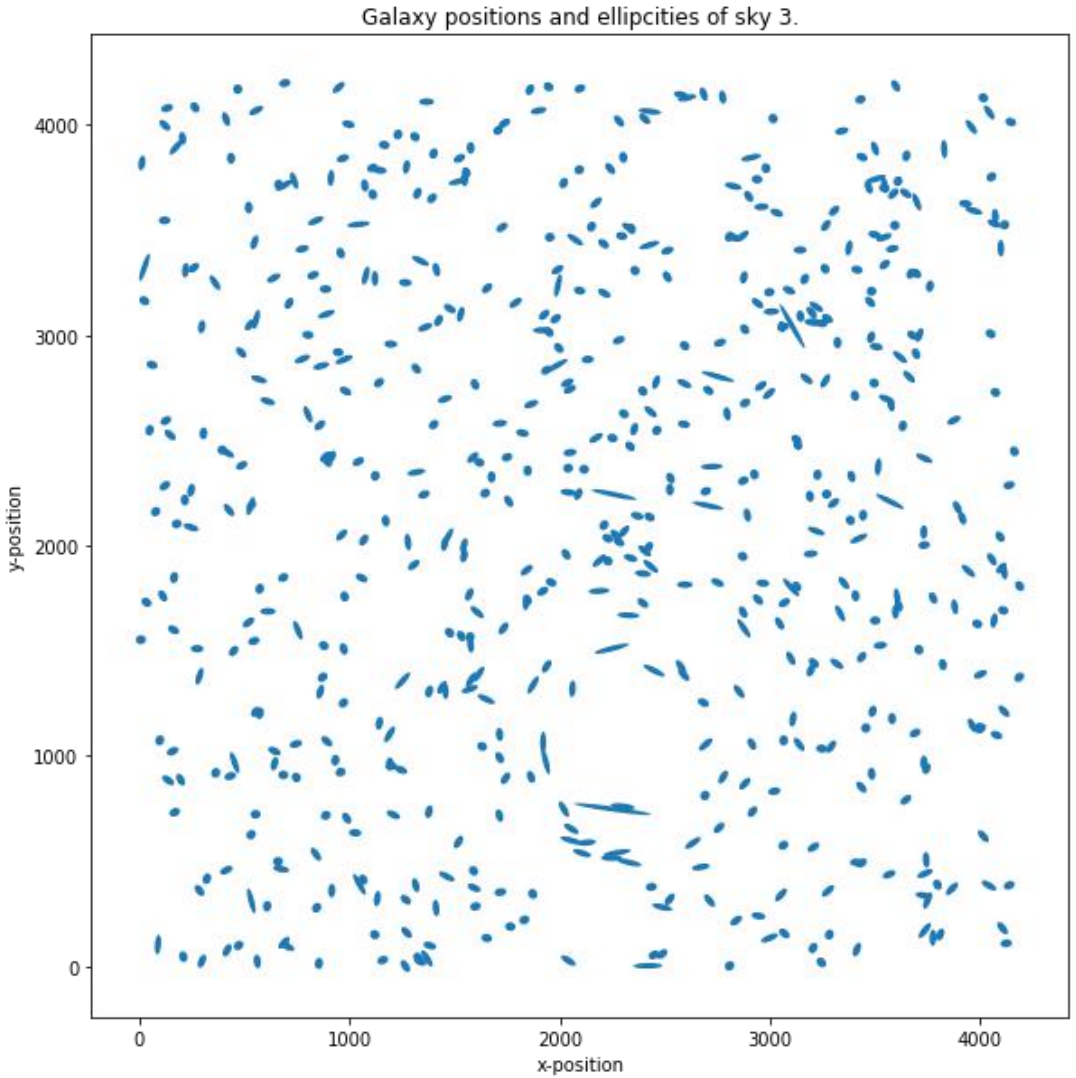
$$e_i|(x, y) \sim Normal(\sum_{j=halo\ positions} d_{i,j} m_j f(r_{i,j}), \sigma^2)$$

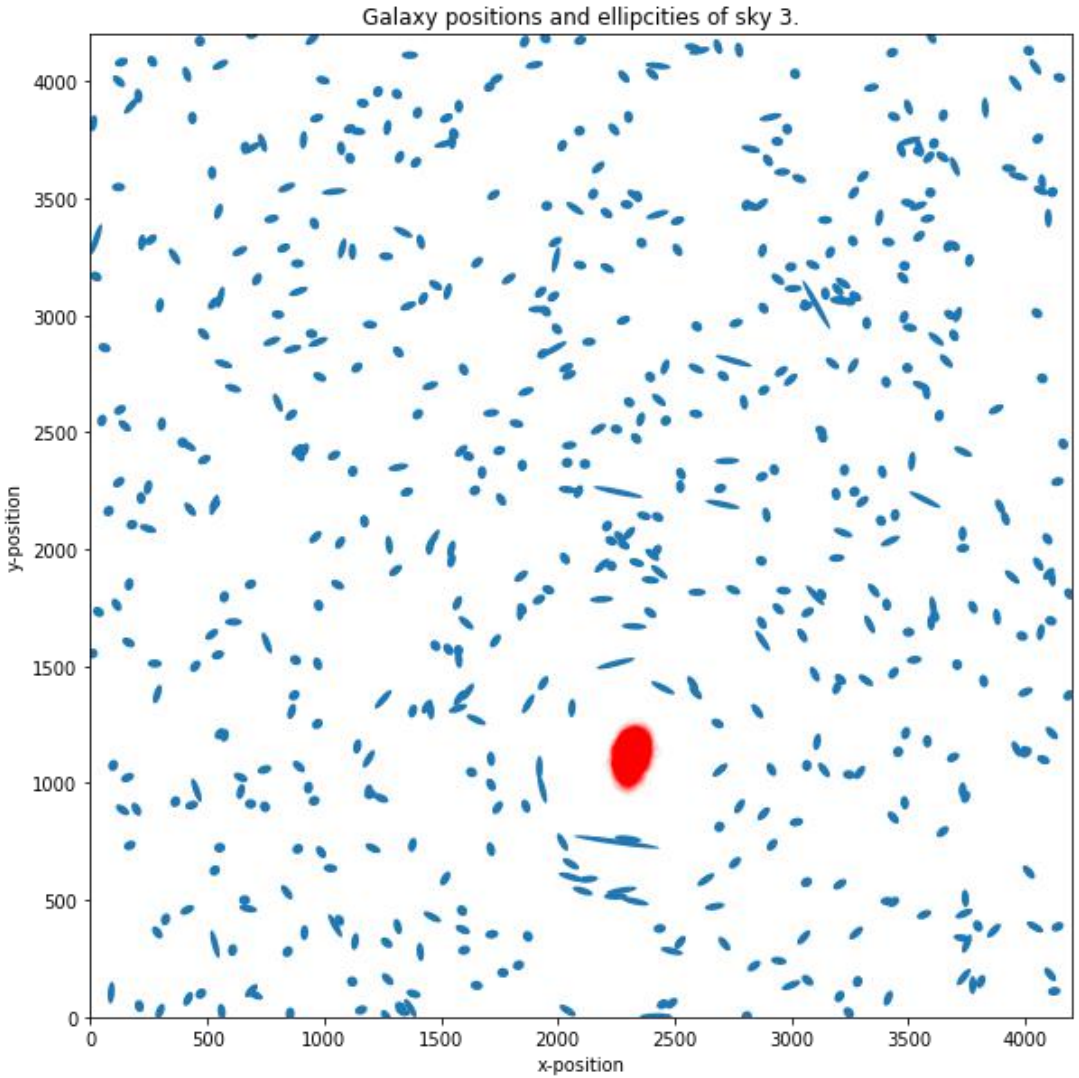
$$f(r_{i,j}) = \frac{1}{min(r_{i,j}, 240)} \qquad f(r_{i,j}) = \frac{1}{min(r_{i,j}, 70)}$$

Predict one dark matter halo



Predict one dark matter halo





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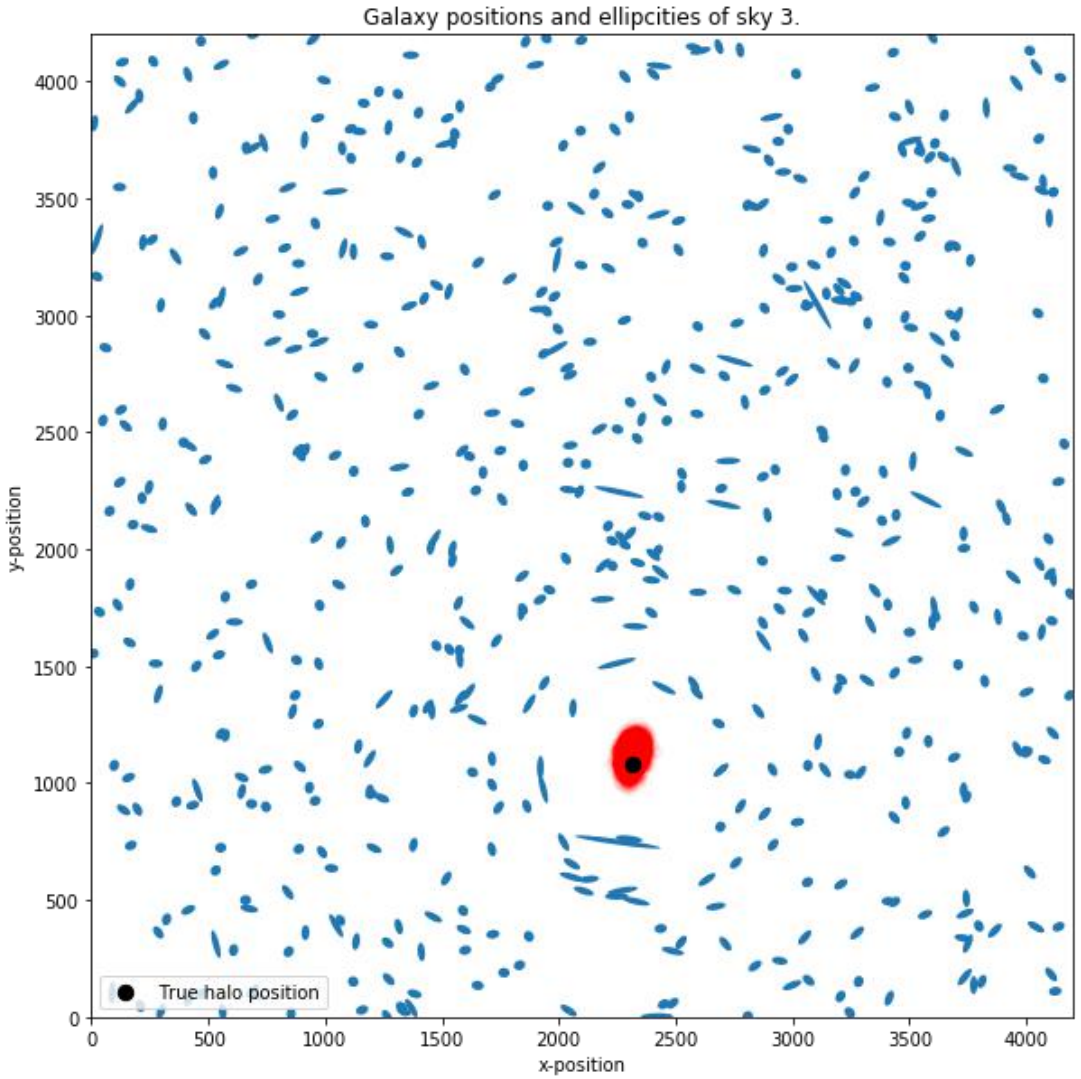


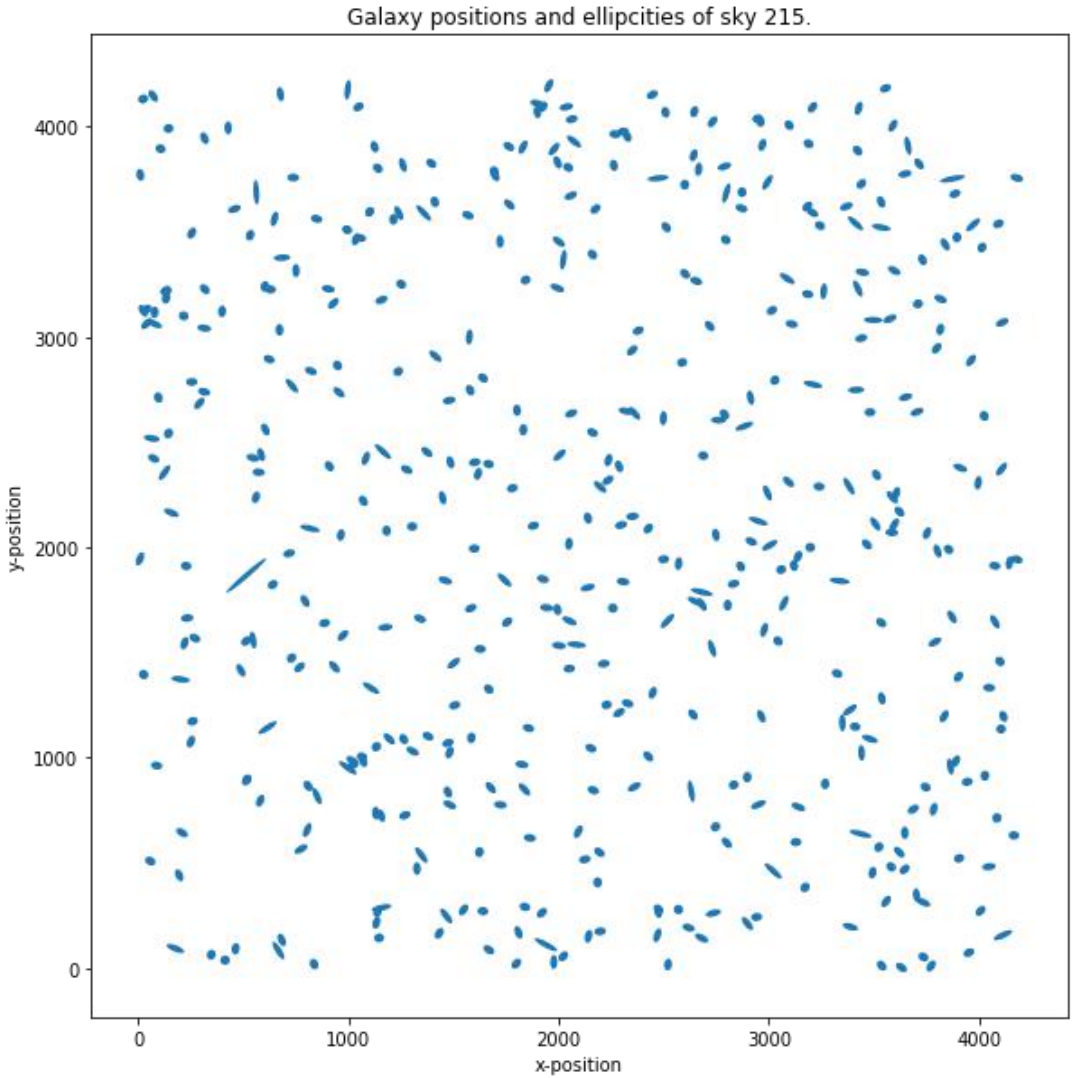
Distance between prediction and true position of halo is 44.90

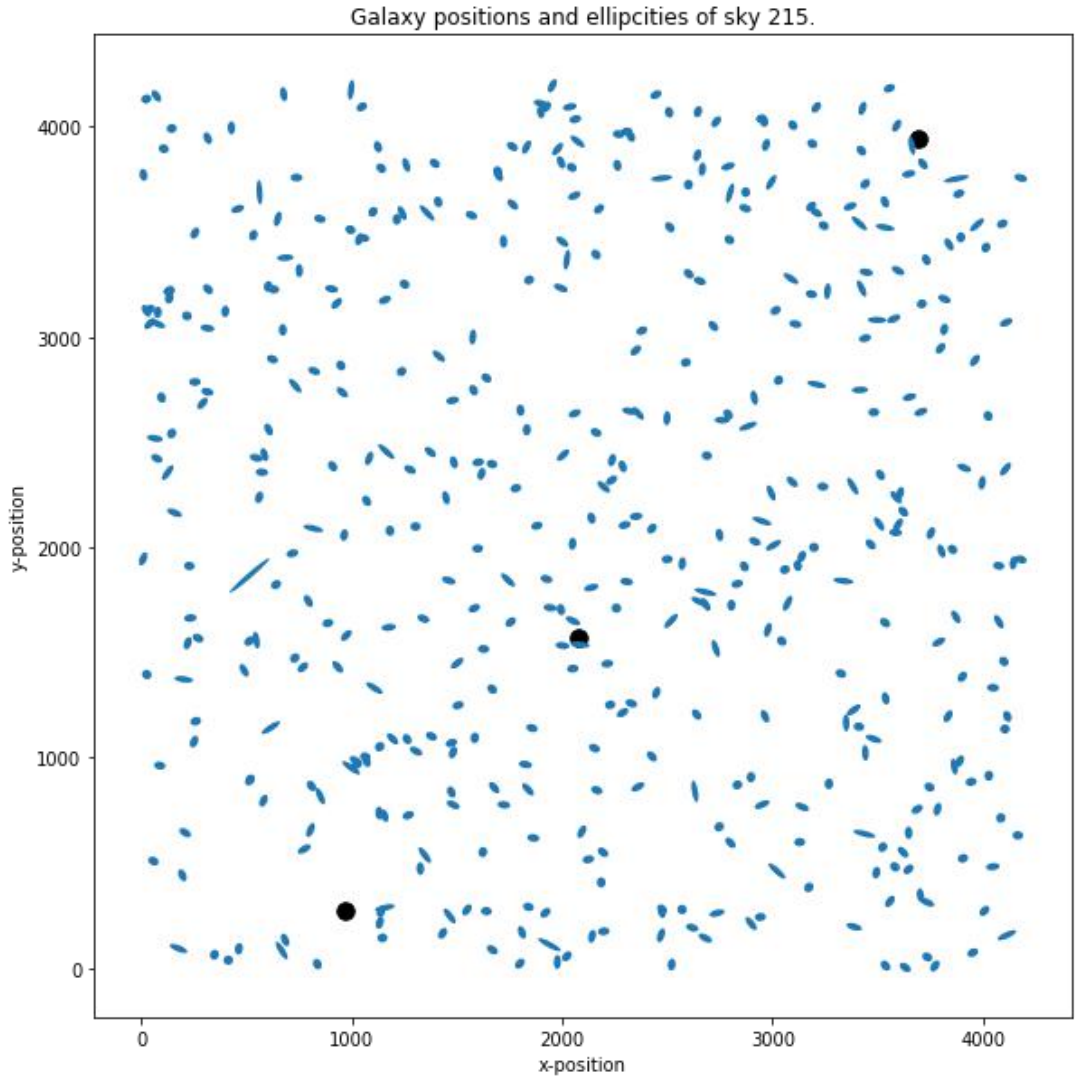
Loss :
1.04

Using a random location: [[2400 3850]]
Distance between prediction and true position of halo is 2769.33

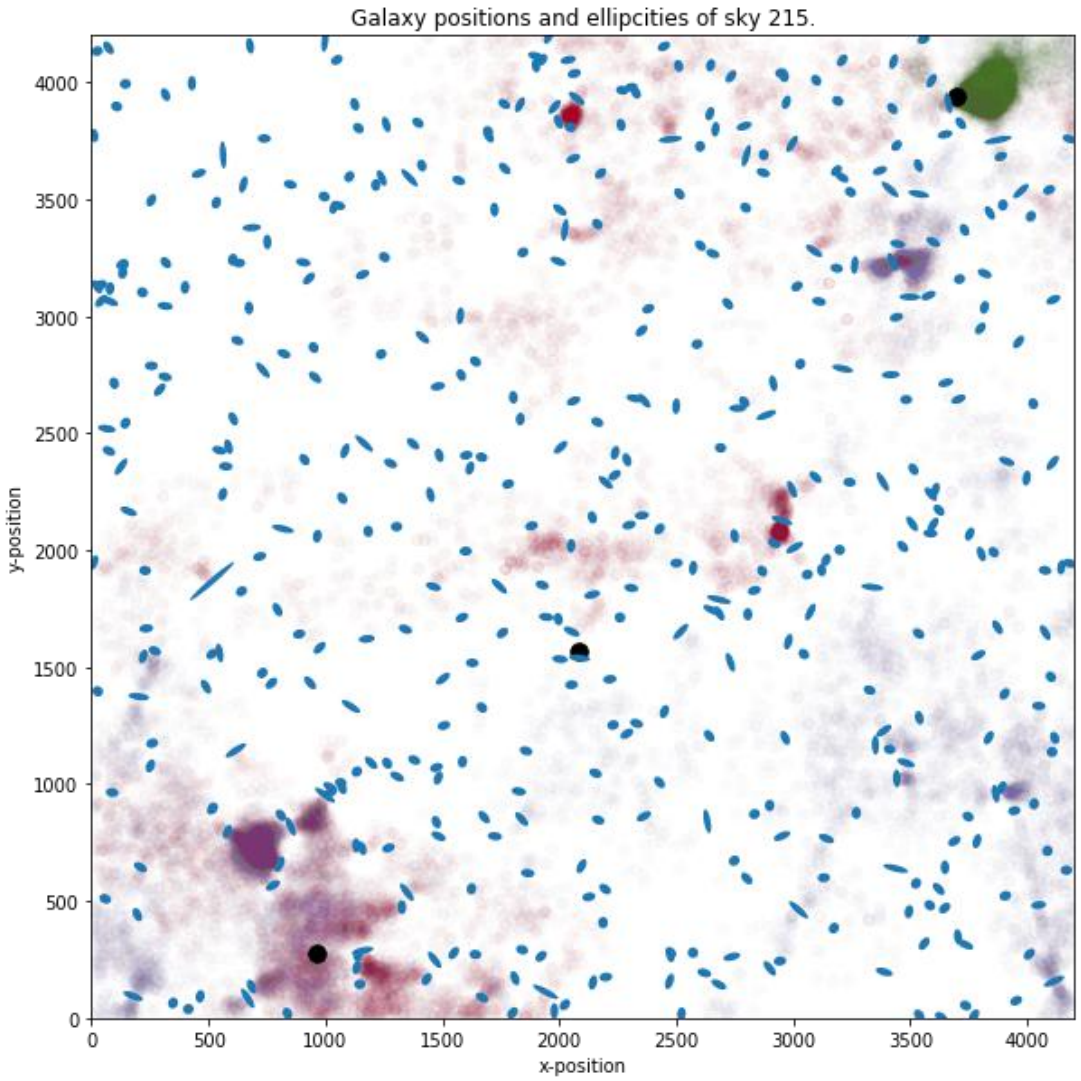
Loss :
3.77

Predict three dark matter halos





Predict three dark matter halos



Distance between prediction and true position of halo is 135.36

Loss :
1.14

Using a random location: [[1775 244]]
Distance between prediction and true position of halo is 4167.42

Loss :
 5.17

Thanks